

Computational magnetohydrodynamics of a Fe_3O_4 -water-based nanofluid flow over a nonlinear stretching surface with velocity and thermal wall slip under variations in the shape factor

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Abstract

Purpose – The shape factor of nanoparticles is a parameter of interest in the variation of the thermophysical properties of nanofluids, and it affects their fluid flow and temperature distribution. Hence, this study aims to focus on analysing the influence of the shape factor on the convective heat and mass transfer over a nonlinear stretching sheet under the influence of magnetohydrodynamics.

Design/methodology/approach – By using similarity transformations, the governing system of partial differential equations was simplified to a nonlinear ordinary differential equation system, which was solved numerically using an explicit finite difference method (Keller box method). The behaviour of the fluid velocity and thermal profile at the boundary, as a result of slip conditions, is studied through a comprehensive parameter exploration.

Findings – The behaviour of the fluid velocity and thermal profile at the boundary, as a result of slip conditions, is studied by a very extensive parameter exploration. The major findings reveal that increasing values of magnetic parameters promote Lorentz's force, which slows down the velocity profile. Increasing the stretching rate parameter causes deformation reducing both the velocity and



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temperature profile. It is found that the temperature rise for elevated values of a thermophoretic parameter is greater for brick-shaped nanoparticles. It is also observed that the heat transfer (Nusselt number) for augmented values of Eckert number is lower in brick-shaped nanoparticles compared to platelet-shaped nanoparticles.

Research limitations/implications – The main limitation of this work has been the two-dimensional nature of the modelling, the analysis of a certain range of parameters that may not suit all the reader interests and the assumption of specific functions for the stretched sheet velocity and temperature, as well as the magnetic field. Also, regarding the nanofluid characteristics, it has been considered as single-phase, with Fe₃O₄ particles only and Newtonian.

Originality/value – This manuscript analyzes mathematically important aspects of the behaviour of a nanofluid with nanoparticles in magnetohydrodynamics of a non-linearly stretched sheet. This work is a detailed parametric exploration (magnetic parameter, stretching parameter, slip parameters, Brownian motion parameter, thermophoretic parameter, Eckert number, Lewis number and solid volume fraction) of the behaviour of two different nanoparticle shapes (brick and platelet), which sheds light on relevant aspects such as, skin friction, heat transfer and mass transfer. These are valuable results for the scientific community for either perform their numerical analysis upon these results and methodology, or perform experimental prototyping according to the behaviour described in this manuscript.

Keywords Nanofluid, FDM, MHD, Shape factor, Slip parameter, Nonlinear stretching sheet

Paper type Research paper

Nomenclature

D_T	= Brownian diffusion coefficient;
Nb	= Brownian parameter;
(x, y)	= Cartesian coordinates (m);
C	= Concentration ($mol\,m^{-3}$);
C_w	= Concentration at wall ($mol\,m^{-3}$);
C_∞	= Concentration at far-field ($mol\,m^{-3}$);
c	= Constant parameter;
A_1, A_2, A_3	= Constants;
Ec	= Eckert number;
Le	= Lewis number;
$B(x)$	= Magnetic field strength (Am^{-1});
M	= Magnetic parameter;
m	= Stretching parameter;
Pr	= Prandtl number;
Nu_x	= Nusselt number;
Re	= Reynolds number;
Sh_x	= Sherwood number;
Cf_x	= Skin friction coefficient;
C_p	= Specific heat ($J\,kg^{-1}\,K^{-1}$);
T	= Temperature (K);
T_w	= Wall temperature (K);
T_∞	= Ambient temperature (K);
k	= Thermal conductivity ($kg\,ms^{-3}\,K^{-1}$);
D_B	= Thermophoresis diffusion coefficient;
Nt	= Thermophoretic parameter;
$U_w(x)$	= Wall velocity (ms^{-1});
(u, w)	= velocity components (ms^{-1}); and
n	= Shape factor parameter.

Greek symbols

η = Dimensionless similarity variable;
 ρ = Density (kgm^{-3});
 σ = Electrical conductivity (Sm^{-1});
 φ = Nanoparticle volume fraction;
 α = Thermal diffusivity (m^2s^{-1});
 ν = Kinematic viscosity (m^2s^{-1});
 (λ, δ) = Slip parameters;
 τ = Effective heat capacity ratio; and
 μ = Dynamic viscosity ($kgm^{-1}s^{-1}$).

Abbreviations

ODEs = Ordinary Differential Equations;
PDEs = Partial Differential Equations;
FDM = Finite Difference Method; and
MHD = Magnetohydrodynamics.

Subscripts

nf = nanofluid; and
 f = base fluid.

1. Introduction

Researchers have vastly investigated the literature on passive techniques for boosting heat transfer rate in natural convection applications and equipment such as shock absorbers, electronic gadgets, vehicular brakes, and so on (Ghazanfari *et al.*, 2024). One of the most efficient passive techniques is to replace conventional fluids with liquids with nanoparticles. Nanofluids are an amalgamation of conventional fluid and nanometric metallic particles that enhance the thermophysical capabilities of the base fluid. The base fluid is usually water, ethylene glycol, oil, biofluids, or polymer solutions. Abbas *et al.* (2024) explored the Marangoni flow of dusty second-grade fluid with magnetohydrodynamics (MHD) in the presence of molybdenum disulphide nanoparticles. Choi and Eastman (1995) did pioneer work on the problem of heat transport through dilute suspensions of solid particles, which garnered renewed attention as it claimed that there was an improvement in the thermal conductivity of suspensions when including nanoparticles. Similarly, Yu *et al.* (2010) discovered a significant enhancement in thermal conductivity in metal nanoparticle-containing nanofluids, although the impact of particle size was not specifically explored in these studies. Shehzad *et al.* (Shehzad *et al.*, 2023) laid down a comparison of numerical and regression analysis for studying the impacts of activation energy on MHD nanofluid flow past an extended surface. The study found that the inclination angle had no influence on heat transfer rate.

However, over the past few decades, challenges with hydromagnetic nanofluid have gained industrial interest in polymer technology and manufacturing processes. In MHD, the induced electric current creates a mechanical force that affects the velocity of the fluid. Thus, it is of strong relevance to study fluid flows with electrical conductivity in a transverse magnetic field in detail by using appropriate mathematical or experimental tools. As a result, many studies on MHD flow and heat transfer with nanofluids have been carried out in recent decades. Mahabaleshwar *et al.* (2024a) studied the effects of MHD on a micropolar ternary nanofluid flow across a stretched surface with velocity slip. The results demonstrate that including fluid nanoparticles increases the heat transmission rate. Siddiq *et al.* (2024) examined the effect of a horizontal magnetic field on viscoelastic fluid flow on a spinning disc through a

Darcy–Forchheimer medium. Under the influence of inclined MHD, [Mahabaleshwar et al. \(2024b\)](#) investigated the micropolar nanofluid's linear stability past the stretching surface. It is perceived that as the strength of the magnetic field increases, fluid velocity decreases and fluid temperature increases. [Manna et al. \(2024\)](#) studied the impacts of enclosure shape and irreversibility on nanofluid flow in different-shaped systems under fluid volume constraint in the presence of MHD. Also, in [Kumar et al. \(2024\)](#), the response surface methodology was used to successfully perform an optimization study of the minimization of drag on a Casson–Carreau nanofluid driven by a magnetic dipole enabled curved sheet, which also paved the way for heat transfer optimization following a similar approach.

Fluid flows on stretched surfaces are found in many engineering applications across sectors such as wire drawing, glass fibre production, extrusion, hot rolling or melt-spring, amongst many others. [Sakiadas \(1961\)](#) was the pioneer to conduct an investigation on linear stretching sheet. [Crane \(1970\)](#) revolutionized the theory of boundary layer flow on a stretching surface. Upon this research, many studies later looked at numerous aspects of the potential issues with stretching surfaces. [Alkinidri et al. \(2024\)](#) investigated the effects of Darcy's law and Newtonian heating on MHD micropolar fluid flow across an expandable disc. The study reveals that thermal flow is also affected by temperature-dependent thermal conductivity. [Adel et al. \(2024\)](#) used the Adomian decomposition method to obtain a solution for convective nanofluid flow past a stretching surface with MHD, Thermophoresis and Brownian motion effects.

A relevant specific parameter in the mathematical modelling of MHD is the slippiness of the wall-type boundary conditions. The slip wall condition is used when there are minor viscous effects on the wall, hence the boundary layer is not developed. The impact of slip condition is especially relevant in various fluids that display wall slides, such as when softening valves of artificial hearts and their inner cavities in technological applications. [Beavers and Joseph \(1967\)](#) were the first to suggest a boundary layer with slip conditions. [Patel et al. \(2024\)](#) studied the effects of velocity and thermal slip on MHD hybrid nanofluid across exponentially stretching/shrinking surface combining radiation and joule heating. According to the findings, the rate of heat transmission reduced as thermal slip elevated. Also, by using a rotating disc, [Hayat et al. \(2017\)](#) studied slip effects over MHD nanofluid flow, confirming the effect of the slip condition on the velocity profiles in their investigation. [Ragupathi et al. \(2021\)](#) examined the impacts of second-order slip and thermal jump over MHD nanofluid flow over a stretching sheet with second-order fluid.

The shape effect of nanoparticles on heat and mass transfer is essential since it influences how the shape and size of nanoparticles interact with the fluid medium impacting rheological attributes of the nanofluid, thermal conductivity, convective heat transfer, and mass transfer rates. Recently, [Rashid et al. \(2020\)](#) investigated the effect of shape factor on MHD nanofluid flow past a stretched sheet with entropy generation. It is observed that, in comparison to sphere-shaped nanoparticles, laminar-shaped particles have improved heat transfer. [Berrehal et al. \(2022\)](#) explored the particle shape effect on nanofluid flow past a stretching surface in the presence of MHD and heat source/sink. Results from the study reveal that disc-shaped particles display enhancement in heat transfer whereas cylindrical-shaped particles exhibit greater entropy generation. Studies showing the impact of shape effect on nanofluid flows are addressed by [Lisha et al. \(2024\)](#) and [Kumar et al. \(2023\)](#). For a comprehensive summary, [Table 1](#) shows the comparison of the novel work of the present study with respect to pre-existing literature.

According to the aforementioned literature review, there is a very few studies on nanofluid flow with shape effects. Actually, there are no studies including a combination of

Table 1. Effects considered in the present investigation over previous related literature

Work	Sharma <i>et al.</i> (2023)	Akgül and Kaya (2023)	Patel <i>et al.</i> (2024)	Ramya <i>et al.</i> (2018)	Present study
MHD	✓	×	✓	✓	✓
Velocity slip	×	×	✓	✓	✓
Thermal jump	×	×	✓	×	✓
Thermophoresis and Brownian motion	✓	×	×	✓	✓
Shape factor	×	✓	×	×	✓
Stretching sheet	✓	×	×	✓	✓
<i>Fluid type</i>					
Mono	×	✓	×	×	✓
Hybrid	×	×	×	×	×
Ternary	×	×	✓	×	×
<i>Methodology</i>					
Keller box	×	×	×	✓	✓

Source(s): Authors' own work

MHD with velocity slip, thermophoresis, shape factor and a stretching sheet, despite the usefulness of this combination to represent actual industrial situations. Therefore, this work investigates the influence of the shape factor on MHD nanofluid flow across a nonlinear stretching sheet coupled with all the abovementioned effects. The Keller box approach (explicit finite difference method [FDM]) is used to yield numerical solutions. This study provides a comparative significant insight into the influence of many different parameter variations on the behaviour of the nanofluid flow. These results provide valuable information for maximizing nanofluid performance and stability in a variety of heat transfer applications, such as heat transfer enhancement, energy transmission and manufacturing processes.

2. Mathematical section

A steady two-dimensional incompressible viscous laminar flow (with water as a base fluid and Fe_2O_3 as a nanoparticle of two different shape factors) over a nonlinear stretching surface is considered here, as illustrated in Figure 1. $U_w(x) = ax^m$ is a velocity at the wall along the x -axis, where m is the stretching parameter. Furthermore, the magnetic effect and solid volume fraction are considered in the modelling. The stretched sheet surface is kept at $T_w(x) = T_\infty(x) + b(x/l)^{2m}$ temperature, where ($b > 0$) and $T_\infty(x)$ is the ambient temperature, and (C_w, C_∞) nanoparticle concentration at the sheet and far-field from the sheet, respectively. Thus, the governing equations are as follows (see Sharma *et al.*, 2023; Akgül and Kaya, 2023; Patel *et al.*, 2024):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu_{nf} \left(\frac{\partial^2 u}{\partial y^2} \right) - \frac{\sigma_{nf}}{\rho_{nf}} B^2 u, \quad (2)$$

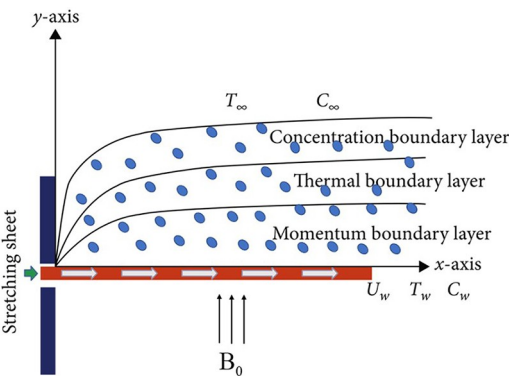


Figure 1. Physical model and coordinate system
Source: Authors' own work

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \left(\frac{\partial^2 T}{\partial y^2} \right) + \frac{\nu_{nf}}{C_p} \left(\frac{\partial u}{\partial y} \right)^2 + \tau \left\{ \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 + D_B \frac{\partial T}{\partial y} \frac{\partial C}{\partial y} \right\}, \tag{3}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \left(\frac{\partial T}{\partial y} \right)^2 \frac{D_T}{T_\infty}. \tag{4}$$

The boundary conditions imposed are:

$$\text{at } y = 0: u - U_w = P \nu_{nf} \left(\frac{\partial u}{\partial y} \right), \quad v = 0, \quad T = T_w + E \left(\frac{\partial T}{\partial y} \right), \quad C = C_w, \tag{5}$$

Table 2. Thermophysical properties of nanofluid

Properties	Nanofluid
Viscosity	$\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}$
Density	$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s$
Thermal conductivity	$\frac{k_{nf}}{k_f} = \frac{k_s + (n - 1)k_f - (n - 1)\phi(k_f - k_s)}{k_s + (n - 1)k_f + (n - 1)\phi(k_f - k_s)}$
Electrical conductivity	$\frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{3 \left(\frac{\sigma_s}{\sigma_f} - 1 \right) \phi}{\left(\frac{\sigma_s}{\sigma_f} - 2 \right) - \phi \left(\frac{\sigma_s}{\sigma_f} - 1 \right)}$
Heat capacity	$(\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s$
Source(s): Table by authors	

$$\text{as } y \rightarrow \infty: u \rightarrow 0, v \rightarrow 0, T = T_{\infty}, C = C_{\infty}. \quad (6)$$

Here, $P = P_1 x^{\left(\frac{-m+1}{2}\right)}$ and $E = E_1 x^{\left(\frac{-m+1}{2}\right)}$ are the velocity and thermal slip conditions that vary with x , P_1 and E_1 are the initial velocity and thermal slip factor. A value of $P = E = 0$ indicates the non-slip condition. These boundary conditions control how the fluid interacts with the surface. When the velocity slip increases, the fluid velocity decreases near the wall due to the uneven surface. On the other hand, the thermal jump condition consists of a constant wall temperature plus a component proportional to the local temperature gradient. This effect occurs when the fluid temperature at the surface differs from the wall temperature due to thermal resistance at the interface, as well as an inadequate thermal contact between the fluid and the solid wall.

The following similarity variables were incorporated into the governing equations above, and the thermophysical properties of the nanofluid is given in [Table 2](#):

$$\left. \begin{aligned} \frac{u}{ax^m} &= f'(\eta), \quad v = - \left(\frac{(1+m)av_f}{2} \right)^{1/2} {}^{(m)l}\sqrt{x} \left[f(\eta) - \eta \left(\frac{m-1}{1+m} \right) f'(\eta) \right], \\ \eta &= y \sqrt{a(1+m)/2\nu_f} {}^{(m)l}\sqrt{x}, \\ \frac{T - T_{\infty}}{b(x/l)^{2m}} &= \theta(\eta), \\ \varphi(\eta) &= (C - C_{\infty}) / (C_w - C_{\infty}) \end{aligned} \right\} \quad (7)$$

with the aid of [equation \(7\)](#), the [equations \(1\)–\(4\)](#) are transformed into:

$$\frac{f'''}{A_2 A_1} + f f'' - \left(\frac{2m}{m+1} \right) f'^2 - \frac{A_4}{A_2} M f' = 0, \quad (8)$$

$$\frac{k_{nf}/k_f}{Pr A_3} \theta'' - \left(\frac{4m}{m+1} \right) f' \theta + f \theta' + Ec f''^2 + Nt \theta'^2 + Nb \varphi' \theta' = 0, \quad (9)$$

$$\varphi'' + \left(\frac{Nt}{Nb} \right) \theta'' + Le Pr f \varphi' = 0, \quad (10)$$

and the boundary conditions in [equations \(5\)](#) and [\(6\)](#) now becomes:

$$\begin{aligned} \text{as } \eta = 0: f'(0) &= 1 + \frac{\lambda}{A_1 A_2} f''(0), f(0) = 0, \varphi(0) = 1, \theta(0) = 1 + \delta \theta'(0), \\ \text{as } \eta \rightarrow \infty: f'(\infty) &= 0, \varphi(\infty) = 0, \theta(\infty) = 0. \end{aligned} \quad (11)$$

These equations have been also simplified by including some parameters. The new terms appearing in [equations \(8\)–\(11\)](#) are listed in the following:



Sl.no.	Parameter	Description
01	$Nb = \frac{(\rho C_p) \tau D_B (C_w - C_\infty)}{(\rho C_p) \nu_f}$	Brownian parameter
02	$A_1 = \frac{1}{(1 - \phi)^{2.5}}$	Constant
03	$A_2 = (1 - \phi) + \phi \frac{\rho_s}{\rho_f}$	Constant
04	$A_3 = (1 - \phi) + \phi \frac{(\rho C_p)_s}{(\rho C_p)_f}$	Constant
05	$A_4 = 1 + \frac{3 \left(\frac{\sigma_s}{\sigma_f} - 1 \right) \phi}{\left(\frac{\sigma_s}{\sigma_f} + 2 \right) - \left(\frac{\sigma_s}{\sigma_f} - 1 \right) \phi}$	Constant
06	$Ec = \frac{U_w^2}{C_p (T_w - T_\infty)}$	Eckert number
07	$Le = \frac{\alpha_f}{D_B}$	Lewis number
08	$M = \frac{2 \sigma_f B_0^2}{a \rho_f (m + 1)}$	Magnetic parameter
09	$Pr = \frac{\nu_f (\rho C_p)_f}{k_f}$	Prandtl number
10	λ, δ	Slip parameters
11	$Nt = \frac{(\rho C_p) \tau D_T (T_w - T_\infty)}{(\rho C_p) T_\infty \nu_f}$	Thermophoretic parameter

2.1. Engineering coefficients

Due to the nature of the problem, there are several outputs of engineering interest in the simulation. These parameters are the skin-friction Cf_x , Nusselt number Nu_x and Sherwood number Sh_x , which are described below with their reduced form:

$$Cf_x = \frac{\mu_{nf}}{\rho_f U_w^2} \left(\frac{\partial u}{\partial y} \right)_{y=0}, \tag{12}$$

$$Nu_x = - \frac{x}{k_f (T_w - T_\infty)} k_{nf} \left(\frac{\partial T}{\partial y} \right)_{y=0}, \tag{13}$$

$$Sh_x = - \frac{x}{D_B (D_w - D_\infty)} D_B \left(\frac{\partial C}{\partial y} \right)_{y=0}, \tag{14}$$

which is transformed into:

$$\text{Re}^{1/2} Cf_x = A_1 \sqrt{\frac{m+1}{2}} f''(0),$$

$$\begin{aligned} \text{Re}^{-1/2}Nu_x &= -A_4\sqrt{\frac{m+1}{2}}\theta'(0), \text{ and} \\ \text{Re}^{-1/2}Sh_x &= -\sqrt{\frac{m+1}{2}}\varphi'(0) \end{aligned} \quad (15)$$

3. Numerical methodology

Many factors influence the technique selection, such as the kind of equations, boundary conditions, desired accuracy and available computing resources. The Keller box technique (see [Singh et al., 2024](#)) has benefits in terms of stability and accuracy for certain types of problems, while other approaches like Runge–Kutta ([Nihaal and Mahabaleshwar, 2024](#)), FDM ([Das and Ahmed, 2024](#)) and homotopy analysis method ([Dinarvand et al., 2016](#)) provide useful choices too. Because our experience with the Keller box technique for problems such as the present one is that the method can be efficiently implemented and provides trustworthy results, no other methods have been explored. However, this does not mean that they are not suitable, and future work could be oriented on a comparison of their performance.

[Equations \(8\)–\(10\)](#) are nonlinear, and it is not feasible to obtain closed-form solutions. As a result, these are numerically solved by using the Keller box technique, which is an explicit finite difference approach. The flow chart of the methodology is displayed in [Figure 2](#). The ordinary differential equations (ODE) plus the boundary conditions are to be solved with this method, which is not a straightforward approach.

The main steps of the Keller box algorithm to get a numerical solution are the following:

- It requires first converting the provided first-order ODE equations into a system.
- Then, by using the FDM, these equations are converted into finite differences terms.
- In this new scenario, the Newton's method can be applied for the linearization of the algebraic equations and their transformation into a vectorial form.
- The last step requires employing block tridiagonal elimination to solve, and the linear system solved. The procedure will be explained in detail in the next subsection.

3.1 Finite difference method

To solve the transformed differential [equations \(8\)–\(10\)](#) constrained to the boundary condition (11), these are initially transformed into system of seven first order equations by following. Due to this, those equations are now represented by means of central differences. To this aim, new parameters $p(\eta)$, $q(\eta)$, $\theta(\eta)$, $t(\eta)$, $\varphi(\eta)$ and $v(\eta)$ were introduced. After this process, [equations \(8\)–\(10\)](#) are revisited as:

$$p(\eta) \leftrightarrow f'(\eta), \quad (16)$$

$$q(\eta) \leftrightarrow p'(\eta), \quad (17)$$

$$t(\eta) \leftrightarrow \theta'(\eta), \quad (18)$$

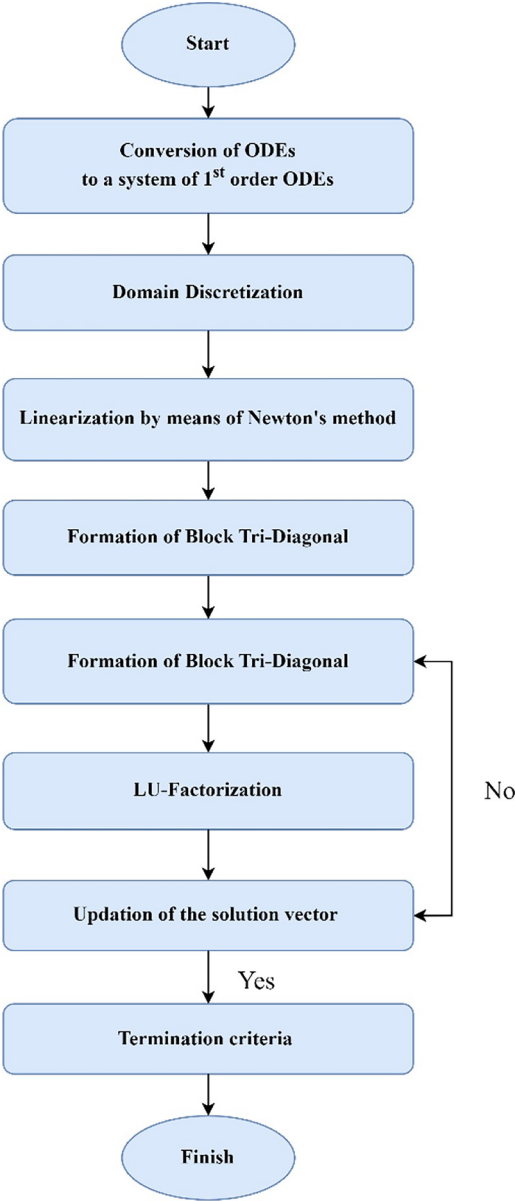


Figure 2. Flow chart of the Keller box method
Source: Authors' own work

$$v(\eta) \leftrightarrow \varphi'(\eta), \quad (19)$$

and thus,

$$\frac{q'}{A_1 A_2} + qf - p^2 \left(\frac{2m}{m+1} \right) - \frac{A_4}{A_2} Mp = 0, \quad (20)$$

$$\frac{k_{nf}}{k_f A_3 Pr} t' + ft - \left(\frac{4m}{m+1} \right) p\theta + Ecq^2 + Nb(tv) + Nt(t^2) = 0, \quad (21)$$

$$v' + Le Pr f v + \left(\frac{Nt}{Nb} \right) t' = 0. \quad (22)$$

with the new boundary conditions given by:

$$p(0) - 1 = \frac{\lambda}{A_1 A_2} q(0), f(0) = 0, \varphi(0) - 1 = 0, \theta(0) - 1 = \delta t(0), \quad (23)$$

$$p(\infty) \rightarrow 0, \theta(\infty) \rightarrow 0, \varphi(\infty) \rightarrow 0. \quad (24)$$

Here, the segment $\eta_{i-1}\eta_i$ along $\eta_{j-1/2}$ midpoint is considered:

$$\eta_0 = 0, \eta_i - h_i = \eta_{i-1}, \eta_i = \eta_\infty, \quad (25)$$

where, h_i is $\Delta\eta$ -space.

The following given equations are approximations of finite difference (Sakiadas, 1961; Crane, 1970; Alkinidri *et al.*, 2024; Adel *et al.*, 2024) for $\eta_{i-1/2}$ midpoint:

$$\frac{f_i - f_{i-1}}{h_i} = \frac{1}{2} (p_i + p_{i-1}), \quad (26)$$

$$\frac{p_i - p_{i-1}}{h_i} = \frac{1}{2} (q_i + q_{i-1}), \quad (27)$$

$$\frac{\theta_i - \theta_{i-1}}{h_i} = \frac{1}{2} (t_i + t_{i-1}), \quad (28)$$

$$\frac{\varphi_i - \varphi_{i-1}}{h_i} = \frac{1}{2} (v_i + v_{i-1}), \quad (29)$$

$$\frac{q_i - q_{i-1}}{A_1 A_2} - h_i (fq)_{i-1/2} - (h_i) \frac{2m}{1+m} \left(p_{i-1/2}^2 \right) - \frac{A_4}{A_2} h_i Mp_{i-1/2} = 0, \quad (30)$$

$$\begin{aligned} & (t_i - t_{i-1}) \frac{k_{nf}}{Pr k_f A_3} + h_i (tf)_{i-1/2} - h_i \left(\frac{4m}{m+1} \right) (\theta p)_{i-1/2} \\ & + Nb h_i (tv)_{i-1/2} + h_i Nt (t_{i-1/2}^2) + h_i Ec (q_{i-1/2}^2) = 0 \end{aligned} \quad (31)$$

$$(v_i - v_{i-1}) + (fv)_{i-1/2} h_i \text{Pr} Le + (t_i - t_{i-1}) \frac{Nt}{Nb} = 0. \quad (32)$$

with the boundary conditions:

$$\left. \begin{aligned} f_0 = 0, \quad p_0 = 1 + \frac{\lambda}{A_1 A_2} q(0), \quad \theta_0 = 1 + \delta t(0), \\ \varphi_0 = 1, \quad p_i = 0, \quad \theta_i = 0, \quad \varphi_i = 0. \end{aligned} \right\} \quad (33)$$

As aforementioned, the nonlinear system (28–33) is linearized with the following expressions by means of the Newton's method:

$$\left. \begin{aligned} f_i^{(1+k)} - f_i^{(k)} &= \delta f_i^{(k)}, \quad q_i^{(1+k)} - q_i^{(k)} = \delta q_i^{(k)}, \\ p_i^{(1+k)} - p_i^{(k)} &= \delta p_i^{(k)}, \quad t_i^{(1+k)} - t_i^{(k)} = \delta t_i^{(k)}, \\ \theta_i^{(1+k)} - \theta_i^{(k)} &= \delta \theta_i^{(k)}, \quad v_i^{(1+k)} - v_i^{(k)} = \delta v_i^{(k)}, \\ \varphi_i^{(1+k)} - \varphi_i^{(k)} &= \delta \varphi_i^{(k)} \end{aligned} \right\} \quad (34)$$

where, $k = 0, 1, 2, 3, \dots$. By including the expressions given in [equation \(34\)](#) in the equations developed before, for the quantities δp^k , δq^k , $\delta \theta^k$, δt^k , $\delta \varphi^k$ and δv^k , the following linear system is obtained (we get rid of the superscript k for the sake of ease):

$$\frac{\delta f_i - \delta f_{i-1}}{h_i} - \frac{1}{2} (\delta p_{i-1} + \delta p_i) - (r_1)_{i-1/2} = 0, \quad (35)$$

$$\frac{\delta p_i - \delta p_{i-1}}{h_i} - \frac{1}{2} (\delta q_{i-1} + \delta q_i) - (r_2)_{i-1/2} = 0, \quad (36)$$

$$\frac{\delta \varphi_i - \delta \varphi_{i-1}}{h_i} - \frac{1}{2} (\delta v_{i-1} + \delta v_i) - (r_3)_{i-1/2} = 0, \quad (37)$$

$$\frac{\delta \theta_i - \delta \theta_{i-1}}{h_i} - \frac{1}{2} (\delta t_{i-1} + \delta t_i) - (r_4)_{i-1/2} = 0 \quad (38)$$

$$\begin{aligned} (x_1) \delta q_i + (x_2) \delta q_{i-1} + (x_3) \delta f_i + (x_4) \delta f_{i-1} + (x_5) \delta p_i \\ + (x_6) \delta p_{i-1} - (r_5)_{i-1/2} = 0 \end{aligned} \quad (39)$$

$$\begin{aligned} (y_1) \delta t_i + (y_2) \delta t_{i-1} + (y_3) \delta f_i + (y_4) \delta f_{i-1} + (y_5) \delta p_i + (y_6) \delta p_{i-1} + (y_7) \delta \theta_i \\ + (y_8) \delta \theta_{i-1} + (y_9) \delta v_i + (y_{10}) \delta v_{i-1} + (y_{11}) \delta q_i + (y_{12}) \delta q_{i-1} - (r_6)_{i-1/2} = 0 \end{aligned} \quad (40)$$

$$(z_1) \delta v_i + (z_2) \delta v_{i-1} + (z_3) \delta f_i + (z_4) \delta f_{i-1} + (z_5) \delta t_i + (z_6) \delta t_{i-1} - (r_7)_{i-1/2} = 0, \quad (41)$$

where

$$\begin{aligned}
 (x_1)_i &= \frac{1}{A_1 A_2} + \left(\frac{1}{2}\right) h_i f_{i-1/2} & (x_2)_i &= (x_1)_i - \frac{2}{A_1 A_2} \\
 (x_3)_i &= \left(\frac{1}{2}\right) h_i q_{i-1/2} & (x_4)_i &= (x_3)_i \\
 (x_5)_i &= \frac{-2m}{m+1} h_i p_{i-1/2} - \frac{A_4 M}{A_2 2} h_i & (x_6)_i &= (x_5)_i \\
 (y_1)_i &= \frac{1}{\text{Pr}} \left(\frac{k_{nf}}{A_3}\right) + \left(\frac{1}{2}\right) h_i f_{i-1/2} + Nb \left(\frac{h_i}{2}\right) v_{i-1/2} + Nt h_i t_{i-1/2} & (y_2)_i &= (y_1)_i - \frac{2}{\text{Pr}} \left(\frac{k_{nf}}{A_3}\right) \\
 (y_3)_i &= \left(\frac{1}{2}\right) h_i t_{i-1/2} & (y_4)_i &= (y_3)_i \\
 (y_5)_i &= -\left(\frac{1}{2}\right) h_i \left(\frac{4m}{m+1}\right) p_{i-1/2} & (y_6)_i &= (y_5)_i \\
 (y_7)_i &= -\frac{h_i}{2} \left(\frac{4m}{m+1}\right) \theta_{i-1/2} & (y_8)_i &= (y_7)_i \\
 (y_9)_i &= \frac{h_i}{2} t_{i-1/2} Nb & (y_{10})_i &= (y_9)_i \\
 (y_{11})_i &= E c h_i q_{i-1/2} & (y_{12})_i &= (y_{11})_i \\
 (z_1)_i &= 1 + \frac{h_i}{2} L e f_{i-1/2} & (z_2)_i &= (z_1)_i - 2 \\
 (z_3)_i &= \text{Pr} L e v_{i-1/2} \frac{h_i}{2} & (z_4)_i &= (z_3)_i \\
 (z_5)_i &= \frac{Nt}{Nb} & (z_6)_i &= -(z_5)_i \\
 (r_1)_i &= (f_{i-1} - f_i) + h_i p_{i-1/2} & (r_2)_i &= (p_{i-1} - p_i) + h_i q_{i-1/2} \\
 (r_3)_i &= (\theta_{i-1} - \theta_i) + h_i (t_{i-1/2}) & (r_4)_i &= (\varphi_{i-1} - \varphi_i) + h_i v_{i-1/2} \\
 (r_5)_i &= (q_{i-1} - q_i) - h_i (f q)_{i-1/2} + \frac{2m}{m+1} h_i p_{i-1/2}^2 - \frac{A_4}{A_2} h_i M p_{i-1/2} \\
 (r_6)_i &= (t_{i-1} - t_i) - \text{Pr} h_i \left(Nb (v t)_{i-1/2} + Nt t_{i-1/2}^2 + E c q_{i-1/2}^2 - \frac{4m}{m+1} (\theta p)_{i-1/2} + (f t)_{i-1/2} \right) \\
 (r_7)_i &= (v_{i-1} - v_i) - L e \text{Pr} h_i (f v)_{i-1/2} - \frac{Nt}{Nb} (t_i - t_{i-1})
 \end{aligned}$$

The boundary conditions become:

$$\begin{aligned}
 \delta f_0 &= 0, \quad \delta p_0 = 0, \quad \delta q_0 = 0, \quad \delta \theta_0 = 0, \quad \delta \varphi_0 = 0, \\
 \delta p_i &= 0, \quad \delta \theta_i = 0, \quad \delta \varphi_i = 0,
 \end{aligned} \tag{42}$$

The elimination method is used to solve the obtained linearized difference equations (39)–(42). For computational reasons, the physical parameter values are set to $\text{Pr} = 6.3$,

$Ec = 0.1$, $Le = 0.8$, $M = 0.5$, $Nb = 0.1$, $Nt = 0.1$ and $\varphi = 0.01$. In addition, the thermophysical properties of both water and nanoparticles are provided in Table 3. The behaviour of velocity, temperature and concentration profiles for significant parameters is investigated in the following section for two particles with distinct shape factors, i.e. brick with 3.7 and platelet with 5.7 shape factors.

4. Results and discussion

This section addresses the behaviour of the nanofluid velocity f' , temperature θ and concentration φ profiles for different parameters. The studied ranges for the dimensionless parameters that define the problem under investigation have been chosen as representative intervals such as: $0.1 \leq M \leq 1.0$, $1 \leq m \leq 2$, $0.1 \leq \lambda \leq 0.5$, $0.1 \leq \delta \leq 0.5$, $0.1 \leq Ec \leq 1.0$, $0.1 \leq Nb \leq 1.0$, $0.1 \leq Nt \leq 1.0$ and $0.8 \leq Le \leq 1.5$.

Because validation is also very important to trust the results from this investigation, the numerical results of the engineering quantities of interest of the present study are first validated with results by Ramya *et al.* (2018), as shown in Table 4. The outstanding match with their data confirms the accuracy of our computation.

The first property to be analysed in this work is how increasing the magnetic field affects the nanofluid flow performance, which has strong relevance in practice for adapting control algorithms to modify the flow behaviour. Figure 3 shows the variations in the velocity profile f' for varying the magnetic field M . It shows that the increase in M slows down the fluid velocity. From a physical point of view, increasing the value of M increases the Lorentz force, which induces resistance among fluid particles, thereby reducing fluid velocity. This provides an easy way to control the fluid velocity by controlling the magnetic field. As seen in the figure, velocity is quite sensitive to the changes in the magnetic field, especially at low values of η . At high values of η the impact of variations in M is negligible, so if the aim is to control the flow velocity at these regions, the variation of the magnetic field might not be the most efficient way.

The impact of the non-linear stretching sheet parameter m on the f' and θ is depicted in Figures 4(a) and 4(b). From Figure 4(a), it is found that the fluid velocity decreases when increasing m . Physically, increasing the values of m deform the fluid notably, resulting in less mobility to the fluid particles. However, the decrease in the velocity profile is minimal for mid/high values of η . From the figure, it is perceived that the brick-shaped particles exhibit a more notable decrease than the platelet-shaped ones. This is because the rigid shape of brick particles significantly increases nanofluid viscosity, which amplifies shear stress and flow resistance as m grows. On the other hand, platelet-shaped particles align with the flow, decreasing blockage and keeping the viscosity low in consequence.

From Figure 4(b), it is observed that when m is increased, the temperature profile θ decreases. Physically, as the non-linear stretching sheet parameter increases, the shear rate rises, enhancing fluid motion and boosting heat transfer. This minimizes the thermal

Table 3. Thermophysical properties of water and nanoparticle [see Sheikhholeslami and Shehzad (2018)]

Nanoparticle and base fluid	Density (kgm^{-3})	Specific heat capacity ($Jkg^{-1}K^{-1}$)	Electric conductivity (Sm^{-1})	Thermal conductivity ($kgms^{-3}K^{-1}$)
H_2O	997.1	4179	0.05	0.613
Fe_3O_4	5180	670	25000	9.7

Source(s): Table by authors

Table 4. Values of $-f''(0)$, $-\theta'(0)$ and $-\varphi'(0)$ for a parametric validation with previous study by varying the magnetic field M , velocity parameter λ and thermal slip parameter δ

M	ζ_1	ζ_2	$\text{Ramya et al. (2018)} - f''(0)$	Present study $-f''(0)$	$\text{Ramya et al. (2018)} - \theta'(0)$	Present study $-\theta'(0)$	$\text{Ramya et al. (2018)} - \varphi'(0)$	Present study $-\varphi'(0)$
0.1	0.2	0.2	0.878734	0.878732	1.942967	1.942967	0.759485	0.759484
0.3	0.2	0.2	0.954825	0.954824	1.886203	1.886202	0.741617	0.741617
0.4	0.2	0.2	0.990667	0.990668	1.858135	1.858136	0.731665	0.731666
0.5	0.2	0.2	1.025242	1.025241	1.830267	1.830265	0.721117	0.721117
1.0	0.3	0.1	1.070591	1.070590	1.940784	1.940783	0.919926	0.919923
1.0	0.4	0.1	0.976827	0.976827	1.925530	1.925532	0.946561	0.946561
1.0	0.5	0.1	0.897480	0.897478	1.904185	1.904184	0.963265	0.963262
1.0	0.1	0.3	1.319597	1.319595	1.502959	1.502958	0.447819	0.447820
1.0	0.1	0.4	1.319597	1.319595	1.352161	1.352159	0.320172	0.320173
1.0	0.1	0.5	1.319597	1.319595	1.219905	1.219901	0.208106	0.208108
Source(s): Table by authors								

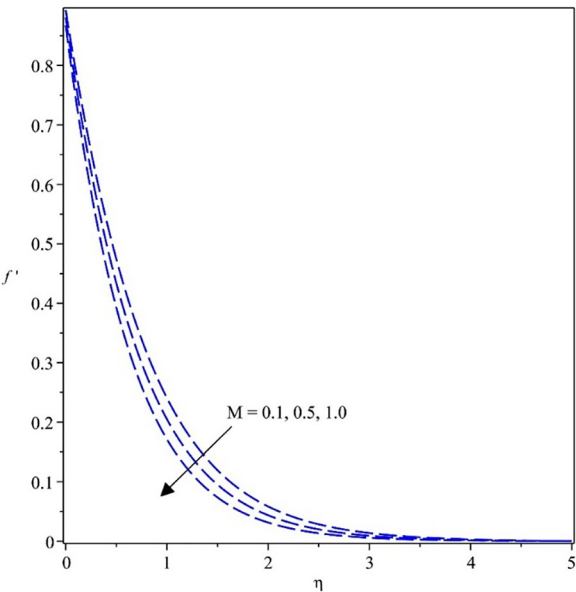


Figure 3. The influence of M on f'
Source: Authors' own work

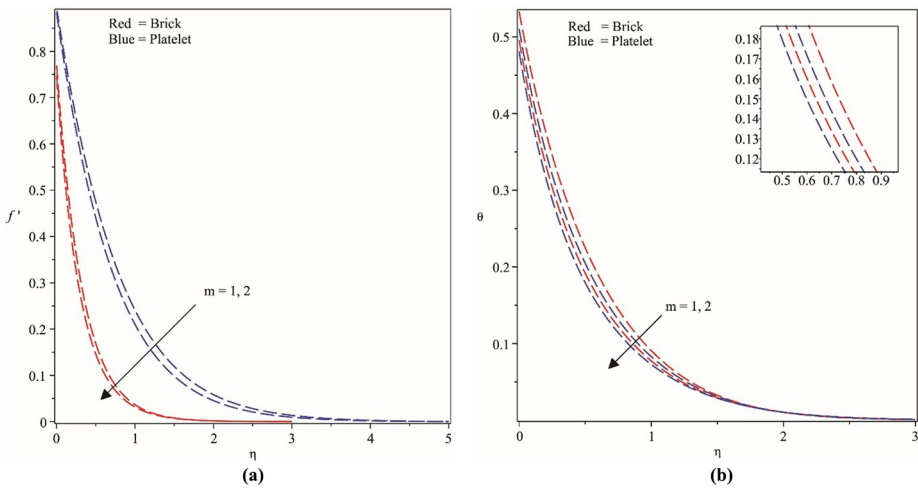


Figure 4. (a) The impact of m over f' , (b) Effect of m on θ
Source: Authors' own work

resistance and promotes heat dissipation, resulting in a reduction in the temperature profile. From the figure, it is found that platelet-shaped particles exhibit a greater reduction in the temperature profile θ . This is due to the higher surface area to volume ratio of platelet-shaped particles, which boosts heat transfer. Their superior thermal conductivity allows faster heat dissipation, which also reduces the thermal boundary layer thickness.

In Figure 5, the influence of the velocity slip parameter λ on the velocity profile f' is depicted. It is observed that as the slip parameter λ increases, the velocity of the stretched sheet will be different from the fluid velocity near the surface, in contrast to a non-slip condition. Physically, the increase in velocity slip parameter λ allows for a greater portion of the fluid to slip past the surface, which in turn leads to a lower average velocity. Furthermore, it is observed that platelet-shaped particles show more reduction compared to brick-shaped particles. This is because platelet-shaped particles tend to align with the flow, resulting in a greater surface area and higher viscous drag and flow resistance. This causes the velocity to decrease further.

In Figure 6 it is shown the impact of δ on the temperature profile θ . It is clear from the figure that θ elevates for augmented values of δ . The thermal slip parameter increases the effective thermal diffusivity, which leads to a rise in the temperature profile θ . Moreover, it is seen that brick-shaped particles show a more remarkable rise when compared to platelet-shaped particles. This is because brick-shaped particles increase thermal boundary layer thickness, allowing for increased heat transfer from the wall into the fluid. As a result, the temperature gradient is enhanced.

The impact of the Eckert number Ec on the temperature profile θ is analysed in Figure 7, where can be seen that there is an increase in θ as Ec increases.

The explanation behind this behaviour is that, as the Ec is increased, the heat dissipation in the boundary layer region improves notably, and hence, the temperature distribution is

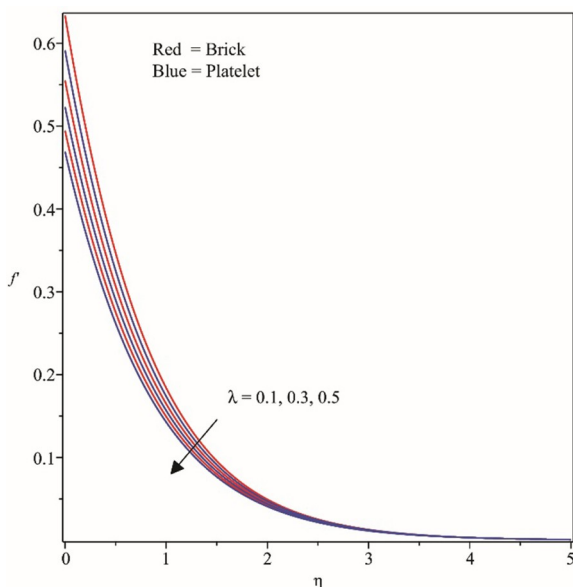


Figure 5. Variation in curves of f' for numerous values of λ
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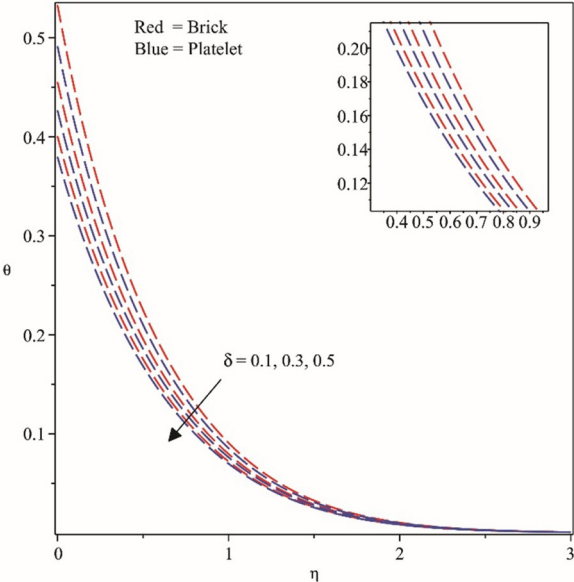


Figure 6. Variation in θ for numerous values of δ
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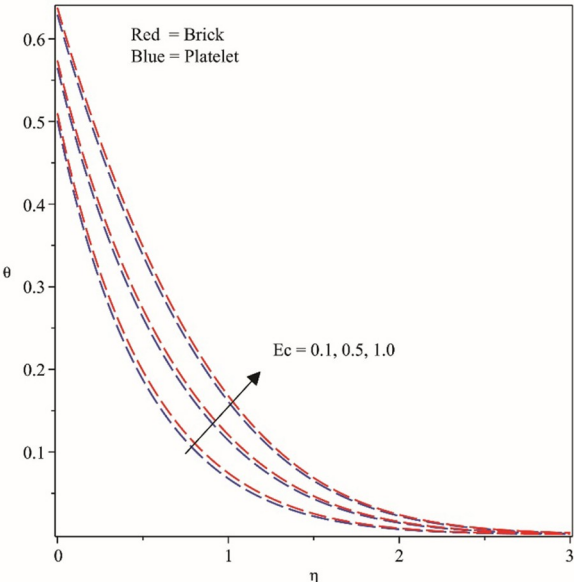


Figure 7. Effect of the Ec on the θ
Source: Authors' own work

improved. It is also observed in the figure that the rise in temperature is greater in brick-shaped particles than in platelet-shaped particles. This occurs because brick-shaped particles have a lower surface area to volume ratio, resulting in lower heat dissipation and more heat retention.

In Figure 8 it is displayed the impact of the Brownian motion parameter Nb over temperature profile θ . It is observed that θ increases for augmented values of Nb . The physical reasoning behind this effect is that, as the Brownian motion parameter is increased, the kinetic energy of the particles increases too, which results in more collisions and energy transfer between the particles and the base flow. This, in turn, leads to a rise in the temperature profile. Furthermore, it can be seen in the figure that the temperature rise is greater for brick-shaped particles than platelet-shaped particles. This is because brick-shaped particles have a smaller surface area to volume ratio, resulting in a lower heat transfer. In addition, a stronger Brownian motion creates more particle collisions, which enhance heat generation in brick-shaped particles.

Also, the effect of the thermophoretic parameter Nt on the temperature profile θ is plotted in Figure 9. The temperature gradient in the boundary region rises as the Nt is increased. As formal explanation, when the thermophoretic effect intensifies, the nanoparticles flow towards a hotter region of the fluid. This migration increases the nanoparticle concentration surrounding the hot surfaces, which boosts thermal energy in the system, rising the overall temperature of the fluid. Due to the smaller surface area to volume ratio of brick-shaped particles, the temperature rise is more remarkable for them compared to platelet-shaped particles.

The impact of the Lewis number on the concentration profile φ is depicted in Figure 10. In this figure can be observed a local peak in concentration at very low values of η . From a physical viewpoint, this is due to the fact that nanoparticles have limited motion near the surface (the stretched wall sheet represents a physical constrain), thus the slight

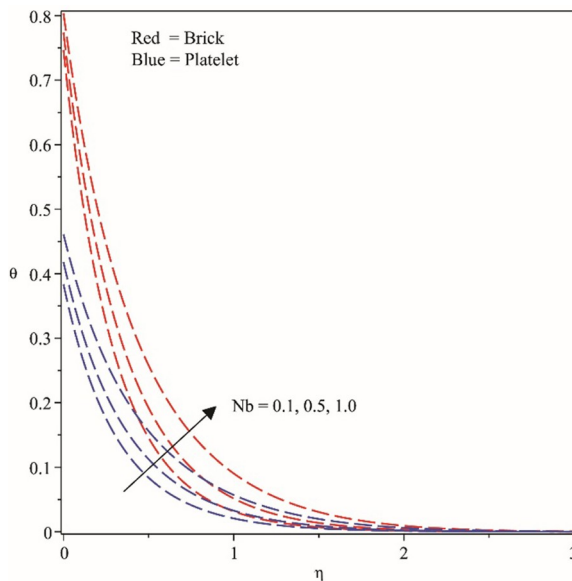


Figure 8. The effect of the Nb on the θ
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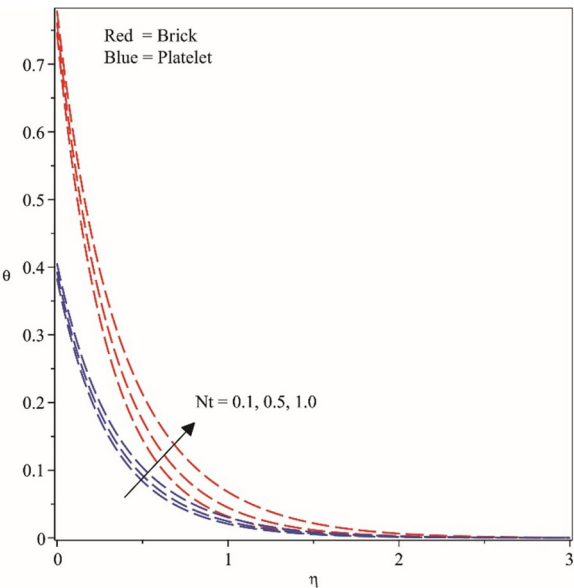


Figure 9. The effect of the Nt on the θ
Source: Authors' own work

concentration peak near the surface makes sense. The fact that φ is slightly over the unity at the peak means that local concentration (C) is slightly greater than the concentration exactly at the wall (C_w). In addition to this observation, the Lewis number indicates the ratio of thermal diffusivity to mass diffusivity. So as the Lewis number is increased, this means that the role of mass transfer is less significant in contrast to heat diffusion, thus the concentration profile is decreased substantially, as observed in Figure 10. Platelet-shaped particles have lower mass transfer resistance, which facilitates fast diffusion of species away from the surface, and hence platelet-shaped particles have a greater reduction in concentration compared to brick-shaped particles.

The behaviour of engineering quantities of interest (skin friction, Nusselt number and Sherwood number) upon variation of solid volume fraction, in conjunction with other input parameters is analysed in Figures 11–13. In Figure 11 it is shown the effect of the magnetic field M and solid volume fraction ϕ on the skin friction Cf_x . In this figure is observed that as M is increased, Cf_x decreases, but the curves are in an upward direction for an augmented value of ϕ . From a physical point of view, as the magnetic field parameter increases, skin friction reduces due to the Lorentz force. As the magnetic field intensifies, it promotes a resistive strain force against fluid motion, slowing down velocity near the surface. This momentum suppression minimizes the velocity gradient at the wall, resulting in a reduced shear stress and, consequently, decreasing skin friction. In contrast, increasing the solid volume fraction causes the curves to rise. This is because a higher concentration of solid particles rises the effective viscosity of the nanofluid, causing greater resistance to motion (more viscous motion). This increase in viscosity causes a greater velocity gradient near the surface, which increases shear stress, and thus, increases skin friction. In addition, brick-shaped particles exhibit a more pronounced deceleration effect compared to platelet-shaped

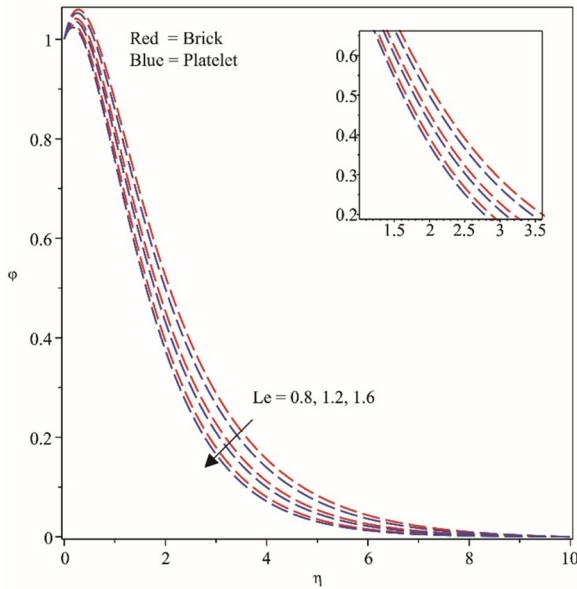


Figure 10. The effect of Le on the ϕ
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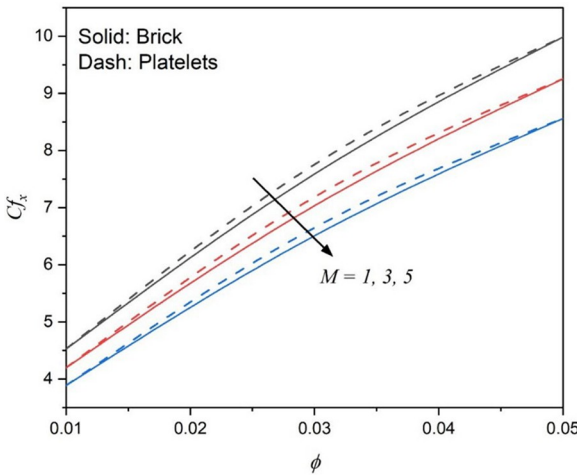


Figure 11. Behaviour of Cf_x for augmented values of ϕ and M
Source: Authors' own work

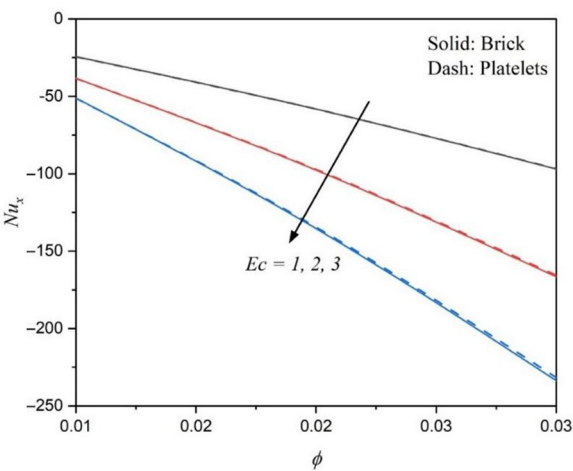


Figure 12. Behaviour of Nu_x for various values of Ec and ϕ
Source: Authors' own work

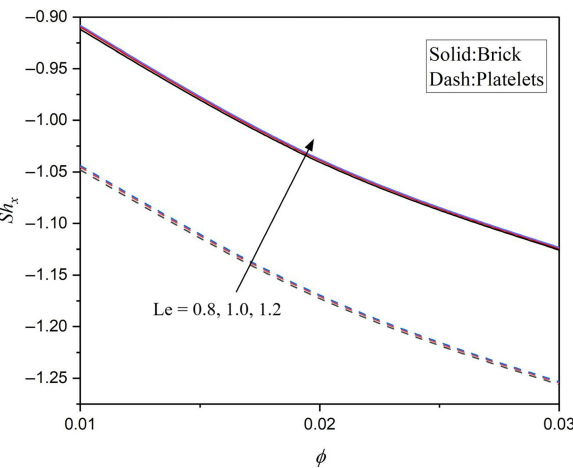


Figure 13. Behaviour of Sh_x for various value of Le
Source: Authors' own work

particles, due to their lower surface-to-volume ratio, resulting in weaker contact with the surrounding fluid. This causes platelet-shaped nanofluids to be more affected by the Lorentz force, which reduces velocity and velocity gradient at the surface, resulting in a larger reduction in shear stress, and thus, reducing skin friction.

The effect of Eckert number Ec and solid volume fraction ϕ on the Nusselt number Nu_x is displayed in Figure 12. Physically, the Nusselt number from equation (13) represents the ratio of convective to conductive heat transfer in a fluid. As the Ec rises, indicating that

viscous dissipation dominates, more energy is transformed into internal energy rather than increasing convective heat transmission. This leads to a decreased temperature gradient at the surface, which lowers the Nusselt number. Similarly, raising the solid volume fraction boosts thermal conductivity of the nanofluid, reducing the temperature difference between the solid and the fluid. This reduces convective effects, which results in a lower Nusselt number. Furthermore, the temperature drop is more significant in the brick shape than in the platelet shape particles. This is because of higher viscous dissipation and flow blockage, brick-shaped particles have a lower Nusselt number than platelets as the Eckert number and solid volume fraction increase. Also, bricks form enhanced frictional heating while thickening the thermal boundary layer, decreasing heat transfer. Their bulky form also disturbs fluid flow more than platelets, reducing the efficiency of convective heat transfer.

Figure 13 illustrates the behaviour of the Sherwood number Sh_x for varying values of Lewis number Le and solid volume fraction ϕ . With an increase in Le , the Sh_x is also increased. This is because an increase in the Le indicates that mass diffusion is lower than thermal diffusion, whilst mass diffusion also appears in the denominator of Sh_x . Thus, a low mass diffusion means a high Lewis and Sherwood number. The reduction in mass diffusivity slows down mass transfer rates, resulting in a decrease in concentration curves of Sh_x . It is found that mass transfer in brick-shaped particles is more relevant than in platelet-shaped particles. Physically, due to the larger surface area of brick-shaped particles, it offers slightly lower diffusion of solute particles, thus exhibiting lower mass transfer in contrast to platelet-shaped particles. Hence, higher values of Sh_x are seen in brick-shaped particles compared to the platelet-shaped particles.

5. Conclusions

This study provides insights into the impact of different shape factors (brick and platelets) on MHD nanofluid flow over a nonlinear stretching sheet with thermophoresis and Brownian motion along slip boundary conditions. A suitable similarity approach was used to transform the governing equations into ODEs. The Keller box method was used to obtain a numerical solution. The impact of magnetic, stretching rate, slip, thermophoretic, brownian motion, Eckert number over velocity, temperature and concentration profiles, as well as skin friction, Nusselt number and Sherwood number, are studied. The present results were also validated with previous literature, with an outstanding match. The main findings from this study are as follows:

- The velocity profile shows decreasing behaviour as the magnetic parameter is increased, due to the presence of augmented Lorentz's force, which induces resistance through a magnetic field among fluid particles, thereby reducing fluid velocity.
- By increasing the stretching rate parameter, the increased deformation leads to both velocity and temperature decrease. Physically, increasing the values of m deforms the fluid notably, resulting in less mobility to the fluid particles.
- When the velocity slip parameter is increased, the fluid velocity decreases. The increase in velocity slip parameter allows for a greater portion of the fluid to slip past the surface, which in turn leads to a lower average velocity. The inverse behaviour is observed in the temperature profile for the thermal slip parameter is increased, since the thermal slip parameter increases the effective thermal diffusivity.
- The temperature gradient increases as the thermophoretic parameter is increased, and the brick-shaped particles show a greater rise in the temperature profile. As the thermophoretic effect intensifies, the nanoparticles flow towards a hotter area of the fluid. This migration rises the nanoparticle concentration at the hotter surfaces.

- It has been observed that the heat transfer for brick-shaped particles is lower than for platelet-shaped particles when increasing the Eckert number.
- The reduction in mass diffusivity for higher values of the Lewis number slows down mass transfer rates, resulting in a decrease in concentration curves related to the Sherwood number. This is a coherent result by taking into account that both the Lewis and Sherwood number are inversely proportional to mass diffusion.

In terms of the impact of this investigation, this work shows a representative and detailed parametric exploration of the behaviour of two different nanoparticle shapes (brick and platelet) under a range of combined conditions not considered before in the literature, which sheds light on relevant aspects such as velocity profile, skin friction, heat transfer and mass transfer. These are valuable results for the scientific community for either performing their computational fluid dynamic analyses upon these results, or even performing experimental industrial prototyping by following the behaviour described in this manuscript.

5.1 Limitations and future scope

The main limitation of this work has been the two-dimensional nature of the modelling, the analysis of a certain range of parameters that may not suit all the reader interests and the assumption of specific functions for the stretched sheet velocity and temperature, as well as the magnetic field. Also, regarding the nanofluid characteristics, it has been considered as single-phase, with Fe_3O_4 particles and Newtonian. In addition, if some terms, e.g. those related to the velocity and thermal slip conditions, were modelled as more complex terms (for instance, with non-analytical piecewise term equations), then the manipulation of the equations might be much more complicated or even impossible by means of similarity variables. Thus, the selection of known boundary conditions is a limitation of this study.

Future research can be oriented to examine the behaviour of non-Newtonian fluids like viscoelastic, Casson and Reiner–Rivlin fluid models over different geometries including hybrid/ternary nanoparticles with external forces like effects of a magnetic dipole, thermal radiation, Joule heating and activation energy which will be considered in the future.

Data availability

The datasets used and/or analysed during the current study available upon reasonable request.

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