

Full Length Article

An irreversibility analysis of ternary hybrid nanofluid flow with thermal radiation and porous medium: A sensitivity analysis approach

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ABSTRACT

The integration of three nanoparticles in blood has considerable possibilities for improving thermal transfer and biomedical engineering applications. The present study offers novel insights on the impact of dissipative force and radiation on bio-transport effects over a permeable wedge-shaped artery with ternary nanoparticles. Inclusion of entropy generation calculates the thermodynamic irreversible nature, allowing for the measurement of energy efficiency in the model. The mathematical model is governed by partial differential equations and is converted to ordinary differential equations through suitable similarity transformation. Further, the reduced model is solved numerically via the Runge-Kutta-Fehlberg-45th method for better computational accuracy and stability. The obtained numerical solution is discussed graphically. The results show that fluid velocity decreases for growing values of Casson and porosity parameters. The thermal profile improves as the values of radiation and the Eckert number increase. The entropy generation number exhibits a decreasing trend for both Casson and radiation. The heat transmission rate across the boundary is found to be increasing under the influence of larger values of radiation and the Eckert number. Quantitative analysis demonstrated that augmented values of radiation lower entropy by 28.5% and preserve heat transfer loss at 1.7%. Sensitivity analysis revealed that the radiation parameter is the most influencing factor over both heat transfer and entropy generation. The results from the current research have significant potential in improving the practical results in blood-based biomedical applications.

1. Introduction

The fluid flow with nanoparticles exhibits better thermophysical attributes when compared to traditional fluid flows. This led to the motivation of many researchers to concentrate on the significance of nanofluid flows in industrial and medical applications. Das and Duari [1] explored the MHD ternary nanofluid flow across a wedge under the influence of the shape factor and radiation. The results reveal that the inclusion of ternary nanofluid has outperformed compared with nanofluids. Jubair et al. [2] examined the effects of heat source and activation energy on chemically reactive non-Newtonian ternary nanofluid flow past a stretched sheet under the consideration of the Darcy–Forchheimer model. Obalalu et al. [3] investigated the influence of the Hamilton and Crosser model on ternary nanofluid flow across a chemically reactive

Riga wedge in the presence of radiation. Results show that use of these models is better for understanding ternary nanofluid flow. Ouyang et al. [4] studied the consequences of dissipation and Joule heating on fluid flow of ternary nanofluid across a shrinking/expanding wedge. In his study, Gangadhar et al. [5] investigated how radiation and magnetic field affected the ternary nanofluid flow over a moving wedge. The study demonstrates the importance of ternary nanoparticles in drag force compared to hybrid nanoparticles.

In wedge-shaped arteries, the concept of a porous medium is explored to study the blood's correlation with neighbouring tissues and porous artery walls and to understand the resistance, shear stress, and energy dissipation in fluid flow through these arteries. Shafiq et al. [6] studied the artificial neural network of nanofluid flow across a permeable wedge in the presence of thermophoresis and Brownian motion. The findings suggest that skin friction rises due to the rise in the porosity

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Nomenclature			
A_1, A_2, A_3, A_4	Constants	β	Pressure gradient parameter
Br	Brinkman number	γ	Wedge angle parameter (rad)
C_p	Specific heat ($Jkg^{-1}K^{-1}$)	η	Similarity variable
Da	Darcy number	λ	Porosity parameter
k	Thermal conductivity ($Wm^{-1}K^{-1}$)	λ^*	permeability factor (m^2)
l	Characteristic length (m)	μ	Dynamic viscosity ($kgm^{-1}s^{-1}$)
N_G	Volumetric entropy generation ($Wm^{-3}K^{-1}$)	ν	Kinematic viscosity (m^2s^{-1})
N_O	Characteristic entropy generation ($Wm^{-3}K^{-1}$)	ρ	Density (kgm^{-3})
Nr	Radiation parameter	σ	Stefan-Boltzmann constant ($kg^{-3}K^{-4}$)
Nu_x	Nusselt number	Ω	Temperature ratio parameter
Pr	Prandtl number	ϕ_1, ϕ_2, ϕ_3	Nanoparticle volume fractions
q_w	Wall heat flux (Wm^{-2})	Subscripts	
Re_x	Local Reynolds number	f	Base fluid
S_G	Entropy generation number	tnf	Ternary nanofluid
T	Fluid temperature (K)	s_1, s_2, s_3	Nanoparticles
T_w	Wall temperature (K)	w	Wall condition
T_∞	Ambient temperature (K)	∞	Free stream condition
u, v	Velocity components (ms^{-1})	Abbreviation	
U_∞	Free stream function	PDE	Partial Differential Equation
x, y	Cartesian coordinates (m)	ODE	Ordinary Differential Equation
Greek symbols		RKF	Runge-Kutta-Fehlberg Method
α	Casson parameter	MHD	Magnetohydrodynamics
		RSM	Response Surface Methodology

Table 1
Comparison between already published works and current work.

Work	[26]	[27]	[29]	[31]	Current work
Wedge	✓	✓	✓	✓	✓
Casson fluid	×	×	×	×	✓
Porous medium	×	×	×	×	✓
Radiation	✓	✓	×	✓	✓
Entropy generation	✓	×	×	×	✓
Fluid category					
Mono	✓	×	✓	×	×
Hybrid	×	×	×	✓	×
Ternary	×	×	×	×	✓
Optimization techniques					
RSM-based sensitivity analysis	×	×	×	✓	✓

parameter. Bharathi and Prakash [7] analyzed the behavior of non-Newtonian nanofluid flow with the Darcy model past a wedge under the influence of suction/injection and activation energy. The results display that growing values of Forchhiemer reduced fluid velocity. Jyothi and Kudenatti [8] examined the consequences of mass transpiration on fluid flow across a wedge in the presence of porous medium. The study results demonstrate that an upsurge in permeability causes the boundary layer to become thinner. Poornima et al. [9] examined the MHD radioactive nanofluid flow across a movable permeable wedge with Brownian motion and thermophoresis. This study highlights the role of porous media in fluid flow. The study outcomes reveal that an increase in porosity has a positive effect on fluid flow velocity. Akbar et al. [10] scrutinized the optimization and entropy generation of permeable peristaltic nanofluid flow under the impacts of MHD, Thermophoresis and Brownian motion with a combination of response surface methodology and neural network approach.

Mahabaleshwar et al. [11] investigated nanofluid flow across an expanding/shrinking surface under the implications of a heat sink/source and radiation. The study results indicate that the higher radiation

leads to enhancement in the temperature profile. Chaudhry et al. [12] investigated the impact of radiation, Thermophoresis, and Brownian motion on MHD bioconvective nanofluid flow past a porous stretching surface. The study results reveal that temperature profile rises for augmented values of radiation. Akbar and Huang [13] studied the combined effect of radiation, MHD, and ohmic heating on electro-osmotic peristaltic flow. The results demonstrate that less heat transfer was obtained for raising the radiation parameter. The nanoscale heat transfer analysis on MHD ternary nanofluid flow across a 3D wedge due to the effect of Joule heating and thermal radiation was analyzed by Almheidat et al. [14]. Choudhary et al. [15] explored the impacts of porosity and radiation on the flow of a hybrid nanofluid on a movable wedge. The results demonstrated that as radiation values increase, the temperature profile also increases. The influence of radiation on MHD blood-based Casson nanofluid flow in a channel was explained by Akbar et al. [16].

Many thermal and fluid systems rely on entropy analysis to evaluate and enhance their performance. In fluid flow, entropy signifies the irreversibility associated with transport phenomena, including viscous dissipation, radiation, and other energy transfer processes that occur across a finite temperature gradient. Sindhu et al. [17] examined the consequences of entropy analysis on radioactive nanofluid film across a wedge with gyrostatic microorganisms. Khan et al. [18] studied entropy generation on Jeffery nanofluid flow over a bent, extending sheet under the influence of CC heat flux and artificial neural networks. Considering the Buongiorno model, Hayat et al. [19] addressed the rheological attributes of MHD non-Newtonian fluid flow across an extending surface with entropy generation. Akbar et al. [20] investigated the impact of radiation and entropy on peristaltic flow between symmetrical channels. The study results indicate that the Bejan number increases as the solid volume fraction rises. Entropy generation works on many surfaces; this phenomenon can be found in these studies [21–24].

The entropy analysis on both hybrid nanofluid and carbon nanotube fluid flow over a wedge in the presence of radiation was conducted by Berrehal et al. [25] and Berrehal and Maougal [26]. Rashidi et al. [27] explored viscoelastic fluid flow past a permeable wedge with impacts of

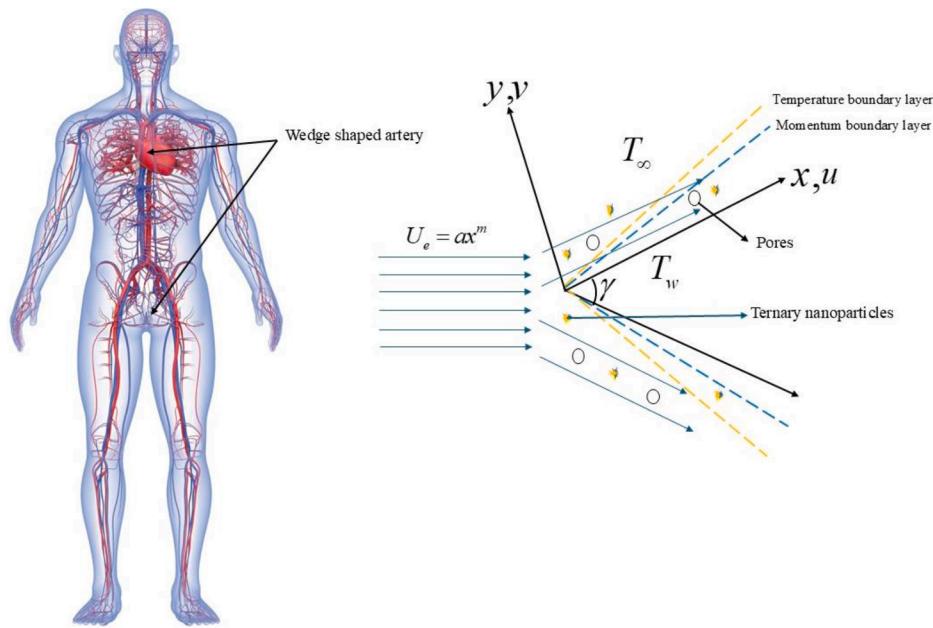


Fig. 1. Physical model of wedge-shape geometry.

radiation and magnetic field. The findings demonstrate that heat transmission of fluid is greater for growing values of angle of wedge. Rafique et al. [28] demonstrated the optimization of heat transfer over radioactive hybrid nanofluid flow on a moving wedge in the presence of a magnetic field and Joule heating. Kulkarni and Shankar [29] examined the effect of mixed convection and double diffusion on chemical reactive Williamson nanofluid flow across a variable wedge. Results show that in both cases of shrinking and stretching of a wedge, augmented values of the mixed convection parameter increase fluid velocity. Basit et al. [30] explored the influence of radiation and activation energy on chemically nanofluid flow across a wedge.

1.1. Novelty

Table 1 displays the novelty of current work when compared with previous works in terms of combined investigation of blood-based ternary non-Newtonian fluid flow across a wedge in the presence of porous medium, radiation, and entropy generation. Further, the addition of RSM-based sensitivity analysis offers understanding of the impact of controlling factors. The mathematical model is portrayed via a nonlinear PDE model and reduced to a nonlinear ODE model through a proper similarity approach. The RKF-45th approach is used for getting a numerical solution for this nonlinear ODE model.

The current research offers a unique dynamic and circulatory-based approach for studying blood-based nanofluid flow with three nanoparticles across a wedge-shaped artery, which resembles artery divisions and stenosed sections. Incorporating Casson blood mechanics, radiation, viscous dissipation, and entropy generation through porous artery walls offers an integrated view on loss of energy and flow resistance in damaged arteries.

Taking everything into account, the current study connects fluid mechanics and biomedical energy efficiency issues by providing a new mathematical tool to improve nanoparticle-assisted methods in wedge-shaped artery geometries.

1.2. Objectives

The current work addresses the above-stated research gaps and objectives by focusing on these aspects.

- To study the impact of Casson and porosity parameters on fluid flow of ternary nanofluid across a wedge geometry.
- To analyze the heat transfer rates and temperature distribution under the influence of radiation and viscous dissipation.
- To examine entropy generation in the presence of Casson fluid, porous medium, radiation, and viscous dissipation.
- To identify the most significant parameter influencing heat transfer and entropy generation with the aid of an RSM-based sensitivity approach.

2. Mathematical modeling

Consider a non-Newtonian, laminar, and incompressible 2D fluid flow with blood-based three nanoparticles (Al_2O_3 , Cu , TiO_2) across a permeable wedge-shaped artery. Fig. 1 displays the physical model with coordinates. The velocity components along axes (x, y) are denoted by (u, v) and the moving velocity of the wedge and ambient velocity flow are respectively given by $U_w(x) = U_0 x^m$ and $U_e(x) = U_\infty x^m$ with m as constant with $(0 \leq m \leq 1)$. Here, $m = \beta/(2 - \beta)$, where β represents the Hartree pressure gradient corresponding to the angle of wedge $\gamma = \pi\beta$. (T_w, T_∞) denotes wall temperature and temperature at far-field. The heat transfer analysis is done in the presence of radiation and viscous dissipation. In inclusion of radiation raises the possibility of its practical uses in fields where radioactive heat transfer affects the thermal behaviour in biological fluids such in laser heating and energy therapy. Further, to enhance this analysis across artery, entropy generation is employed. The non-Newtonian rheological behaviour of blood is reflected due to variations of Casson parameter, which measures the yield stress. The following relation provides rheological state property for an isotropic, incompressible Casson nanofluid flow.

$$\tau = \left(\mu_b + \frac{p_y}{\sqrt{\pi 2}} \right) e_{ij}, \pi > \pi_c, \text{ and } \tau = \left(\mu_b + \frac{p_y}{\sqrt{\pi 2}} \right) e_{ij}, \pi < \pi_c$$

where, π is the product of the deformation rate (e_{ij}) component, π_c is the assessment of π based on non-Newtonian modeling, and (p_y, μ_b) represents yield stress and dynamic viscosity of the plastic respectively.

The vector representation of governing equations is as follows:

$$\nabla \cdot \mathbf{V} = 0, \quad (1)$$

Table 2
Thermo-physical characteristic value of base fluid and nanoparticles.

Properties	Density (ρ)	Thermal conductivity (k)	Specific heat (C_p)
Blood	1050	0.52	3617
Cu	8933	401	385
Al_2O_3	3970	40	765
TiO_2	4250	8.9538	686.2

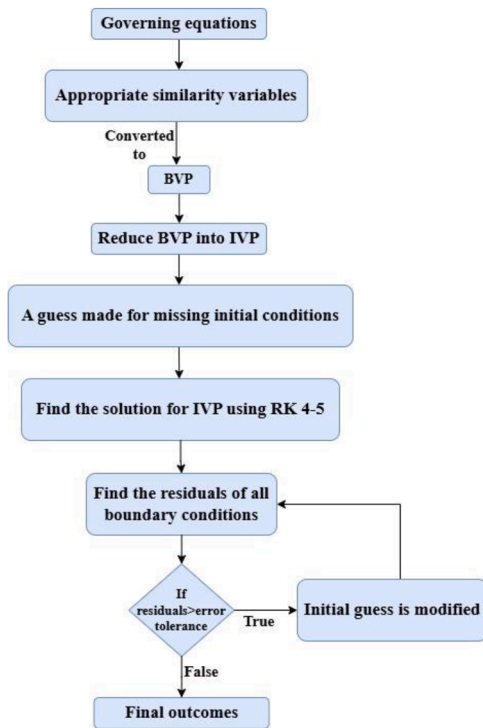


Fig. 2. Flow chart of RKF-45th method.

Table 3
Comparison of $f''(0)$ between current study and published work for distinct values of (m) with limiting cases $A_1 \rightarrow 1$, $A_2 \rightarrow 1$, $\left(1 + \frac{1}{\beta}\right) \rightarrow 1$, $\lambda = 0$.

m	Berrehal [25]	Current work
0	0.469599	0.469598
0.2	0.802126	0.802124
0.5	1.038903	1.038905
1	1.232588	1.232590

Table 4
Comparison of $-\theta'(0)$ between current study and published work for distinct values of (m) with limiting cases $A_3 = A_4 = Pr = 1$, $Ec = Br = \lambda = 0$, and $Bi \rightarrow \infty$.

m	Berrehal [25]	Current work
0	0.876898	0.876899
1	1.127959	1.127961

$$(\mathbf{V} \cdot \nabla) \mathbf{V} = -\frac{1}{\rho_{mf}} \nabla p + v_{mf} \left(1 + \frac{1}{\alpha}\right) \nabla^2 \mathbf{V} - \frac{v_{mf}}{\lambda^*} (\mathbf{V} - \mathbf{V}_e), \quad (2)$$

$$(\mathbf{V} \cdot \nabla) T = \alpha_{mf} \nabla^2 T + \alpha_{eff} \nabla^2 T + \Phi_v, \quad (3)$$

where, $\mathbf{V}$ is the function of (u, v) , ∇ represents $\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)$, $\mathbf{V}_e = (U_e, 0)$, α_{eff}

Table 5
Grid independence study for selecting mesh points.

Mesh grid points	$f''(0)$	$-\theta'(0)$
100	-0.41123	-1.68891
200	-0.41115	-1.68908
400	-0.41110	-1.68957
800	-0.41105	-1.69977
1000	-0.41101	-1.69975

Table 6
Domain independence study for computational boundary η_{∞} .

η_{∞}	$f''(0)$	$-\theta'(0)$
2	-0.45776	1.45776
3	-0.41636	1.68696
5	-0.40125	1.71236
7	-0.40127	1.71232

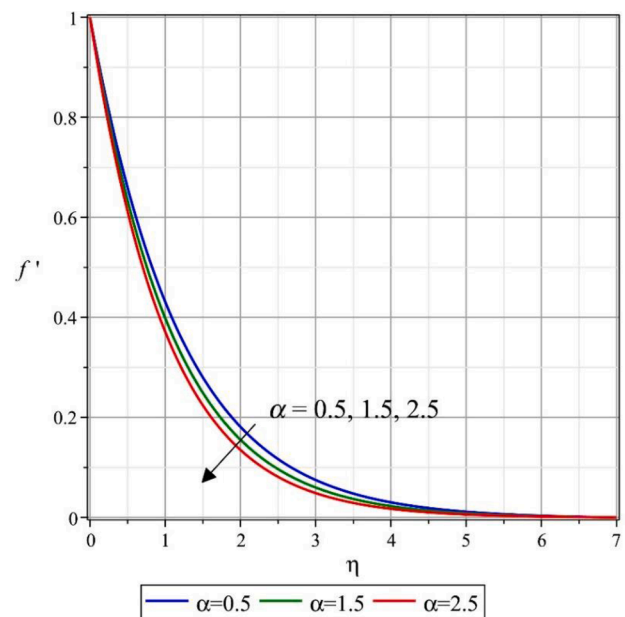


Fig. 3. Effect of α on f' .

$$= \frac{k_{mf}}{(\rho C_p)_{mf}} + \frac{16\sigma^* T_{\infty}^3}{3k^* (\rho C_p)_{mf}}, \text{ and } \Phi_v = \frac{\mu_{mf}}{(\rho C_p)_{mf}} \left(\frac{\partial u}{\partial y}\right)^2.$$

The PDE model from above assumptions with respective boundary conditions are as follows ([8, 26, 27]):

$$\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} = 0, \quad (4)$$

This is continuity equation which ensured conservation of mass.

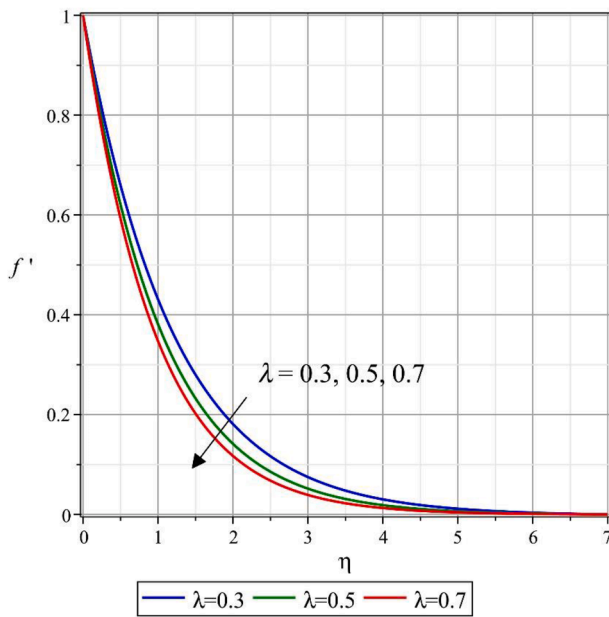
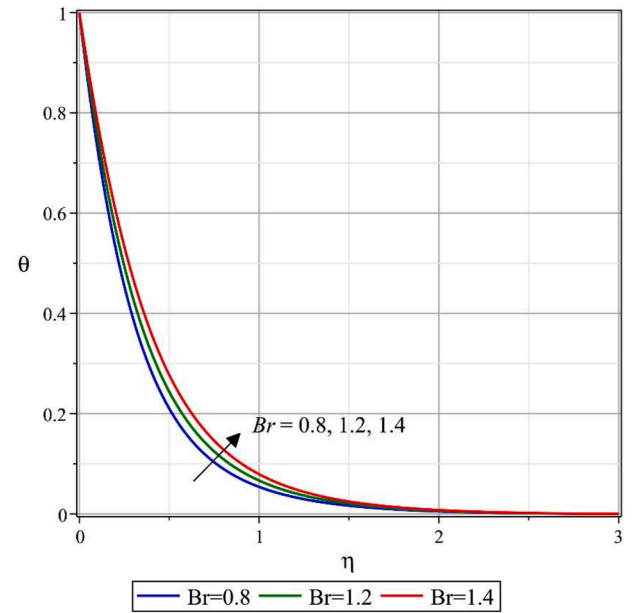
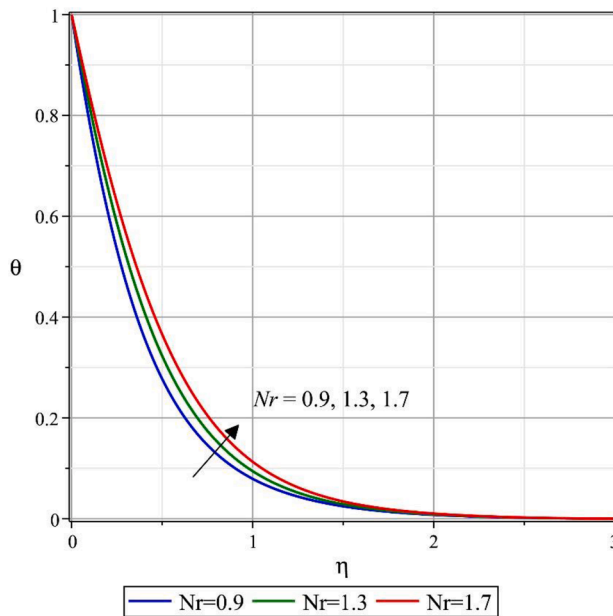
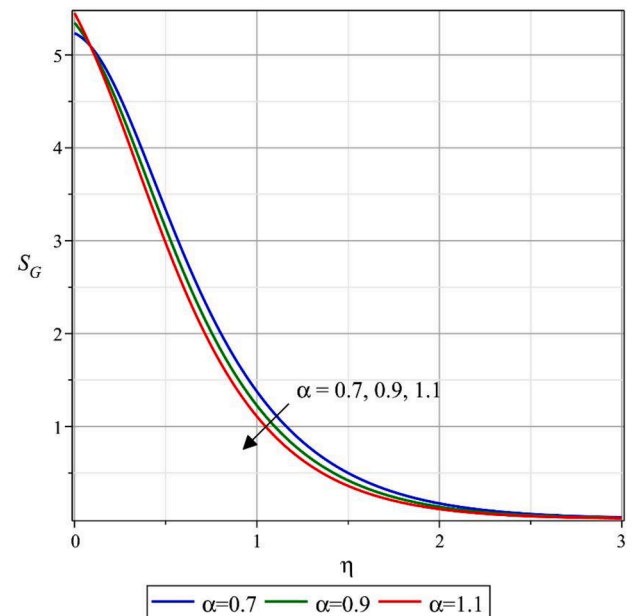
$$v \frac{\partial u}{\partial y} + u \frac{\partial u}{\partial x} = U_e \frac{dU_e}{dx} + v_{mf} \underbrace{\left(1 + \frac{1}{\alpha}\right) \frac{\partial^2 u}{\partial y^2}}_{\text{Casson}} - \underbrace{\frac{v_{mf}}{\lambda^*} (u - U_e)}_{\text{Porous}}, \quad (5)$$

This is momentum equation consisting of Casson parameter and porous medium term.

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_{mf}}{(\rho C_p)_{mf}} \frac{\partial^2 T}{\partial y^2} + \underbrace{\frac{16\sigma^* T_{\infty}^3}{(\rho C_p)_{mf} 3k^*} \frac{\partial^2 T}{\partial y^2}}_{\text{Radiation}} + \underbrace{\frac{\mu_{mf}}{(\rho C_p)_{mf}} \left(\frac{\partial u}{\partial y}\right)^2}_{\text{Viscous dissipation}}, \quad (6)$$

This is temperature/energy equation consisting of radiation and viscous dissipation terms.

The corresponding boundary conditions are,

Fig. 4. Influence of porosity parameter λ on f' .Fig. 6. Impact of Br on θ .Fig. 5. Impact of Nr on θ .Fig. 7. Impact of α on S_G .

$$u = U_w(x) = U_0 x^m, v = 0, T = T_w, \text{ at } y = 0 \quad (7)$$

$$u \rightarrow U_e(x) = U_\infty x^m, T \rightarrow T_\infty, \text{ as } y \rightarrow \infty \quad (8)$$

Here, at $y = 0$: $u = U_w$ denotes no slip condition, where fluid particles are in direct contact with wedge surface and travel at the same speed as that of wedge surface, $v = 0$ indicates that fluid is flowing along the surface, and $T = T_w$ is the constant wall temperature denoting that wedge surface is maintained at uniform temperature T_w .

Further, as $y \rightarrow \infty$: $u \rightarrow U_e(x)$ denotes free stream velocity, and $T \rightarrow T_\infty$ denotes that temperature of the fluid reaches ambient temperature.

The employed similarity variables are,

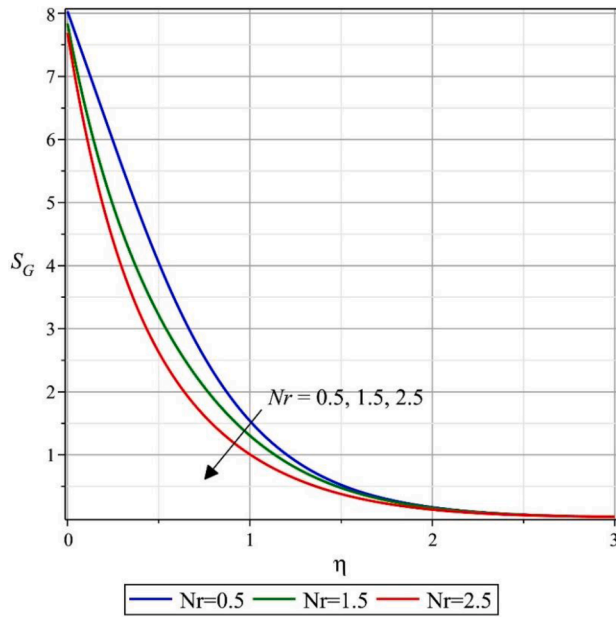
$$\eta = y \sqrt{a \frac{(m+1)}{2\nu_f} x^{\frac{m-1}{2}}}, \quad (9)$$

$$u = U_e f', v = -\sqrt{a\nu_f \frac{(m+1)}{2}} x^{\frac{m-1}{2}} \left\{ f + f' \eta \left(\frac{m-1}{m+1} \right) \right\}, \text{ and } \theta = \frac{T - T_\infty}{T_w - T_\infty}. \quad (10)$$

The thermophysical characteristics of ternary nanofluid are [32],

$$k_{nf} = \left(\frac{k_{S1} + 2k_{hnf} - 2\phi_1(k_{hnf} - k_{S1})}{k_{S1} + 2k_{hnf} + \phi_1(k_{hnf} - k_{S1})} \right) k_{hnf}, k_{hnf} = \left(\frac{-2\phi_2(k_{nf} - k_{S2}) + k_{S2} + 2k_{nf}}{\phi_2(k_{nf} - k_{S2}) + k_{S2} + 2k_{nf}} \right),$$

$$k_{hnf} = \left(\frac{-2\phi_3(k_f - k_{S3}) + k_{S3} + 2k_f}{\phi_3(k_f - k_{S3}) + k_{S3} + 2k_f} \right), \mu_{nf} = \frac{\mu_f}{(1 - (\phi_1 + \phi_2 + \phi_3))^{2.5}},$$

Fig. 8. Impact of Nr on S_G .

$$\rho_{mf} = (1 - \phi_1)((1 - \phi_2)[(1 - \phi_3)\rho_f + \rho_{s3}\phi_3]\rho_{s2}\phi_2) + \rho_{s1}\phi_1,$$

$$(\rho C_p)_{mf} = \left(\frac{(\rho C_p)_{s1}\phi_1}{(\rho C_p)_f} + (1 - \phi_1) \left[(1 - \phi_2) \left[(1 - \phi_3) + \frac{(\rho C_p)_{s3}\phi_3}{(\rho C_p)_f} \right] + \frac{(\rho C_p)_{s2}\phi_2}{(\rho C_p)_f} \right] \right) (\rho C_p)_f.$$

The nonlinear PDEs are converted into nonlinear ODEs using similarities (9–10) and thus the reduced ODEs are,

$$A_1 A_2 \left(1 + \frac{1}{\alpha} \right) f''' - \frac{2m}{(m+1)} (f'^2 - 1) + f f'' - A_1 A_2 \frac{2m}{(m+1)} \frac{\lambda}{Da} (f' - 1) = 0, \quad (11)$$

$$(A_3 + Nr)\theta'' + A_4(\text{Pr}f\theta') + \left(\frac{A_1}{A_4} \right) Br f'^2 = 0, \quad (12)$$

and

$$f' = \delta, f = 0, \theta = 1, \quad (13)$$

$$f' \rightarrow 0, \theta \rightarrow 0. \quad (14)$$

2.1. Equation for entropy generation

The assessment of irreversibility in a system can be understood by studying entropy generation monitored by thermodynamics' second law. Entropy must be minimized for the system to improve its energy. Hence, an irreversibility study has been done on non-Newtonian ternary nanofluid flow across a wedge-shaped artery. For this study, this analysis takes into account the impact of variables such as porous medium, viscous dissipation, and thermal radiation. The respective equation is as follows:

$$N_G = \underbrace{\frac{1}{T_\infty^2} \left[\left\{ \frac{k_{mf}}{k_f} + \frac{16\sigma^* T_\infty^3}{3k^*} \right\} \left(\frac{\partial T}{\partial y} \right)^2 \right]}_{\text{Thermal irreversibility}} + \underbrace{\left(1 + \frac{1}{\alpha} \right) \frac{\mu_{mf}}{T_\infty} \left(\frac{\partial u}{\partial y} \right)^2}_{\text{Fluid friction irreversibility}} + \underbrace{\frac{\mu_{mf}}{T_\infty \lambda^*} u^2}_{\text{Porous medium irreversibility}}. \quad (15)$$

The obtained reduced form of the Eq. (15) is as follows:

$$S_G = \frac{N_G}{N_O} = \left\{ \frac{m+1}{2} \right\} \text{Re}_x \left[(A_3 + Nr)\theta'^2 + \left(1 + \frac{1}{\alpha} \right) A_1 \frac{Br}{\Omega} f'^2 \right] + A_1 \frac{Br}{\Omega Da} f'^2. \quad (16)$$

The Bejan number (Be) obtained from the ratio of overall entropy to entropy by irreversibility by heat transport is given by

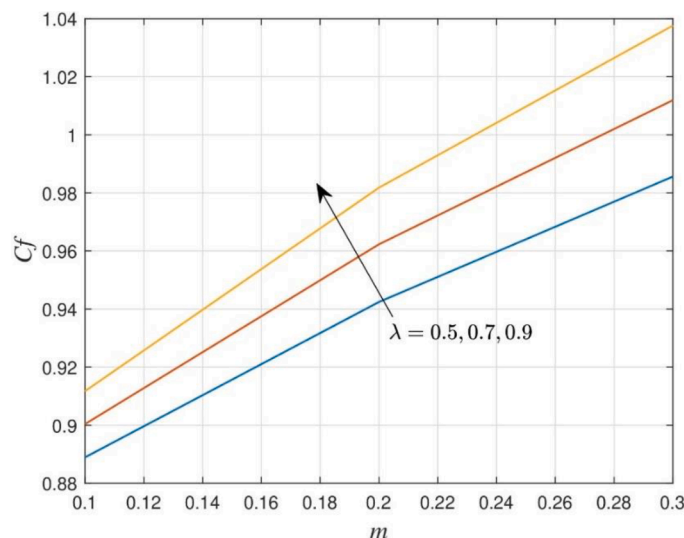
$$Be = \frac{\text{Thermal irreversibility}}{\text{Entropy generation}} = 0.7672.$$

When the value of $Be \geq 0.5$, thermal irreversibility dominates and when $Be \leq 0.5$, the fluid friction irreversibility dominates. Since the value is $0.7372 > 0.5$, the thermal irreversibility dominates. where,

$$A_1 = \frac{\mu_{mf}}{\mu_f}, A_2 = \frac{\rho_{mf}}{\rho_f}, A_3 = \frac{k_{mf}}{k_f}, A_4 = \frac{(\rho C_p)_{mf}}{(\rho C_p)_f}, \lambda = \frac{v_f}{U_\infty l}, Da = \frac{\lambda^*}{l^2}$$

$$\text{Pr} = \frac{v_f (\rho C_p)_f}{k_f}, Br = \frac{\mu_f U_\infty^2}{k_f \Delta T}, Nr = \frac{16\sigma^* T_\infty^3}{3k^* k_f}, \Omega = \frac{T_\infty}{\Delta T}, N_O = \frac{k_f \Delta T^2}{x^2 T_\infty^2}, \text{Re} = \frac{U_\infty x}{v_f}$$

$$\delta = \frac{U_w}{U_e}$$

Fig. 9. Change in the Skin friction C_f for various values of λ and m .

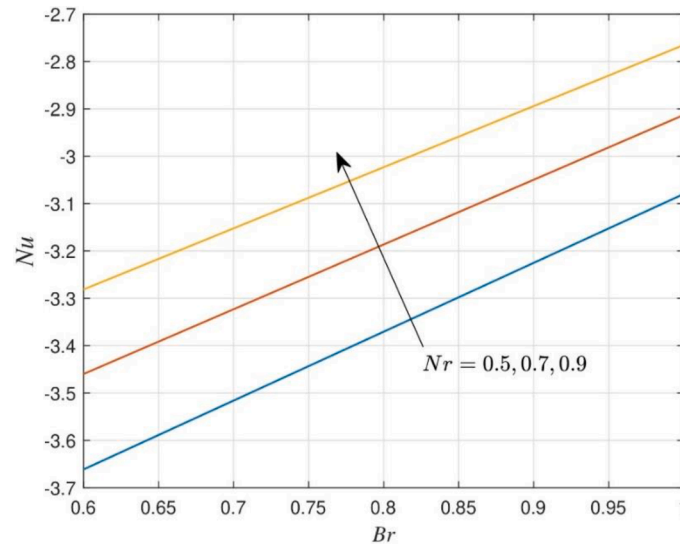


Fig. 10. Change in the Nusselt number Nu for various values of Nr and Br .

Table 7

Comparison of Skin friction and Nusselt number for mono, hybrid, and ternary nanofluids.

α	K	δ	Br	Skin friction		
0.1				Mono nanofluid $\phi_1 = 0.01,$ $\phi_2 = 0,$ $\phi_3 = 0$	Hybrid nanofluid $\phi_1 = 0.01,$ $\phi_2 = 0.01,$ $\phi_3 = 0$	Ternary nanofluid $\phi_1 = 0.01,$ $\phi_2 = 0.01,$ $\phi_3 = 0.01$
0.1	0.1	0.5	0.1	-0.07410	-0.07409	-0.07500
0.3	0.1	0.5	0.1	-0.23462	-0.23456	-0.23775
0.5	0.1	0.5	0.1	-0.41050	-0.41039	-0.41636
0.5	0.3	0.5	0.1	-0.06715	-0.06713	-0.38531
0.5	0.4	0.5	0.1	-0.07070	-0.07068	-0.40109
0.5	0.5	0.5	0.1	-0.07410	-0.07409	-0.41636
0.5	0.1	1.5	0.1	-0.07410	-0.07409	-0.41636
0.5	0.1	2.0	0.1	-0.07410	-0.07409	-0.41636
0.5	0.1	2.5	0.1	-0.07410	-0.07409	-0.41636
0.5	0.1	0.5	0.2	-0.07410	-0.07409	-0.07500
0.5	0.1	0.5	0.4	-0.07410	-0.23456	-0.23775
0.5	0.1	0.5	0.6	-0.07410	-0.41039	-0.41636
λ	δ	Nr	Br	Nusselt number		
0.1				Mono nanofluid $\phi_1 = 0.01,$ $\phi_2 = 0,$ $\phi_3 = 0$	Hybrid nanofluid $\phi_1 = 0.01,$ $\phi_2 = 0.01,$ $\phi_3 = 0$	Ternary nanofluid $\phi_1 = 0.01,$ $\phi_2 = 0.01,$ $\phi_3 = 0.01$
0.1	0.1	2.5	0.1	1.81433	-1.76646	-1.72127
0.3	0.1	2.5	0.1	1.84121	-1.79425	-1.74844
0.5	0.1	2.5	0.1	1.76508	-1.72167	-1.67633
0.5	0.3	2.5	0.1	1.81645	1.76856	-1.69686
0.5	0.4	2.5	0.1	1.81537	1.76749	-1.68653
0.5	0.5	2.5	0.1	1.81434	1.76646	-1.67633
0.5	0.1	1.5	0.1	2.51266	2.44499	-2.26818
0.5	0.1	2.0	0.1	2.10456	2.04833	-1.92394
0.5	0.1	2.5	0.1	1.81434	1.76646	-1.67633
0.5	0.1	2.5	0.2	1.81521	-1.76646	-1.72127
0.5	0.1	2.5	0.4	1.81499	-1.79425	-1.74844
0.5	0.1	2.5	0.6	1.81477	-1.72167	-1.67633

2.2. Engineering coefficients

The Engineering factors obtained in this study are Skin friction Cf , and Nusselt number Nu . The expressions with their reduced form are derived as follows:

Table 8

Quantitative analysis of Nusselt number and Entropy generation.

β	m	Nr	Br	Ω	Nusselt number	Entropy generation
0.5	0.1	1.5	0.1	0.3	7.66	3.62
0.7	0.1	1.5	0.1	0.3	7.66	3.62
0.9	0.1	1.5	0.1	0.3	7.67	3.62
0.5	0.2	1.5	0.1	0.3	7.66	3.62
0.5	0.3	1.5	0.1	0.3	7.66	3.62
0.5	0.4	1.5	0.1	0.3	7.66	12.29
0.5	0.1	1.7	0.1	0.3	7.64	3.35
0.5	0.1	2.0	0.1	0.3	7.60	3.02
0.5	0.1	2.5	0.1	0.3	7.53	2.59
0.5	0.1	1.5	0.3	0.3	7.66	3.63
0.5	0.1	1.5	0.5	0.3	7.66	3.64
0.5	0.1	1.5	0.7	0.3	7.66	3.65
0.5	0.1	1.5	0.1	0.5	7.66	3.62
0.5	0.1	1.5	0.1	0.6	7.66	3.62
0.5	0.1	1.5	0.1	0.7	7.66	3.62

Table 9

Input parametric values with corresponding coded values.

Parameters	Coded symbols	Levels		
		Low (-1)	Medium (0)	High (1)
Nr	A	1.5	2.0	2.5
Br	B	0.1	0.3	0.5
Ω	C	0.5	0.7	0.9

$$Cf = \left(1 + \frac{1}{\alpha}\right) \frac{\tau_w}{\rho_f U_\infty^2(x)}, \text{ where } \tau_w = \mu_{mf} \left. \frac{\partial u}{\partial y} \right|_{y=0}. \quad (17)$$

$$Re^{1/2} Cf = A_1 \sqrt{\frac{m+1}{2}} \left(1 + \frac{1}{\alpha}\right) f''(0). \quad (18)$$

$$Nu = \frac{-xq_w}{k_f \Delta T}, \text{ where } q_w = \left\{ k_{mf} + \frac{16\sigma^* T_\infty^3}{3k^*} \right\} \left. \frac{\partial T}{\partial y} \right|_{y=0}. \quad (19)$$

$$Re^{-1/2} Nu = -\sqrt{\frac{m+1}{2}} (A_3 + Nr) \theta'(0). \quad (20)$$

Table 10
Experimental runs with their corresponding responses.

Exp. runs	Coded values			Actual values			Nu	N_G
	A	B	C	Nr	Br	Ω		
1	0	0	0	2	0.3	0.7	2.117	7.736
2	0	0	0	2	0.3	0.7	2.117	7.736
3	-1	-1	-1	1.5	0.1	0.5	2.528	6.479
4	1	-1	-1	2.5	0.1	0.5	1.825	8.977
5	-1	0	0	1.5	0.3	0.7	2.528	6.487
6	-1	1	1	1.5	0.5	0.9	2.527	6.491
7	1	0	0	2.5	0.3	0.7	1.825	8.985
8	0	0	0	2	0.3	0.7	2.117	7.736
9	0	1	0	2	0.5	0.7	2.117	7.746
10	0	0	0	2	0.3	0.7	2.117	7.736
11	-1	-1	1	1.5	0.1	0.9	2.528	6.476
12	-1	1	-1	1.5	0.5	0.5	2.527	6.507
13	1	1	-	2.5	0.5	0.5	1.825	9.005
14	0	0	0	2	0.3	0.7	2.117	7.736
15	0	0	1	2	0.3	0.9	2.117	7.732
16	1	1	1	2.5	0.5	0.9	1.825	8.989
17	0	0	-1	2	0.3	0.5	2.117	7.742
18	0	0	0	2	0.3	0.7	2.117	7.736
19	1	-1	1	2.5	0.1	0.9	1.825	8.974
20	0	-1	0	2	0.1	0.7	2.117	7.726

3. Numerical methodology

In this study, the mathematical techniques used are shooting and RKF-45th which is well suited to handle nonlinear problems with ease.

Table 11
ANOVA data for Nu .

Sources	Degrees of freedom	Adj. sum of squares	Adj. mean squares	F-value	P-value
Model	9	1.25222	0.13914	1.56E + 10	0
Linear	3	1.23463	0.41154	4.60E + 10	0
Nr	1	1.23463	1.23463	1.38E + 11	0
Br	1	0	0	76,930.51	0
Ω	1	0	0	0	1
Square	3	0.01758	0.00586	6.56E + 08	0
Nr *Nr	1	0.00967	0.00967	1.08E + 09	0
Br *Br	1	0	0	0	1
Ω * Ω	1	0	0	0	1
2-way interaction	3	0	0	620.18	0
Nr *Br	1	0	0	1860.53	0
Nr * Ω	1	0	0	0	1
Br * Ω	1	0	0	0	1
Error	10	0	0	0	1
Lack of fit	5	0	0	*	*
Pure error	5	0	0		
Total	19	1.25222			

Table 12
ANOVA data for N_G .

Sources	Degrees of freedom	Adj. sum of squares	Adj. mean squares	F-value	P-value
Model	9	15.6056	1.734	13,803,385	0
Linear	3	15.6055	5.2018	41,409,928	0
Nr	1	15.6041	15.6041	1.24E + 08	0
Br	1	0.0011	0.0011	9075.16	0
Ω	1	0.0002	0.0002	1722.66	0
Square	3	0	0	23.44	0
Nr *Nr	1	0	0	0	1
Br *Br	1	0	0	0	1
Ω * Ω	1	0	0	38.67	0
2-way interaction	3	0.0001	0	204.17	0
Nr *Br	1	0	0	0	1
Nr * Ω	1	0	0	0	1
Br * Ω	1	0.0001	0.0001	612.5	0
Error	10	0	0		
Lack of fit	5	0	0	*	*
Pure error	5	0	0		
Total	19	15.6056			

Initially, these highly nonlinear Eqs. (11–14) with a higher order are converted into nonlinear equations with a first order using new variables as follows:

$$f = p_1, f' = p_2, f'' = p_3,$$

$$f''' = p_3' = - \left(1 + \frac{1}{\alpha} \right)^{-1} \left(\frac{1}{A_1 A_2} \right) \left\{ - \frac{2m}{(m+1)} p_2^2 + p_1 p_3 - A_1 A_2 \frac{2m}{(m+1)} \frac{\lambda}{D \alpha} p_2 \right\}$$

$$\theta = p_4, \theta' = p_5,$$

$$\theta'' = p_5' = - \left(\frac{1}{(A_3 + Nr)} \right) \left\{ A_4 (Pr p_1 p_5) + \left(\frac{A_1}{A_4} \right) Br p_3^2 \right\} \quad (21)$$

and

$$p_2 = 1, p_1 = 0, p_4 = 1, \quad (22)$$

$$p_2 \rightarrow 0, p_4 \rightarrow 0.$$

Still, this is a Boundary Value Problem (BVP), which is converted into an Initial Value Problem with the help of the shooting technique, and then this IVP is solved using the RKF-45th method. During the simulation, the error tolerance, step size, and convergence criteria is set for $(10^{-8}, 0.01, 10^{-5})$ respectively and the values from the Table 2 are considered [33–34]. The flow chart of the RKF-45 method is displayed in Fig. 2. Table 3 and 4 demonstrates the comparison of values of $f''(0)$ and $-\theta'(0)$ with already published results considering limiting cases with good matching.

Table 5 and 6 shows the grid independence and domain

Table 13
Sensitivity analysis for Nu .

Cases	Coded values			Sensitivity values for Nu	
	A	B	C	$\partial Nu / \partial A$	$\partial Nu / \partial B$
Case 1		-1	-1	-2.12952	-0.002679
	-1	-1	0	-2.12952	-0.002679
		-1	1	-2.12952	-0.002679
		0	-1	-2.12906	-0.002679
	-1	0	0	-2.12906	-0.002679
		0	1	-2.12906	-0.002679
		1	-1	-2.12861	-0.002679
	-1	1	0	-2.12861	-0.002679
		1	1	-2.12861	-0.002679
		-1	-1	-1.65513	-0.002223
Case 2	0	-1	0	-1.65513	-0.002223
		-1	1	-1.65513	-0.002223
		0	-1	-1.65467	-0.002223
	0	0	0	-1.65467	-0.002223
		0	1	-1.65467	-0.002223
	0	1	-1	-1.65421	-0.002223
		1	0	-1.65421	-0.002223
		1	1	-1.65421	-0.002223
		-1	-1	-1.18074	-0.002223
	1	-1	0	-1.18074	-0.001767
Case 3		-1	1	-1.18074	-0.001767
		0	-1	-1.18028	-0.001767
	1	0	0	-1.18028	-0.001767
		0	1	-1.18028	-0.001767
		1	-1	-1.17982	-0.001767
	1	1	0	-1.17982	-0.001767
		1	1	-1.17982	-0.001767

Table 14
Sensitivity analysis for N_G .

Cases	Coded values			Sensitivity values for N_G		
	A	B	C	$\partial Nu / \partial A$	$\partial Nu / \partial B$	$\partial Nu / \partial C$
Case 1		-1	-1	2.49833	0.18519	-0.03545
	-1	-1	0	2.49833	0.10766	0.03101
		-1	1	2.49833	0.03013	0.09747
		0	-1	2.49833	0.18519	-0.11298
	-1	0	0	2.49833	0.10766	-0.04652
		0	1	2.49833	0.03013	0.01994
		1	-1	2.49833	0.18519	-0.19051
	-1	1	0	2.49833	0.10766	-0.12405
		1	1	2.49833	0.03013	-0.05759
		-1	-1	2.49833	0.18519	-0.03545
Case 2	0	-1	0	2.49833	0.10766	0.03101
		-1	1	2.49833	0.03013	0.09747
		0	-1	2.49833	0.18519	-0.11298
	0	0	0	2.49833	0.10766	-0.04652
		0	1	2.49833	0.03013	0.01994
	0	1	-1	2.49833	0.18519	-0.19051
		1	0	2.49833	0.10766	-0.12405
		1	1	2.49833	0.03013	-0.05759
		-1	-1	2.49833	0.18519	-0.03545
	1	-1	0	2.49833	0.10766	0.03101
Case 3		-1	1	2.49833	0.03013	0.09747
		0	-1	2.49833	0.18519	-0.11298
	1	0	0	2.49833	0.10766	-0.04652
		0	1	2.49833	0.03013	0.01994
		1	-1	2.49833	0.18519	-0.19051
	1	1	0	2.49833	0.10766	-0.12405
		1	1	2.49833	0.03013	-0.05759

independence study performed for computational purposes. Table 5 reveals that the data (engineering factors) exhibited minimal variations after 400 grid points. This indicates no change in numerical solutions with respect to mesh refinement. Further from Table 6, it can be found that changes in data (engineering factors) after $\eta_\infty = 3$ are also trivial. Hence, $\eta_\infty = 3$ was opted for all computational purposes.

4. Assessment and interpretation of results

This section provides the detailed analysis on the results from Eqs. (8), (11-14) and (16)11 -14 and 16) under the influence of entropy, radiation, and dissipation on flow across a permeable wedge shaped artery with ternary nanofluid. The range of parameters and fixed values of some are set as $0.3 \leq \lambda \leq 0.7$, $0.5 \leq \alpha \leq 2.5$, $0.9 \leq Nr \leq 1.7$, $0.8 \leq Br \leq 1.4$, $0.01 \leq \phi_1 = \phi_2 = \phi_3 \leq 0.03$, $0.2 \leq Br \leq 1.5$, $Pr = 21$, $Da = 0.5$, $m = 0.5$, $\alpha = 1.5$.

4.1. Analysis of results

Fig. 3 shows the effect of the Casson parameter α on the velocity profile f' . For increasing values of the Casson parameter, the velocity profile reduces. Physically, a rise in the Casson parameter correlates to greater yield stress and greater effective viscosity of the blood-based fluid. Hence, the fluid needs greater kinetic energy for movement, which suppresses the velocity field and reduces momentum boundary layer thickness. This indicates strong resistance to fluid movement in the fluid flow region.

Fig. 4 demonstrates the effect of the porosity parameter λ on f' . It is seen that fluid flow velocity decreases due to an upsurge in porosity values. By physically raising the porosity parameter, the porous material provides a strong barrier to the fluid flow. This extra drag force opposes the flow and limits the momentum inside the boundary layer, causing a lesser velocity boundary layer and less fluid velocity. Such a phenomenon represents the dampening effect of porous materials on fluid flow.

Fig. 5 portrays the radiation Nr impact on the temperature profile θ . From the figure, it is observed that the temperature profile elevates for growing values of radiation. Physically, growing values of radiation enhance radiative heat transfer within the fluid, adding up extra thermal energy to the boundary layer area. This slows down escape of heat transfer from fluid and promotes thermal diffusion, resulting in temperature increase and a thicker thermal boundary layer.

The impact of the Brinkman number Br over θ is observed in Fig. 6. The figure shows that θ increases for rising values of Br . From a physical perspective, an augmented Brinkman number indicates promotion of kinetic energy into thermal energy due to viscous force. Then the fluid's temperature is raised by this extra internal heat generation, which increases overall temperature profile. Further, the thermal boundary layer gets thicker and causes more heat transfer inside the fluid system.

Fig. 7 demonstrates the variation in entropy generation number S_G for distinct values of α . It is clear from the figure that S_G declines for augmented values of α . Physically, growing values of Casson parameter indicates more yield stress effect hindering fluid flow. This decrease reduces convective form of heat transmission causing drop in thermal irreversibility. This drop reduces overall entropy generation through the fluid system. This declination in entropy generation indicates irreversible energy losses in the fluid system and enhanced thermodynamic performance of the fluid flow.

Fig. 8 demonstrates the variation in S_G for growing values of radiation parameter Nr . The negative trend is seen due to larger radiation values. From physical point of view, increase in radiation promotes radioactive heat flux and this lowers thermal gradients within the thermal boundary layer and since thermal gradients affect thermal irreversibilities directly, reduction in thermal gradients results in decreasing of entropy generation within the system. Thus, a total entropy generation reduces indicates efficient heat transfer within the fluid system.

Variation in the Skin friction Cf at different values of m for rising values of porosity parameter λ is seen in Fig. 9. From figure it is seen that Cf enhances for both varying values of λ and m . From physical point of view, less permeability leads to a more resistance force in fluid flow. This results and steeper velocity gradient and greater wall shear stress. Thus, skin friction being directly related to shear stress, it also increases. Also, greater values of angle of wedge indicates greater wall shear stress

which contributes in increasing Skin friction.

Variation in the Nusselt number Nu at different values of Brinkman number Br and growing values of radiation parameter Nr is depicted in Fig. 10. It is observed that Nusselt number increases for both values of Br and Nr . From physical point of view, greater values of Br increases viscous dissipation which promotes amplified temperature gradients near wall and increases overall heat transfer and more radiation adds up more thermal energy to the fluid. This combined effect boosts heat exchange near surface, resulting in higher magnitude of Nusselt number and better heat transfer capabilities of the fluid system.

Table 7 demonstrates the analysis of mono, hybrid, and ternary nanofluids to examine the combined impact from multiple nanoparticles. The outcomes show that a better heat transfer is achieved in ternary combination of nanoparticles compared to hybrid and mono. This is due the combined impact from the mixture of Cu/s (strong thermal conductivity), dispersion and stability of (Al_2O_3 and TiO_2) nanoparticles which improves the thermal conductivity and thus enhance universal thermal performance.

Table 8 outcomes show that increasing radiation parameter reduces entropy production by 28.5% while preserving a significant heat transfer rate with 1.7% loss in Nusselt number, thus boosting thermal efficiency in bio medical transport systems.

4.2. Sensitivity analysis

Sensitivity analysis identifies which input variable has dominant impact on response variable. This helps experimental designs by focusing on important elements in the model which affects results. Many works have adopted this analysis to study most influencing parameters affecting flow, heat, and mass transfer properties [35–37]. This section discusses the RSM based sensitivity analysis performed to evaluate the influence of radiation (Nr), brinkman number (Br), and temperature difference ratio parameters (Ω) on heat transfer rate and entropy generation. Table 9 shows the input parametric values with respect to their coded values which is A for Nr , B for Br , and C for Ω . Further Table 10 shows the total number of experimental runs which is calculated based on following equation,

$$q = 2^i + 2i + n_0, \quad (23)$$

where, ($q, i, 2^i, n_0$) denotes experimental runs, no of input parameters, no fractional points, and no center points. The design used for this study is Face Centered Central composite Design model. From this study, the obtained quadratic mathematical model for Nu and N_G is as follows:

$$Nu = 4.47194 - 1.65467Nr - 0.002223Br + 0.000000\Omega + 0.237197Nr * Nr - 0.000000Br * Br - 0.000000\Omega * \Omega + 0.000456Nr * Br - 0.000000Nr * \Omega - 0.000000Br * \Omega \quad (24)$$

and

$$N_G = 2.7392 + 2.49833Nr + 0.10766Br - 0.04652\Omega + 0.000000Nr * Nr - 0.000000Br * Br + 0.03323\Omega * \Omega - 0.000000Nr * Br - 0.000000Nr * \Omega - 0.07753Br * \Omega \quad (25)$$

From Table 11, it is recognised that the terms like $\Omega, Br^2, \Omega^2, Nr * \Omega$, and $Br * \Omega$ are non-significant ($p > 0.05$) and also from Table 12, it is recognised that terms like $Nr^2, Br^2, Nr * Br$, and $Nr * \Omega$ are nonsignificant ($p > 0.05$). Thus, obtained model is as follows:

$$Nu = 4.47194 - 1.65467Nr - 0.002223Br + 0.237197Nr^2 + 0.000456Nr * Br, \quad (26)$$

and

$$N_G = 2.7392 + 2.49833Nr + 0.10766Br - 0.04652\Omega + 0.03323\Omega^2 - 0.07753Br * \Omega. \quad (27)$$

Further sensitivity of Nu and N_G against parameters (Nr, Br, Ω) is examined through partial derivative of Nu and N_G with corresponding to input parameters as follows:

$$\frac{\partial Nu}{\partial Nr} = -1.65467Nr + 2(0.237197)Nr + 0.000456Br, \quad (28)$$

$$\frac{\partial Nu}{\partial Br} = -0.002223Br^2 + 0.000456Nr, \quad (29)$$

$$\frac{\partial N_G}{\partial Nr} = 2.49833, \quad (30)$$

$$\frac{\partial N_G}{\partial Br} = 0.10766 - 0.07753\Omega, \quad (31)$$

and

$$\frac{\partial N_G}{\partial \Omega} = -0.04652\Omega + 2(0.03323)\Omega - 0.07753Br * \Omega. \quad (32)$$

Considering these sensitivity derived functions (Eqs. 28–32), we determine to which input parameter the response variables Nu and N_G are highly sensitive towards. Table 13 and 14 displays the sensitivity of Nu and N_G in three levels which are high (case 3), medium (case 2), and low (case 1). From Table 13, it can be seen that Nu is highly sensitive towards radiation parameter ($Nr = A$) compared to ($Br = B$) in all cases and impact of Nr on Nu in case 1 is more and then reduces when approached to case 3. Also, Brinkman number shows negligible effect on Nu in all cases. From Table 14, it can be seen that entropy production is highly sensitive towards $Nr(A)$ in all cases with sensitive value of 2.49833. The parameter $Br(B)$ offers less influence, where N_G is more sensitive to Br when $\Omega(C)$ is at its lower level. Further, N_G is weakly sensitive towards $\Omega(C)$, since very small changes are observed in N_G in all cases.

With all factors considered, the sensitivity examination demonstrates that radiation parameter is primary factor impacting both heat transfer and entropy generation. However, Brinkman number and temperature ratio show less significant impact on both heat transfer and entropy generation.

5. Conclusion

This study was carried out to analyze the rate of heat transfer and entropy generation of Casson-blood based ternary nanofluid flow across a wedge shaped artery under the coupled influence of porous medium, viscous dissipation, and radiation. The nonlinear PDE model was converted into nonlinear ODE model through similarity transformation and solved numerically with the RKF-45th numerical approach. The analysis and interpretation of numerical results were discussed via graphical plots. The main results drawn from this study are summarized as follows:

- The fluid velocity profile declines for increasing values of Casson and porosity parameter due to respective greater yield stress and porous resistance which slows down fluid velocity within the momentum boundary layer.
- It was found that radiation and Brinkman number has a positive impact on thermal profile adding up extra heat due to radiative heat transfer and viscous effects respectively. This promotes an increase in heat within the thermal boundary layer.
- The entropy generation number has negative impact for rising values of Casson and Brinkman number indicating energy loss and lower irreversibility within the fluid system.
- The drag force enhances for growing values of porosity parameter and wedge angle because of more flow resistance and steeper velocity gradients near the surface.

- An improved heat transfer rate was achieved for growing value of Brinkman and radiation parameter resulting from strong heat transfer and wall heat exchange across surface.
- Based on sensitivity analysis, the radiation and Brinkman have the most impact on heat transfer and entropy generation properties.
- According to Bejan number study, the thermal irreversibility is the primary factor for entropy generation under optimal conditions.

In general, the study findings offer novel insights on how to regulate heat transmission and entropy production in non-Newtonian nanofluid models which possess significant implications for the advancement of efficient thermal transport in bio medical based practical applications.

5.1. Limitations and future scope of the study

The present study is limited to examine heat transfer and entropy generation in a Casson ternary nanofluid flow with base fluid blood across a permeable wedge shaped artery under the combined effect from thermal radiation and viscous dissipation. The analysis provides theoretical insights into the heat transport behaviour and thermodynamic attributes of the fluid system under consideration of assumptions considered in mathematical modeling.

The present work can be modified to integrate other external effects, different geometry surfaces, and more challenging flow cases for better geometrical representations. Further analysis involving different non Newtonian fluid models, nanoparticle shape factor, aggregation of nanoparticles, and advanced numerical, neural networks, and optimisation approaches can provide significant insights into heat and mass transfer, and entropy generation in such boundary layer fluid systems.

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CRedit authorship contribution statement

K.M. Nihaal: Conceptualization, Formal analysis, Methodology, Software, Validation, Writing – original draft, Writing – review & editing. **K. Nagegowda:** Writing – original draft, Software, Methodology, Formal analysis, Conceptualization. **U.S. Mahabaleshwar:** Writing – review & editing, Writing – original draft, Validation, Software, Methodology, Investigation, Formal analysis, Conceptualization. **U. Guevara:** Writing – review & editing, Supervision, Investigation, Formal analysis, Conceptualization. **L.M. Pérez:** Writing – review & editing, Supervision, Investigation, Formal analysis, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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