



Thermal transfer enhancement in a Casson-based hybrid nanofluid flow in a permeable wall jet with suction and injection

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Abstract

Wall jet flows are frequently used in cooling electronic equipment, in which a high-velocity fluid is released from a small opening along a flat plate. This kind of system improves heat transfer rates, making it useful for controlling thermal loads in tiny electrical systems. This research intends to provide a novel numerical solution to a non-Newtonian wall jet hybrid nanofluid flow with heat and mass transfer phenomena implementing a colloidal combination of *Ag* and *Cu* nanoparticles submerged in sodium alginate base fluid. The present study's novelty is that it provides comparative results on wall jet hybrid nanofluid flow past a permeable stretching surface under the effect of Brownian motion and Thermophoresis. The results are calculated in two flow cases: suction ($S > 0$) and injection ($S < 0$). Suitable similarities are used for constructing the non-dimensional model of the governing equations. The bvp-4c technique is used to compute the numerical solutions. The graphical representations reveal the consequences of various non-dimensional constraints on momentum, thermal, and concentration profiles. The novel findings reveal that increased porosity parameter reduces skin friction diminishing resistive force between fluid layers allowing for easy fluid flow. It is found that the reduction is more significant for suction case. Growing values of Thermophoresis parameter reduces Nusselt number due to thick boundary layers reducing convective heat transfer across the boundary and the drop is more in case of injection because the thicker boundary is formed by new fluid entering via injection. A rise in Sherwood number is observed for elevated values of the Thermophoresis parameter and the injection phenomenon offers enhanced mixing and uniform distribution of nanoparticles providing better mass transfer rates. The current research has significant implications for cooling electronic devices, turbine vanes, and improved thermal management systems.

Keywords Hybrid nanofluid · Casson fluid · Thermophoresis and Brownian motion · Suction/Injection · Bvp-4c · Wall jet flow

List of symbols

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Nb Brownian motion parameter.
 D_b Coefficient of Brownian motion (m^2s^{-1}),
 D_T Coefficient of Thermophoretic diffusion (m^2s^{-1})
 C Concentration of the fluid ($molm^{-3}$)
 C_w Concentration at wall ($molm^{-3}$)
 C_∞ Concentration at far field ($molm^{-3}$)
 (x, y) Coordinates (m)
 D_b Diffusivity (m^2s^{-1})

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Le	Lewis number
Nu	Nusselt number
Pr	Prandtl number
Cp	Specific heat capacity ($Jkg^{-1}K^{-1}$)
S	Suction/injection parameter
Cf	Skin friction
Re_x	Local Reynolds number
Sh	Sherwood number
T	Temperature of the fluid (K)
T_w	Temperature at wall (K)
T_∞	Temperature at far field (K)
Nt	Thermophoresis parameter.
k	Thermal conductivity ($kgms^{-3}K^{-1}$)
(u, v)	Velocity components (ms^{-1})
p_y	Yield stress component

Greek symbols

β	Casson parameter
μ	Dynamic viscosity (m^2s^{-1})
ρ	Density (kgm^{-3})
ν	Kinematic viscosity (m^2s^{-1})
$\tilde{\tau}$	Heat capacity ratio
α	Porosity parameter
π	Product of deformation rate component
ϕ	Solid volume fraction

Abbreviations

MHD	Magneto-Hydro-Dynamics.
HNF	Hybrid Nano Fluid.
PDEs	Partial Differential Equations.
ODEs	Ordinary Differential Equations.
bvp	Boundary value problem

Suffixes

f	Base fluid.
nf	Nanofluid.
hnf	Hybrid nanofluid.
ODEs	Ordinary Differential Equations
PDEs	Partial Differential Equations.
BVPs	Boundary Value Problems

1 Introduction

Hybrid nanofluids benefit transformer cooling, thermal energy systems, and the automobile sectors. The hybrid nanofluid's superior thermal capabilities translate to improved energy efficiency, more economical operating costs, and significantly superior execution. Thus, these characteristics drew the attention of many scholars. Heat and mass transfer analysis across various boundary layer flows comprising different hybrid nanoparticles is discussed in these articles (Ganga et al. 2023; Uddin et al. 2021; Kumar et al. 2018a; Upreti et al. 2024). Ganga et al. (2025) studied the consequences of MHD and non-linear thermal radiation over a Maxwell hybrid nanofluid flow past an exponentially expanding surface. According to the study results, hybrid nanofluids exhibit higher heat transfer rates than nanofluids. Kumar et al. (2023a) explored the influence of nanoparticle shape effects on hybrid nanofluid across a wedge. Results reveal that heat transfer is faster from blade-shaped particles than from spherical nanoparticles and brick-shaped particles than from sphere-shaped nanoparticles. Aly et al. (2022) studied the influence of mass transpiration and radiation on the hybrid nanofluid flow of a wall jet across a stretched surface. It was discovered that increasing the volume fraction of nanoparticles in a hybrid nanofluid raises the velocity profile in the suction case. The MHD wall jet flow across a stretched surface was numerically solved by Yaseen et al. (2023) adopting an Eyring-Powell hybrid nanofluid model. The study outcome reveals that when a magnetic field augments, the friction factor drops by over fifty percent.

Casson fluid's distinct rheological features, particularly viscoelasticity, provide enhanced heat transmission properties. Studies (Varatharaj et al. 2024; Mahabaleshwar et al. 2024a) have shown that factors such as radiation and magnetic field have a significant impact on heat and mass transference rates in Casson fluids, resulting in improved convective heat transfer in a variety of applications, such as cooling mechanisms and chemical reactions. With the inclusion of Ohmic heating and mixed convection, Shamshuddin et al. (2023) studied the non-Newtonian hybrid nanofluid flow past an exponentially extending surface. The results show that the velocity of mono/hybrid nanofluids rises with the augmented values of mixed convection. Mahabaleshwar et al. (2024b) investigated the impact of inclined MHD and heat source/sink on Casson hybrid nanofluid flow past a permeable stretching/shrinking surface. Alzahrani et al. (2022) investigated a laminar Glauert-type wall jet on a radioactive Casson nanofluid flow under the influence of suction and boundary slip conditions. From the study's outcome, it is perceived that the rising Casson parameter minimizes the heat transference rate of nanoliquids, whilst increasing radiation parameter values have the opposite impact.

The scientific study of heat and mass transport via Thermophoresis and Brownian motion is essential in nanotechnology for the production of superior materials and technologies and the interaction of Thermophoresis with Brownian motion can significantly affect flow behavior, which is especially important in applications involving complex fluids like bodily fluids and polymeric solutions. Many studies employing these effects in their study can be found here: Babu et al. (2024) examined the joint effects of Brownian Motion and Thermophoresis on a Williamson nanofluid Flow Past a Stretching Sheet, along with the MHD and Chemical Reaction Effects. Thermal distribution was shown to increase when the Thermophoresis and Brownian Motion were raised. The combined effects of Brownian motion and Thermophoresis on radiative Hiemenz flow over a nonlinear extending surface with MHD, chemical reaction, and heat generation were studied by Ibrahim et al. (2023). According to the study, Thermophoresis and Brownian motion stimulate certain thermal impacts. Lone et al. (2023) evaluated Thermophoresis and Brownian motion effects in a bioconvective MHD flow of Casson hybrid nanofluid across an exponential stretching sheet with a porous medium. Sudarmozhi et al. (2024) examined the melting heat impact on Maxwell fluid flow on a stretched sheet in the occurrence of a chemical reaction with the influence of Thermophoresis and Brownian motion. Based on the findings, a rise in the Brownian motion parameter decreases the concentration outline, but increasing the Thermophoresis parameter has the opposite impact.

Suction and injection conditions have significance in fluid flow, heat, and mass transfer applications because they allow for flow field manipulation, heat transfer optimization, boundary layer modification, and interaction with other parameters to achieve desired efficiency and outcomes. Several studies (Kumar et al. 2020a; Ram Sharma and Jain 2024; Mahmood et al. 2024) were carried out under this framework to investigate the feasibility and use of suction and injection scenarios in fluid flow problems. Recently, Köten et al. (2024) studied the flow dynamics and heat transmission of ternary water-based nanofluids across a stretching/shrinking sheet while considering suction/injection scenarios. Furthermore, in injection cases, the transition from a shrinking to a stretching sheet has a greater influence on the percentage change in temperature relative to ambient circumstances than in the suction scenario. In research comparing numerical and artificial neural networks, Mahabaleshwar et al. (2024c) examined the effects of heat sources, MHD, chemical reactions, and suction/injection on ternary nanofluid flow via a radially expanding sheet. The investigation reveals that in the blowing scenario, heat distribution is more pronounced, while in the injection scenario, mass transfer is much less.

Wall jet flow occurs when a fluid flows from an opening or gap and flows along to a surface. The flow transforms into

a radial wall jet, with an inner and outer layer. Understanding these factors is critical for optimizing wall jet applications in various technical domains. Mahros et al. (2023) obtained theoretical and numerical solutions for the wall jet flow of hybrid nanofluids across a stretched sheet influenced by MHD. It was discovered that the HNF is driven into a void area by wall suction, resulting in the velocity decline. Aly et al. (2024) investigated laminar wall jet convective flow across a shrinking/stretching surface and the consequences of suction and injection on fluid flow behavior were also examined. Khan et al. (2023a) assessed Eyring-Powell nanofluid flow through a wall jet on a vertical stretchable sheet, including the effects of a non-uniform heat source/sink. Furthermore, the current article discusses the aiding and opposing effects of fluid flow and heat transfer via a wall jet. Khan et al. (2023b) adopted the Brinkman-extended Darcy Model to investigate the radiation and activation energy impacts on Glauert Wall Jet nanofluid flow comprising nano lubricants. It was discovered that the altered porosity parameter and the permeability parameter had a preliminary impact on the motion of the wall jet flow before significantly increasing the velocity.

The bvp-4c approach offers researchers an effective tool for solving fluid dynamics boundary value problems. Its great accuracy, ease of use in MATLAB, and ability to handle nonlinear equations make it an effective option for simulating a variety of physical phenomena associated with fluid flow, heat, and mass transfer. Using this approach, researchers can get better information into the complicated behavior of the fluid. Chaurasiya et al. (2024) used the bvp-4c approach to provide a numerical solution to the effect of induced magnetic field on the Marangoni convective flow of self-rewetting fluid over an expanded disk. Kumar and Ray (2022) investigated the nanoparticle shape effects and hall effects on MHD hybrid nanofluid flow in the presence of entropy generation across a spinning disk. The bvp-4c method was used to obtain a numerical solution. Nasir et al. (2024) explored chemically reactive ternary nanofluid flow past a stretching Riga plate in the presence of viscous dissipation and heat source/sink. Numerical solutions for the present study were archived using the bvp-4c approach. Kumar et al. (2023b) explored the effects of the shape factor on the mixed convective unsteady flow of hybrid nanofluid across a rotating sphere in the presence of heat absorption/generation. Table 1 provides a detailed review of the unique work, demonstrating the research gap between the current study and the pre-existing literature.

The literature surveys stated above showed that no attempt has been made to examine the comparative research of wall jet flow in hybrid nanofluid under the impact of the porous medium, Thermophoresis, and Brownian motion with hybrid nanofluid in two circumstances: 1. suction and 2. injection. Furthermore, wall jet flows are important in a wide range of heat and mass transfer applications, including electronic

Table 1 Comparison of the present study with earlier publications

Work	Zaidi et al. (2017)	Usafzai et al. (2023)	Jafarimoghaddam and Shafizadeh (2019)	Nayak et al. (2024)	Present study
Porous medium	×	×	×	✓	✓
Casson fluid	×	×	×	×	✓
Thermophoresis and Brownian motion	×	×	✓	✓	✓
Suction/Injection case	×	×	×	✓	✓
Wall jet flow	✓	✓	✓	×	✓
Fluid type					
Mono	✓	✓	×	×	×
Hybrid	×	×	×	×	✓
Ternary	×	×	×	×	×
Methodology					
bvp-4c	×	×	×	×	✓

cooling, vehicle systems, jet cooling, and proficient thermal management methods. The hybrid nanofluid has further uses in burning and works efficiently for cooling in a wide temperature range. Thus, in the current problem, the impact of injection and suction over a hybrid nanofluid via a wall jet on a stretched surface are investigated in the presence of a porous medium, Thermophoresis, and Brownian motion employing hybrid nanofluid. The flow problem is described using PDEs. Then Glauert variables are used to convert them into ODEs, which are then analyzed numerically using a bvp-4c solver.

2 Mathematical section

Figure 1 displays the schematic outline of the laminar wall jet nanofluid model. A two-dimensional wall jet flow with mass and heat transfer phenomena employing sodium alginate-based hybrid nanofluid including Ag and Cu nanoparticles over a stretched surface are investigated under the significant impact of the porous medium, Thermophoresis, and Brownian motion. Furthermore, these effects on velocity, temperature, and concentration profiles are studied in two scenarios, suction ($S > 0$), and injection ($S < 0$), for the objective of this investigation. The two velocities associated with the wall jet are u and v in the x and y directions. T is the fluid temperature, which receives heat at a constant rate T_w at the wall, while T_∞ is the fluid's ambient temperature. Assuming C_w and C_∞ be the fluid's uniform wall and far-field concentrations. For an isotropic and incompressible Casson fluid flow, the rheological representation of the state is given below:

$$\tau = \left(\mu_b + \frac{p_y}{\sqrt{\pi_2}} \right) e_{ij} 2, \quad \pi_c < \pi; \quad \tau = \left(\mu_b + \frac{p_y}{\sqrt{2\pi_c}} \right) 2e_{ij}, \quad \pi_c > \pi. \text{ where, } \pi - \text{represents the product of the deformation rate component and } (\pi_c, \mu_b) \text{ is a significant assessment of } \pi$$

and plastic dynamic viscosity of a non-Newtonian model, e_{ij} is the $(i, j)^{th}$ component of the deformation rate, p_y represents yield stress of fluid. The following governing equations are possibly formed using the aforementioned assumptions see (Ali Zaidi et al. 2017; Khan Usafzai et al. 2023; Jafarimoghaddam and Shafizadeh 2019; Uddin et al. 2024; Kumar et al. 2020b).

$$\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} = 0, \quad (1)$$

$$v \frac{\partial u}{\partial y} + u \frac{\partial u}{\partial x} = v_{hnf} \left(1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2} - \frac{v_{hnf} u}{K(x)}, \quad (2)$$

$$v \frac{\partial T}{\partial y} + u \frac{\partial T}{\partial x} = \frac{\partial^2 T}{\partial y^2} \left(\frac{k_{hnf}}{(\rho C p)_{hnf}} \right) + \tau \left(D_b \frac{\partial T}{\partial y} \frac{\partial C}{\partial y} + \left(\frac{\partial T}{\partial y} \right)^2 \frac{D_T}{T_\infty} \right), \quad (3)$$

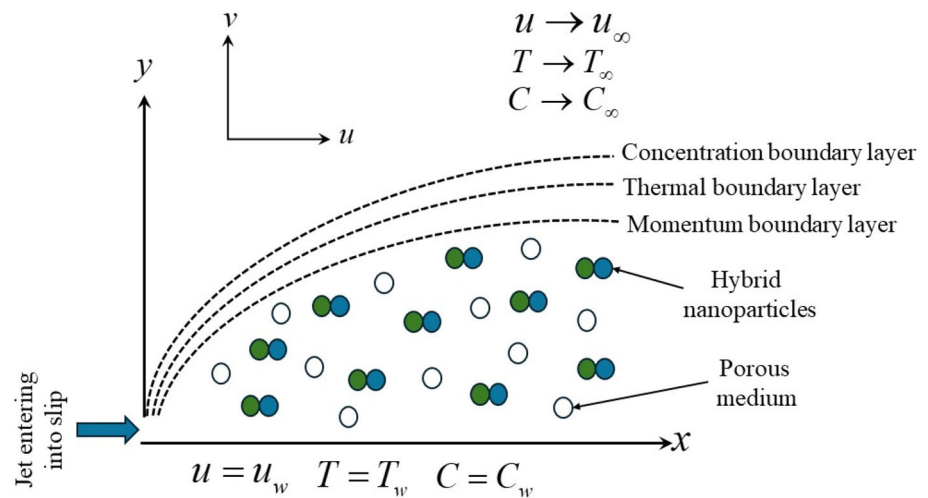
$$v \frac{\partial C}{\partial y} + u \frac{\partial C}{\partial x} = D_b \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2}, \quad (4)$$

and following are the corresponding boundary conditions

$$\text{at } y = 0 \quad u = u_w, \quad v = v_w, \quad T = T_w, \quad C = C_w, \quad (5)$$

$$\text{at } y \rightarrow \infty : \quad u \rightarrow u_\infty, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty. \quad (6)$$

Here, (μ, ν) represents dynamic and kinematic viscosity, β denotes Casson parameter, ρ represents fluid's density, k is the thermal conductivity, Cp means specific heat capacity, $K(x) = \hat{k}_0 x^{3/2}$ denote variable permeable constraint, τ signifies heat capacity ratio, (D_b, D_T) is the coefficient of Brownian diffusion and thermophoretic diffusion,

Fig. 1 Geometry of flow problem**Table 2** Thermophysical features of base fluid and hybrid nanoparticle (Kumar et al. 2018b, 2023c; Maboodet al. 2020)

Physical properties	Sodium Alginate	Ag	Cu
$\rho (\text{kgm}^{-3})$	989	10,500	8933
$Cp (\text{Jkg}^{-1}\text{K}^{-1})$	4175	235	385
$k (\text{kgms}^{-3}\text{K}^{-1})$	0.6376	429	401

Table 3 Thermal characteristics of hybrid nanoparticles (Uddin et al. 2022)

(μ)	$\frac{\mu_{hnf}}{\mu_f} = \frac{1}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}},$
Dynamic viscosity	
(ρ)	$\frac{\rho_{hnf}}{\rho_f} = \left[\left(\phi_1 \frac{\rho_1}{\rho_f} + (1-\phi_1) \right) (1-\phi_2) + \phi_2 \frac{\rho_2}{\rho_f} \right],$
Density	
(ρCp)	$\frac{(\rho Cp)_{hnf}}{(\rho Cp)_f} =$
Heat capacity	$\left[\left(\phi_1 \frac{(\rho Cp)_1}{(\rho Cp)_f} + (1-\phi_1) \right) (1-\phi_2) + \phi_2 \frac{(\rho Cp)_2}{(\rho Cp)_f} \right],$
(k)	$\frac{k_{hnf}}{k_f} = \frac{k_2 + k_f - \phi_2 (k_f - k_2)}{k_2 + k_f + \phi_2 (k_f - k_2)},$
Thermal conductivity	$\frac{k_{nf}}{k_f} = \frac{k_1 + k_f - \phi_1 (k_f - k_1)}{k_1 + k_f + \phi_1 (k_f - k_1)}$

$v_w = -v_f^{1/2} x^{-3/4} S$. Table 2 provides the thermophysical characteristics of conventional fluid (Sodium alginate) and hybrid nanoparticles (Ag and Cu) and the data on the hybrid nanoparticles thermophysical characteristics is given in Table 3.

Here, the similarities followed by ref Khan et al. 2023b were used to facilitate scrutinizing the current wall jet model.

$$\eta = \frac{y}{\sqrt{v_f}} x^{-3/4}, \quad u = \frac{4}{\sqrt{x}} F', \quad v = -\sqrt{v_f} x^{-3/4} (F - 3\eta F'), \quad (7)$$

$$G(T_w - T_\infty) = T - T_\infty, \quad H(C_w - C_\infty) = C - C_\infty. \quad (8)$$

Using Eqs. (7) and (8) with thermophysical properties in Table 2, Eqs. (2)–(4) are transformed as

$$\frac{1}{A_1 A_2} \left(1 + \frac{1}{\beta} \right) F''' + 2F'^2 + FF'' - \frac{1}{A_1 A_2} \alpha F' = 0, \quad (9)$$

$$\frac{k_{hnf}}{k_f} \frac{1}{\text{Pr} A_3} G'' + FG' + NbH'G' + NtG'^2 = 0, \quad (10)$$

$$H'' + LePrFH' + \frac{Nt}{Nb} G'' = 0. \quad (11)$$

and

$$F'(0) = 1, \quad F(0) = S, \quad G(0) = 1, \quad H(0) = 1, \quad (12)$$

$$F'(\infty) = 0, \quad G(\infty) \rightarrow 0, \quad H(\infty) \rightarrow 0. \quad (13)$$

where, α is the porosity parameter, $\text{Pr} = \frac{\nu_f}{\alpha f}$ indicates Prandtl number, $Nb = \frac{D_b \tilde{\tau} \Delta C}{\nu_f}$ and $Nt = \frac{D_T \tilde{\tau} \Delta T}{T_\infty \nu_f}$ indicates Brownian motion and Thermophoresis parameters respectively, $Le = \frac{\alpha f}{D_b}$ is Lewis number and S represents injection /suction parameter.

The engineering quantities with their reduced form obtained in this study are as follows:

$$Cf = \frac{1}{\rho_f u_w^2} \left(1 + \frac{1}{\beta} \right) \mu_{hnf} \frac{\partial u}{\partial y} \bigg|_{y=0} \\ \Rightarrow 2(\text{Re}_x)^{1/2} Cf = \left(1 + \frac{1}{\beta} \right) \frac{1}{A_1} F''(0), \quad (14)$$

$$Nu = -\frac{x}{k_f \Delta T} k_{hnf} \frac{\partial T}{\partial y} \bigg|_{y=0} \Rightarrow 2 \frac{Nu}{\sqrt{\text{Re}_x}} = -\frac{k_{hnf}}{k_f} G'(0), \quad (15)$$

$$Sh = \frac{-x}{D_b \Delta C} D_b \frac{\partial C}{\partial y} \bigg|_{y=0} \Rightarrow 2 \frac{Sh}{\sqrt{\text{Re}_x}} = -H'(0). \quad (16)$$

where, $Re_x = \frac{x u_w}{\nu_f}$ is the local Reynolds number.

3 Methodology section

This section explains the methodology used to generate solutions as well as code validation. The `bvp-4c` technique is a MATLAB function that solves Boundary Value Problems (BVPs) for ODEs. This approach provides for precise approximation of solutions alongside satisfying boundary circumstances at both ends of the interval. Figure 2 shows the flow chart for methodology.

The algorithm is explained in the following steps.

a. Transformed Equations: First consider ODEs transformed from PDEs via similarity transformations.

b. Convert to the first-order system: Amend the ODEs by transforming them into a first-order system through the insertion of additional parameters, facilitating the solution of the model.

$$\langle F = \Theta_1, F' = \Theta_2, F'' = \Theta_3, G = \Theta_4, G' = \Theta_5, H = \Theta_6, H' = \Theta_7 \rangle, \quad (17)$$

c. Write MATLAB syntax: Formulate the MATLAB syntax for the reduced Eqs. (9) to (11) and boundary conditions (12) and (13) to prepare for simulation.

For reduced equations and BCs, the MATLAB syntax is employed as shown below.

$$\Theta_3' = -\frac{A_1 A_2}{\left(1 + \frac{1}{\beta}\right)} \left(2\Theta_2^2 + \Theta_1 \Theta_3 - \frac{1}{A_1 A_2} \alpha \Theta_2\right), \quad (18)$$

$$\Theta_5' = -\frac{\text{Pr } k_f A_3}{k_{hnf}} \left(\Theta_1 \Theta_5 + Nb \Theta_7 \Theta_5 + Nt \Theta_5^2\right), \quad (19)$$

$$\Theta_7' = -\left(Le Pr \Theta_1 \Theta_7 + \frac{Nt}{Nb} \Theta_5'\right). \quad (20)$$

and

$$\Theta_2(0) = 1, \quad \Theta_1(0) = S, \quad \Theta_4(0) = 1, \quad \Theta_6(0) = 1, \quad (21)$$

$$\Theta_2(\infty) = 0, \quad \Theta_4(\infty) \rightarrow 0, \quad \Theta_6(\infty) \rightarrow 0. \quad (22)$$

d. Insertion of reduced equations and boundary conditions: Input the relevant reduced Eqs. (9) to (11) into the `bvp-4c` solver to obtain the numerical solutions, specifying the syntax as “solution = `bvp-4c` (@BVP, @BC, solutioninit)”, where “@BVP and @BC” define the equations and boundary conditions respectively. The preliminary mesh points and the primary solution approximation at these locations are coded using the “solutioninit” function.

Table 4 Comparison of present work with Khan et al. (2023b) and Waini et al. (2021) work in the absence of α , S with $\beta \rightarrow \infty$, $A_1 = A_2 = 1$

ϕ	Waini et al. (2021)	Khan et al. (2023b)	Present study outcomes
0.0	0.2222	0.2222	0.2222
0.035	–	0.3178	0.3180
0.037	–	0.3656	0.3659
0.039	–	0.4132	0.4138

e. Execute solver: After substituting the parametric values $\beta = 1.0$, $S = \pm 0.5$, $\alpha = 1.5$, $Pr = 6$, $0.01 \leq \phi_1 = \phi_2 \leq 0.03$, $Le = 0.8$, $Nt = 0.3$, $Nb = 0.3$, and setting mesh size at 100, execute the solver to compute the solution.

3.1 Validation of the results

Khan et al. (2023b) and Waini et al. (2021) calculations provide constrained values for the solid volume fraction ϕ in the absence of hybrid nanoparticle properties, porosity, and suction/injection parameters (see Table 4). According to the tabular findings, the current and previous works are well-matched.

4 Results and discussion

This section addresses the physical consequence of several dimensionless factors of the present boundary value problem. For wall jet hybrid nanofluid flow over a stretching surface, the porosity parameter ($1.5 \leq \alpha \leq 2.5$), Casson parameter ($0.5 \leq \beta \leq 0.9$), Thermophoresis parameter ($0.1 \leq Nt \leq 0.2$), Brownian motion parameter ($0.5 \leq Nb \leq 0.9$), and by fixing the parametric values of the suction/injection parameter ($S = \pm 0.5$), the study is conducted in two cases i.e., suction ($S > 0$), and injection ($S < 0$). The nonlinear PDE is converted into an ODE with similarity constraints and solved numerically via MATLAB's built-in function `bvp-4c`. Further, the Figs. 3, 4, 5, 6, 7, 8, 9, 10, 11 demonstrate the effects of the above-mentioned parameters on velocity $F'(\eta)$, temperature $G'(\eta)$, and concentration $H'(\eta)$ profiles, as well as engineering coefficients such as the friction factor Cf , Nusselt number Nu , and Sherwood number Sh .

The impact of α on $F'(\eta)$ profile is displayed in Fig. 3. The graphical results clearly show that when the alpha value increases, the velocity curves decrease. Physically, the flow dynamics vary dramatically depending on the porosity parameter. Higher porosity indicates that the fluid has more paths for motion, which will interrupt the flow and reduce fluid velocity. Also, the increasing porosity increases

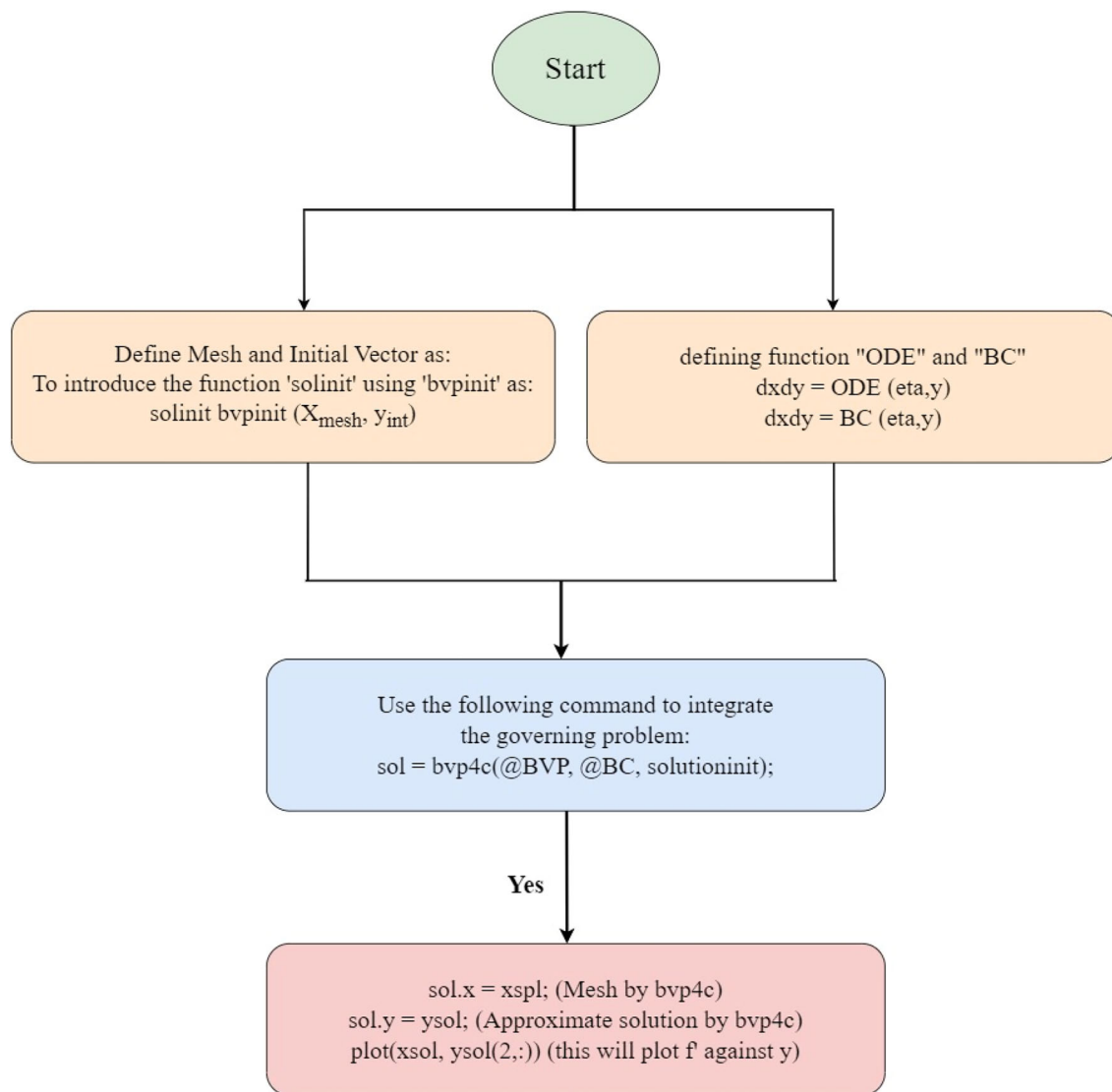


Fig. 2 Flow chart of the method

the thickness of the boundary layer, resulting in a reduced velocity profile near the surfaces.

The same trend is seen in $F'(\eta)$ profile for varying β values in Fig. 4. Physically, the augmentation of the β shows that the fluid's non-Newtonian behavior is more pronounced. Higher viscous forces inside the fluid are caused by this enhanced non-Newtonian character, which lowers fluid velocity and also lessens the boundary layer thickness. In both Figs. 3 and 4, the decrease is more significant in the suction case compared to injection. It is because, the impact of suction increases the boundary layer thickness, slowing fluid velocity near the wall.

Figure 5 showcases the influence of Nt on $G'(\eta)$ profile. From the figure, it is clear that the $G'(\eta)$ profile rises for amplified Nt values. From a physical standpoint, as the Thermophoresis parameter increases, it also raises the heat

gradient in the fluid. This increased gradient facilitates higher heat transfer from the heated surface to the fluid, increasing the $G'(\eta)$ profile. The increased mobility of particles owing to Thermophoresis aids in improved mixing and energy distribution within the fluid, raising the overall temperature.

Figure 6 demonstrates the increasing nature of $G'(\eta)$ profile for amplified values of Nb . It shows that when Nb is given the highest value, $G'(\eta)$ profile achieves its most elevated level. Physically, the relationship between the temperature profile and the Brownian motion component may be understood through assessing particle motion. Brownian motion generates random particle movement, which improves convective heat transfer in fluids. As the particles travel, they share thermal energy, which helps to distribute heat more effectively throughout the fluid. Higher levels of the Brownian motion parameter accentuate this convective process and

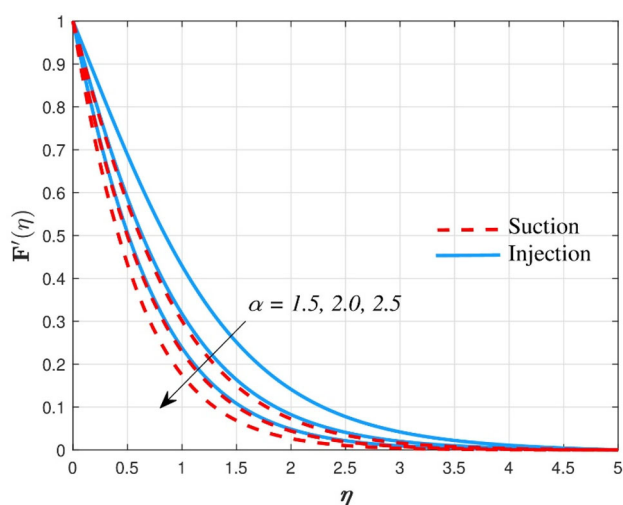


Fig. 3 Effect of α over $F'(\eta)$ profile

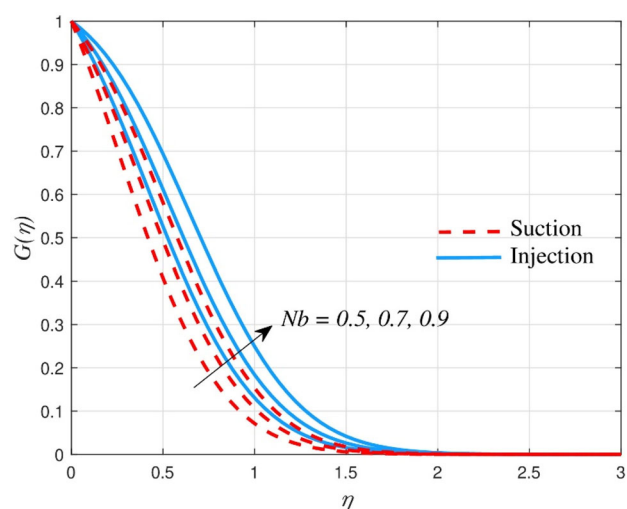


Fig. 6 Impact of Nt over $G'(\eta)$ profile

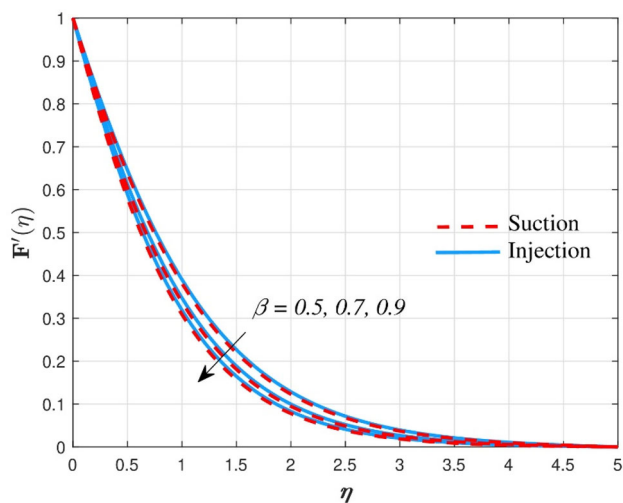


Fig. 4 Effect of β over $F'(\eta)$ profile

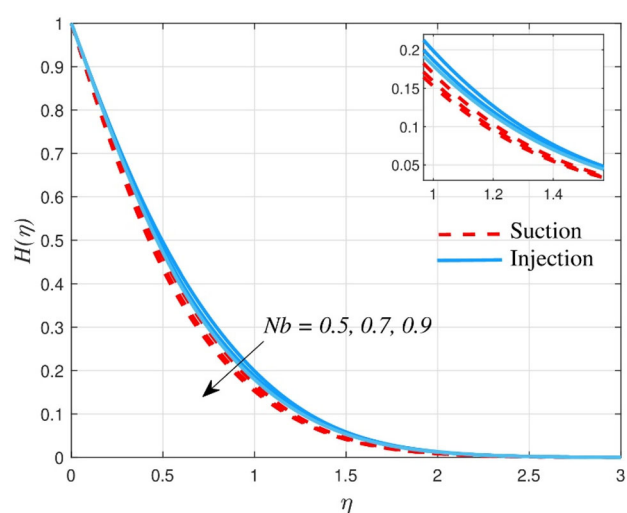


Fig. 7 Impact of Nb over $H(\eta)$ profile

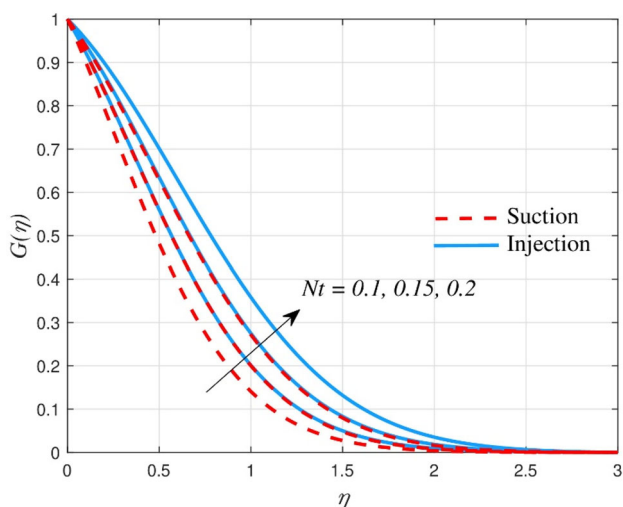


Fig. 5 Impact of Nt over $G'(\eta)$ profile

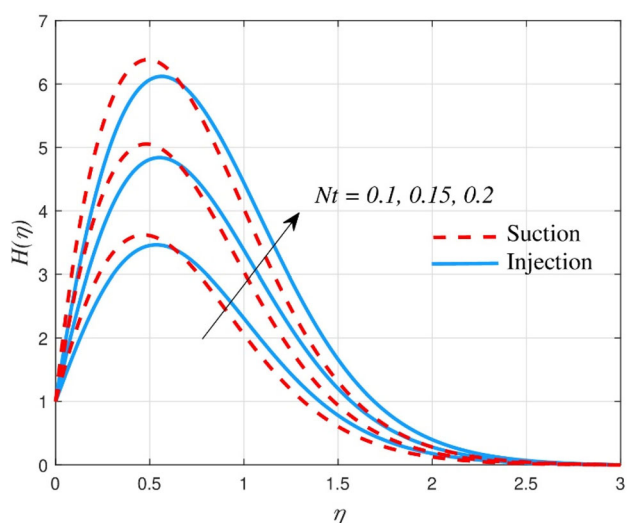


Fig. 8 Impact of Nt over $H(\eta)$ profile

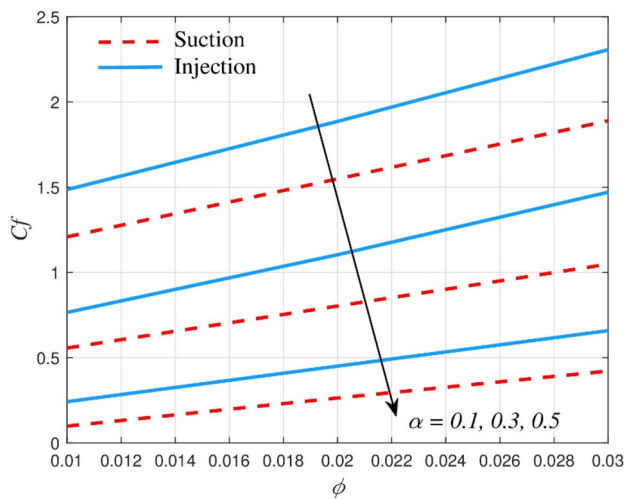


Fig. 9 Variations in C_f for distinct values of α

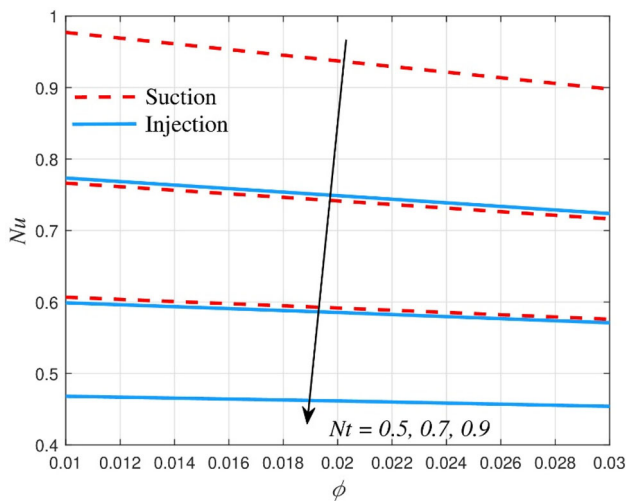


Fig. 10 Variations in Nu for distinct values of Nt

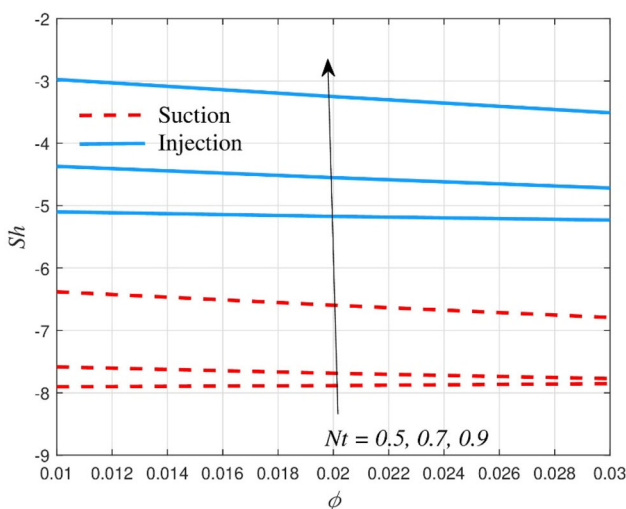


Fig. 11 Variations in Sh for distinct values of Nt

contribute to the temperature profile increase. The temperature rise is greater in the case of injection than in suction. In the suction instance, the fluid moves closer to the surface at ambient temperatures, lowering the thickness of the thermal boundary layer. This restricts the temperature rise in the suction scenario, as opposed to the injection case, where the thickness of the thermal boundary layer grows due to the heated fluid being pulled away from the surface.

Figure 7 displays the effect of Nb over $H'(\eta)$ profile. A downfall behavior was witnessed in $H'(\eta)$ for increased values of Nb . Physically, as the Brownian motion parameter rises, it corresponds to a larger amount of random movement of particles. This enhanced chaotic mobility causes a more uniform dispersion of nanoparticles in the fluid, resulting in a lower concentration profile, particularly at the surface. The drop is more significant in the case of suction than injection. This is because, in the suction situation, the fluid is drawn into a porous material, slowing its velocity. This decrease causes a slower rate of mass transfer and lower solute concentrations near the surface. Suction generates a lower pressure that inhibits particle diffusion, resulting in a greater reduction in concentration than injection, which effectively drives fluid into the entire system.

Figure 8 portrays the influence of Nt parameter over $H'(\eta)$ profile. In the figure, it is clear that the concentration profile rises for elevated values of Nt . Physically, the thermophoretic effect can cause particle distribution, with particles near the hot surface being driven away and those in cooler regions pulled in. This dynamic can cause an overall rise in the concentration profile, as particles are concentrated in certain parts of the fluid. The effect is found to be more substantial in the injection case. The reason is that the injection improves the transfer of concentrated fluid into the domain, leading to a more profound boost in the concentration profile than the suction case.

Figure 9 shows the variations in Skin-friction C_f for distinctive values of α and ϕ . Higher porosity suggests a more permeable medium, allowing fluid to move smoothly through the porous structure. Increased permeability decreases flow resistance, reducing shear stress at the surface and, as a result, less skin friction. As the fluid passes through the pores, it experiences less drag, which results in reduced skin friction C_f . In the suction scenario, increased permeability allows for easier fluid intrusion, reducing flow resistance and thus decreasing skin friction. Engineers in aircraft engineering can prevent boundary layer separation and maintain a laminar flow by applying suction via porous surfaces, lowering drag and boosting fuel economy drastically.

Figure 10 depicts the change in the behavior of Nusselt number Nu for augmented values of Nt and ϕ . From the figure, a reduction in Nu is observed for various values of Nt . Physically, when Nt grows, particle migration advances, causing an overall fall in the convective heat transfer rate and,

Table 5 Variations in Engineering factors (Cf , Nu , Sh) for various values of parameters

Parameters							Engineering factors		
β	S	α	ϕ	Le	Nt	Nb	Cf	Nu	Sh
0.5	0.5	1.5	0.01	0.8	0.3	0.3	– 0.76549	1.57045	– 0.40868
1.0	0.5	1.5	0.01	0.8	0.3	0.3	– 0.40800	1.56542	– 0.42281
1.5	0.5	1.5	0.01	0/8	0.3	0.3	– 0.28506	1.56081	– 0.43096
0.5	– 0.5	1.5	0.01	0.8	0.3	0.3	– 0.48577	1.29595	– 0.21710
0.5	0.0	1.5	0.01	0.8	0.3	0.3	– 0.44557	1.42768	– 0.31728
0.5	0.5	1.5	0.01	0.8	0.3	0.3	– 0.40800	1.56542	– 0.42281
0.5	0.5	1.5	0.01	0.8	0.3	0.3	– 0.22211	1.52554	– 0.43324
0.5	0.5	2.0	0.01	0.8	0.3	0.3	– 0.20780	1.51880	– 0.43449
0.5	0.5	2.5	0.01	0.8	0.3	0.3	– 0.18934	1.51235	– 0.43555
0.5	0.5	1.5	0.01	0.8	0.3	0.3	– 0.38676	1.66537	– 0.45576
0.5	0.5	1.5	0.02	0.8	0.3	0.3	– 0.39424	1.61472	– 0.43920
0.5	0.5	1.5	0.03	0.8	0.3	0.3	– 0.40800	1.56542	– 0.42281
0.5	0.5	1.5	0.01	0.8	0.3	0.3	– 0.40800	1.56542	– 0.42281
0.5	0.5	1.5	0.01	1.0	0.3	0.3	– 0.39599	1.57218	– 0.59178
0.5	0.5	1.5	0.01	1.2	0.3	0.3	– 0.38434	1.57819	– 0.75447
0.5	0.5	1.5	0.01	0.8	0.3	0.3	– 0.40800	1.56542	– 0.42281
0.5	0.5	1.5	0.01	0.8	0.5	0.3	– 0.40978	1.54664	– 0.76006
0.5	0.5	1.5	0.01	0.8	0.7	0.3	– 0.40197	1.52809	– 1.08687
0.5	0.5	1.5	0.01	0.8	0.3	0.5	– 0.40800	1.56542	– 0.42281
0.5	0.5	1.5	0.01	0.8	0.3	0.7	– 0.40195	1.54738	– 0.14204
0.5	0.5	1.5	0.01	0.8	0.3	0.3	– 0.40195	1.52944	0.02635

as a consequence, a drop in Nu is seen. As the Thermophoresis parameter rises, the injection case leads to the thickening of the thermal boundary layer more than the suction case. This thickening permits particles to accumulate more within the boundary layer, resulting in a more significant drop in the Nu in the injection scenario. In electronic cooling applications, such as system processors or power systems, the injection of coolant fluids can be enhanced by taking Thermophoresis into account.

The change in the nature of Sherwood number Sh for distinctive values of Nt and ϕ is seen in Fig. 11. An increasing behavior is observed in mass transfer when Nt is elevated. Physically, as the Thermophoresis parameter grows, it causes strong temperature gradients and leads to a more uniform distribution of nanoparticles potentially improving mass transfer rates. Injection increases fluid motion in the system. This increased velocity improves convective mass transport, resulting in consequently raising the Sherwood number. A higher Sherwood number indicates better mass transfer of heat from surfaces to the coolant in systems that utilize fluid injection for cooling, such as electronic or HVAC systems. This can improve cooling efficiency and reduce energy consumption, making systems more efficient.

The influence of several parameters on engineering factors like Skin friction Cf , Nusselt number Nu and Sherwood number Sh is demonstrated in Table 5.

5 Final remarks

The present work investigates the fluid flow characteristics of hybrid nanofluids of wall jet flow which is driven through a permeable stretching sheet. The novel research on the present study was conducted in two scenarios i.e., injection and suction. The effect of Thermophoresis and Brownian motion have been taken into consideration for heat and mass transfer analysis. The basic PDEs are transformed into ODEs and solved numerically using the bvp-4c. The impacts of interacting parameters on mass, heat, and flow transfer are taken into consideration, and based on these proofs, the study's key findings are listed below:

- A higher Casson parameter β and porosity parameter α resulted in a lower velocity profile. The effect of these parameters is significantly more in the case of suction.
- As the Thermophoresis parameter Nt is increased, both temperature and concentration profile exhibits increasing trend. Here, the rise is improved more in the case of injection.
- The elevated values of the Brownian motion parameter Nb resulted in an upsurge in the temperature profile and the rise is found to be more in the injection case but the concentration profile exhibits an opposite trend.

- d. With the rising values of α parameter, the friction factor Cf declines and the drop is found to be more effective in the suction case.
- e. The variations in the Nusselt number Nu was found to be reduced significantly in injection case for improved values of the Thermophoresis parameter Nt .
- f. The rate of mass transfer was improved upon the influence of raised values of the Thermophoresis parameter Nt . The improvement in the mass transfer was better in the case of injection.

The findings from this study on hybrid nanofluids past a permeable wall jet flow under the influence of Thermophoresis and Brownian motion subjected to suction/injection boundaries have several practical applications like chemical and polymer processing, cooling systems, heat exchangers and energy systems where controlling flow characteristics, heat and mass transfer rates.

6 Future study

This research could potentially be extended to explore the behaviour of different non-Newtonian fluids, such as viscoelastic fluids, in wall jet flows including various nanoparticles and base fluids, specifically how these fluids behave to Thermophoresis and Brownian motion under suction and injection circumstances over various geometries. Using machine learning techniques in coupled with standard numerical approaches can aid in the prediction of flow characteristics and parameter optimization in nanofluid flows.

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Data availability No datasets were generated or analysed during the current study.

Declarations

Conflict of interest The authors declare no competing interests.

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