



Algebraically decaying solutions for magnetohydrodynamic heat and mass transfer in Newtonian flow driven by stretching or shrinking surface: applications to thermal management and cooling technologies

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ABSTRACT

The aim of the current analyses is to examine the thermal and solutal transfer impact on Newtonian fluid flow across permeable expanding/shrinking surfaces, and the impact of solar radiation, magnetic field, and Schmidt number on fluid flow has been extracted. Governing Navier-Stokes partial differential equations are formulated into ordinary differential equations via similarity variables, and closed-form multiple analytic solutions are obtained. The power law heat and mass flux conditions were examined using hyper-geometric series solution. A particular emphasis is placed on the exact analytical treatment of the algebraically decaying flow branch and its associated thermal and solutal transport characteristics under prescribed surface temperature, prescribed heat flux, prescribed surface concentration, and prescribed mass flux conditions. The results of the current study reveal that increasing the porous media, magnetic field, and stretching parameter reduces the velocity of the fluid flow, and increasing the thermal radiation values from 1 to 3 enhances the temperature by approximately 17% under prescribed surface conditions and by 22% under prescribed heat flux conditions. Increasing the Schmidt number from 1 to 3 reduces the concentration by approximately 19% under the prescribed surface concentration conditions and by 26% under prescribed mass flux conditions. The findings of this study are valuable for engineering applications involving heat and mass transfer across permeable surfaces, such as thermal management in cooling systems, chemical reactors, and material processing.

1. Introduction

Research on the impact of a magnetic field on fluid flow across a stretching sheet is of significant importance in engineering and industrial processes. Magnetic effects can be used to control the heat transfer and stability of electrically conducting fluids, which are important in applications that use magnetohydrodynamics. This behavior is particularly important in metal casting, polymer extrusion, and the cooling of electronic devices. They also play a major role in processes such as crystal growth and biomedical device design. Overall, analyzing this interaction boosts the design and optimization of regimes where heat and fluid characteristics are critical. Chen [1] examined the influence of Lorentz force and mass suction on the power-law non-Newtonian fluid movement across an expanding surface. Awais [2] examined the impact

of viscous dissipation and activation energy for the MHD Eyring-Powell fluid flow. Sachhin [3] examined the impact of magnetic field and dissipation on fluid flow. Salahuddin [4] studied the influence of heat and mass transport along with variable viscosity of magnetohydrodynamic Williamson fluid. Turkyilmazoglu [5] examined the effects of Soret and Dufour on convective fluid flow over a moving surface.

The impact of porous media on fluid movement across an expanding surface has a significant impact in different scientific and engineering industries. Porous media alters the heat transfer behavior of the fluid and allows precise control over fluid behavior in complex systems, and is also important in processes such as petroleum recovery, filtration, and thermal insulation. In environmental engineering porous media aid in the analysis of ground-water flow and prediction of pollutant transport. Cooper [6] explored Newtonian fluid movement across porous media

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using experimental analysis. Mahabaleshwar [7] explored the impact of the heat transfer on fluid flow over porous media. Awais [8] studied the impact of Darcy-Forchheimer and variable fluid properties for the flow of Carreau–Yasuda nanofluid. Sochi [9] explored the pore-scale network modelling of fluid movement across porous media. Zhao [10] explored the effect of energy transfer on fluid movement using the fractional Darcy approach.

The study of the influence of radiation on fluid movement across expanding surface is important in analyzing the heat transfer characteristics in high-temperature environments. Radiation significantly impacts the energy distribution within the fluids, especially where conduction and convection are unable to describe energy transport accurately. This effect has many applications in polymer extrusion, and glass manufacturing, where heat exchange plays a major role. Yu [11] explored the impact of radiation and carbon nanotube impact on fluid movement using a collocation approach. Misra [12] examined the impact of the velocity slip and Lorentz force on unsteady fluid flow by expanding the surface. Pop [13] explored the effect of Eckert number and thermal radiation on Casson fluid movement. Turkiymazoglu [14] examined the impact of radiation on the fluid flow across a porous disc. Chinyoka [15] examined the impact of solar radiation and magnetic fields on the fluid movement using a numerical technique.

The Prescribed mass conditions of fluid flow across a stretching surface are important for analyzing the mass transfer process in industrial and engineering systems. These boundary restrictions help in chemical vapor, drying of liquid films, and polymer coating, where precise solutal transfer control on the surface is crucial, and also play a major role in biomedical applications, including drug diffusion and membrane filtration. Awais [16] examined the influence of mixed convection and thermal diffusion on fluid movement. Prado [17] examined the impact of molecular simulations on gas molecules. [18,19] explored the impact of heat transfer on fluid flow. Singh [20] examined the impact of thermal and solutal transfers on viscoelastic fluid movement across an expanding sheet.

The impact of the prescribed heat conditions on the fluid movement across the expanding surface is important for analyzing the controlling heat and solutal transfer in different engineering systems. These energy boundary restrictions alter the real situation where either the surface temperature or the heat supplied is fixed. These fluid restrictions help in glass production, polymer extrusion, and cooling of electrical components, and play a major role in the energy management of solar collectors, heat exchange, and energy storage systems. Vajravelu [21] explored the impact of prescribed heat restrictions on laminar fluid movement across an expanding sheet. Fatunmbi [22] examined the impact of heat dissipation and ohmic heating on micropolar fluid flow. Metri [23] examined the impact of the Lorentz force and Eckert number on mixed convective fluid flow. Bataller [24] explored the influence of heat transfer on fluid flow over an expanding surface using heat boundary restrictions. Rauf [25] explored the impact of prescribed heat and mass flux on fluid flow. Some of the latest research works on magnetohydrodynamic and heat and mass transfer (see [26–30]).

1.1. Motivation and novelty

Despite extensive studies on boundary layer flows over stretching or shrinking surfaces, most investigations have focused on either heat transfer or mass diffusion in isolation, often under simplified conditions such as impermeable walls or purely thermal boundary constraints. However, many practical engineering systems such as cooling channels in electronic devices, catalytic reactors, membrane filtration units, and geothermal collectors operate under simultaneous heat and mass transport in porous and magnetically influenced environments. The above configurations frequently alter the non-uniform wall fluxes, mass transpiration, and radiation effects, which genuinely alters the transport dynamics but remain unexplored in analytical methods.

The current work addresses this research gap by deriving closed-form

similarity solutions for Newtonian fluid flow with concurrent thermal and solutal transport over a permeable stretching/shrinking surface under magnetic and porous-medium effects. A particular emphasis is placed on the identification and analytical treatment of an algebraically decaying flow branch, in addition to the conventional exponentially decaying branch. The algebraically decaying branch provides an exact analytical velocity solution and is subsequently used to formulate the corresponding thermal and concentration transport problems. The study further considers four power-law transport boundary conditions, namely prescribed surface temperature, prescribed heat flux, prescribed surface concentration, and prescribed mass flux, thereby providing a unified analytical framework for heat and mass transfer. The existence and admissibility of the exponential and algebraic branches are explicitly examined through the corresponding root, discriminant, and decay conditions. This formulation provides analytical benchmark solutions for assessing the effects of magnetic field, porous resistance, thermal radiation, mass diffusion, suction/injection, and stretching/shrinking characteristics on coupled heat and mass transfer. The resulting framework extends the existing analytical treatment of MHD transport over permeable surfaces by establishing a direct connection between exponential and algebraic decay regimes under multiple thermal and solutal boundary conditions.

2. Mathematical formulation

Considered 2-dimensional, incompressible, laminar Newtonian fluid flow porous stretching/shrinking, prescribed temperature and mass transfer boundary conditions, u and v are the momentum terms along x and y axes, T and C represent the energy and concentration, magnetic field velocity, thermal radiation temperature, and Schmidt number in the concentration equation obtained closed-form analytic and algebraic decay solutions, and Fig. 1 shows the physical representation of the fluid flow.

2.1. Governing equations

The continuity equation defines the conservation of mass in fluid movement, indicating that the amount of fluid entering a region must be the same as the amount leaving or accumulating within it. It acts as a fundamental constraint in all flow problems, ensuring that no fluid is mysteriously created or lost during the motion. Physically, it explains why the velocity increases in a narrowing pipe or decreases in an expanding pipe to maintain a constant mass flow [1,2].

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \quad (1)$$

The velocity equation defines how fluid movement is governed by

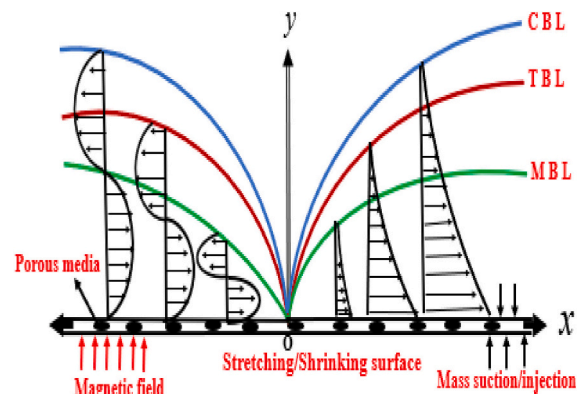


Fig. 1. Physical representation of the flow.

balance forces, incorporating changes in velocity to viscosity, pressure and other effects. When the Lorentz force is applied to capture the opposing influence of magnetohydrodynamics, porous media provide an additional drag force that restricts the flow. This momentum equation is important for examining systems such as the filtration and cooling of reactors. Physically, it portrays how porous media and magnetic effects resist the control of fluid momentum, boundary layer, and overall fluid transport [4,8,9].

$$\hat{u} \frac{\partial \hat{u}}{\partial x} + \hat{v} \frac{\partial \hat{u}}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \left(\frac{\partial^2 \hat{u}}{\partial x^2} + \frac{\partial^2 \hat{u}}{\partial y^2} \right) - \frac{1}{\rho} \left(\sigma B_0^2 + \frac{\mu}{K^*} \right) \hat{u}, \quad (2)$$

$$\hat{u} \frac{\partial \hat{v}}{\partial x} + \hat{v} \frac{\partial \hat{v}}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{\mu}{\rho} \left(\frac{\partial^2 \hat{v}}{\partial x^2} + \frac{\partial^2 \hat{v}}{\partial y^2} \right). \quad (3)$$

The energy equation provides the distribution and exchange of the thermal energy inside the fluid, including convection, conduction, and other heat sources. When the radiation effect is incorporated, it becomes important in high-temperature thin media, and the equation is significant for predicting energy fields in engineering systems such as cooling devices, furnaces, and solar collectors. Physically it explores how energy spreads through the fluid and interacts with the atmosphere, thereby influencing the fluid properties within the system [12–14].

$$\hat{u} \frac{\partial T}{\partial x} + \hat{v} \frac{\partial T}{\partial y} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{1}{\rho C_p} \frac{16\sigma^* T_\infty^3}{3k^*} \frac{\partial^2 T}{\partial y^2}. \quad (4)$$

The considered concentration equation defines how species transfer within the fluid, incorporating diffusion, convection, and other forces. It is important to examine the solutal transfer processes in pollution dispersion and, chemical reactors where precise transport is required. Physically, the considered equation defines how particles or solutes accumulate, spread in response to fluid movement, and directly impact the reaction rate and overall performance of the system [16,17].

$$\hat{u} \frac{\partial C}{\partial x} + \hat{v} \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2}. \quad (5)$$

Boundary restrictions specify the characteristics of fluids at the limits of a system, such as inlets, surfaces, and outlets, defining the choices of momentum, energy, and mass transfer, which are essential for examining the flow equation because they ensure physical and uniquely realistic solutions. Physically, boundary restrictions reflect how fluids interact with walls, enforcing no-slip at the surfaces, prescribed heat, and mass conditions. Properly stated boundary restrictions are important for accurately predicting flow patterns and, energy transfer in natural and engineering systems [15,16,21,23].

$$\begin{aligned} \hat{u} &= u_w = cx, \quad \hat{v} = v_w, \text{ at } y = 0. \\ \hat{u} &= 0, \hat{v} = 0, \text{ as } y \rightarrow \infty. \end{aligned} \quad (5a)$$

2.2. Similarity transformation

Similarity terms are important for reducing partial differential equation to ordinary differential equations. These are important because they simplify complex fluid flow problems, making analytical or numerical solutions more tractable. Physically, similarity variables reveal underlying self-similar behavior in the flow, indicating that velocity, temperature, or concentration profiles at different locations can collapse onto a single universal curve. This approach provides insight into the scaling and structure of the fluid and facilitates the prediction of fluid, thermal, and mass transport characteristics [12–14].

$$\begin{aligned} \hat{u} &= axf'(\eta), \quad \hat{v} = -f(\eta)\sqrt{av_f}, \\ \psi(x, y) &= \sqrt{av}xf(\eta), \quad \eta = \sqrt{\frac{a}{v}}y, \end{aligned} \quad (6)$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}.$$

The wall mass transfer velocity becomes $v_w(x) = -\sqrt{av}f(0)$. By using the above similarity variables the non-dimensional form of velocity Eq. (2) is defined as

$$\frac{d^3 f(\eta)}{d\eta^3} + f(\eta) \frac{d^2 f(\eta)}{d\eta^2} - \left(\frac{df(\eta)}{d\eta} \right)^2 - M \frac{df(\eta)}{d\eta} - L \frac{df(\eta)}{d\eta} = 0. \quad (7)$$

with the modified boundary restrictions.

$$f(0) = S, \quad \frac{df(0)}{d\eta} = d, \quad \frac{df(\infty)}{d\eta} = 0. \quad (8)$$

where $S = -\frac{v_w}{\sqrt{av}}$ is the mass transpiration, and d is the stretching/shrinking parameter. Using the above boundary conditions, the analytical solution for the momentum equation is defined as [12–14].

$$f(\eta) = S + \frac{d}{\beta} (1 - e^{-\beta\eta}), \quad \beta > 0, \quad (9)$$

Here, β can be obtained by substituting the assumed solution into the governing non-dimensional equation as

$$\beta = \frac{S \pm \sqrt{S^2 + 4[M + L + d]}}{2}, \quad \beta > 0. \quad (10)$$

The nature of the exponential solutions is determined by the discriminant $S^2 + 4(M + L + d)$. For $S^2 + 4(M + L + d) > 0$, two real exponential branches $\beta_{\pm}$ are obtained. A decaying exponential solution requires $\beta > 0$. In particular, two decaying exponential branches exist when $S > 0$, $M + L + d < 0$ and $S^2 + 4(M + L + d) > 0$. When $M + L + d > 0$ and the discriminant is positive, one root is positive and other is negative, so only one decaying branch exists. When $S^2 + 4(M + L + d) = 0$, the repeated root is $\beta = \frac{S}{2}$, and a decaying solution exists only when $S > 0$. For $S^2 + 4(M + L + d) < 0$, no real exponential decay solution exists. Thus, the condition $\beta > 0$ is the explicit criterion for exponential decay as $\eta \rightarrow \infty$.

For the algebraically decaying solution, we consider the following special parameter choice: $M = L = 0, S = \sqrt{6}, d = -1$.

The exact algebraically decaying solution is given by

$$f(\eta) = \frac{6}{\eta + \sqrt{6}}. \quad (11)$$

2.3. Verification of analytical solutions

The analytical solutions were verified by direct substitution into the corresponding similarity ordinary differential equations and boundary conditions. For the exponentially decaying branch, substitution of the assumed exponential velocity profile into the reduced momentum equation yields the characteristic relation governing the admissible decay rates. The thermal and concentration solutions are then obtained by transforming the corresponding equations into Kummer-type equations, and their boundary conditions are imposed to determine the solution constants. For the algebraically decaying branch, the special parameter choice introduced above reduces the momentum equation to an equation admitting a power-law solution. Substitution of the power-law form produces the corresponding characteristic equation, while the requirement of decay at large similarity distance imposes a negative admissible exponent. The same procedure is subsequently applied to the thermal and concentration equations under the four prescribed boundary conditions. Thus, each analytical solution satisfies both the governing similarity equation and its associated boundary restrictions.

3. Analytical solutions for temperature equation for PST case

Analytical solutions for the temperature equation under the Prescribed Surface Temperature (PST) condition provide exact expressions for the temperature distribution in a fluid when the surface temperature

is fixed. These solutions are important because they allow the precise prediction of heat transfer rates and thermal profiles without relying solely on numerical approximations. For the prescribed surface temperature (PST) condition with a power-law wall temperature, the boundary conditions are [12–14].

$$T = T_w = T_\infty + A \left(\frac{x}{l} \right)^{\delta_T} \text{aty} = 0, \quad (12)$$

$$T = T_\infty, \text{asy} \rightarrow \infty.$$

Here, δ_T is the power index, T_w and T_∞ are the temperature, and the neglected term $\frac{\kappa}{\rho C_p} \frac{\partial^2 T}{\partial x^2}$ gives $\frac{\nu \delta_T (\delta_T - 1)}{a x^2} \theta(\eta)$ after non-dimensionalities of the temperature equation, which is negligible for large x values, by using the similarity for temperature to obtain Eq. (4) as

$$(1 + Nr) \frac{d^2 \theta(\eta)}{d\eta^2} + Pr f(\eta) \frac{d\theta(\eta)}{d\eta} - Pr \delta_T \frac{df(\eta)}{d\eta} \theta(\eta) = 0, \quad (13)$$

with corresponding boundary restrictions [12,13].

$$\theta(0) = 1, \theta(\infty) = 0, \quad (14)$$

incorporating the solutions of momentum equation obtains

$$(1 + Nr) \frac{d^2 \theta(\eta)}{d\eta^2} + Pr \left(S + \frac{d}{\beta} (1 - e^{-\beta \eta}) \right) \frac{d\theta(\eta)}{d\eta} - Pr \delta_T (d e^{-\beta \eta}) \theta(\eta) = 0, \quad (15)$$

To solve the above equation, we introduce the novel variable $\Omega = \frac{Pr d}{\beta^2 (1 + Nr)} e^{-\beta \eta}$, incorporating the variable Eq. (15) obtains

$$\Omega \frac{d^2 \theta}{d\Omega^2} + (1 - b + \Omega) \frac{d\theta}{d\Omega} - \delta_T \theta(\Omega) = 0, \quad (16)$$

with the boundary restrictions Eq. (16) as

$$\theta \left(\frac{dPr}{\beta^2 (1 + Nr)} \right) = 1, \theta(0) = 0, \quad (17)$$

here $b = \frac{Pr(S\beta + d)}{(1 + Nr)\beta^2}$, and δ_T is the power index

Using the above series solution, the temperature solution as a function of Ω is

$$\theta(\Omega) = \left(\frac{\Omega}{\Omega_0} \right)^b \frac{\text{Hypergeometric}(b - \delta_T, b + 1, -\Omega)}{\text{Hypergeometric}(b - \delta_T, b + 1, -\Omega_0)}, \quad (18)$$

The solution as a function of η reads

$$\theta(\eta) = e^{-\beta b \eta} \frac{\text{Hypergeometric} \left(b - \delta_T, b + 1, -\frac{Pr d}{\beta^2 (1 + Nr)} e^{-\beta \eta} \right)}{\text{Hypergeometric} \left(b - \delta_T, b + 1, -\frac{Pr d}{\beta^2 (1 + Nr)} \right)}, \quad (19)$$

here $\Omega_0 = \frac{Pr d}{\beta^2 (1 + Nr)}$.

Energy gradient defined as

$$\theta_\eta(0) = -\beta b + \frac{Pr d}{\beta (1 + Nr)} \frac{b - \delta_T}{b + 1} \frac{\text{Hypergeometric} \left(b - \delta_T + 1, b + 2, -\frac{Pr d}{\beta^2 (1 + Nr)} \right)}{\text{Hypergeometric} \left(b - \delta_T, b + 1, -\frac{Pr d}{\beta^2 (1 + Nr)} \right)}, \quad (20)$$

For the algebraically decaying solution above equation calculated as

$$(1 + Nr) \frac{d^2 \theta(\eta)}{d\eta^2} + \frac{6Pr}{\eta + \sqrt{6}} \frac{d\theta(\eta)}{d\eta} + \frac{6\delta_T Pr}{(\eta + \sqrt{6})^2} \theta(\eta) = 0. \quad (21)$$

with the same BCs as equation. The solution gives an algebraically decaying function as $\theta(\eta) = D((\eta + \sqrt{6}))^r$.

Incorporate it into the Eq. (21) obtains

$$(1 + Nr)r^2 + (6Pr - (1 + Nr))r + 6Pr\delta_T = 0. \quad (22)$$

The above equation formulated as

$$r = \frac{(1 + Nr) - 6Pr \pm \Xi}{2(1 + Nr)}, \quad (23)$$

where

$$\Xi = \sqrt{(6Pr - (1 + Nr))^2 - 24Pr\delta_T(1 + Nr)}, \quad (24)$$

to obtain the value r , it requires

$$(6Pr - (1 + Nr))^2 - 24Pr\delta_T(1 + Nr) \geq 0, \quad (25)$$

which is same as below

$$\delta_T \leq \frac{(6Pr - (1 + Nr))^2}{24Pr(1 + Nr)}. \quad (26)$$

Using wall B. Cs, we obtain $D = (6)^{-r/2}$. If $y \rightarrow \infty$, then r must be negative. The energy gradient was formulated as follows:

$$\frac{d\theta(0)}{d\eta} = \frac{r}{\sqrt{6}}. \quad (27)$$

For the upper branch solution

$$1 + Nr - 6Pr + \Xi < 0. \quad (28)$$

For the Lower solution branch, it is obtained that

$$1 + Nr - 6Pr - \Xi < 0. \quad (29)$$

4. Analytical solutions for temperature equation for PHF case

Analytical solutions for the energy equation under the Prescribed Heat Flux (PHF) condition provide exact expressions for the temperature distribution when the rate of heat exchange at the sheet is specified rather than the energy itself. These solutions are important because they allow for an accurate evaluation of the heat transfer characteristics and thermal gradients in the fluid. The corresponding boundary restrictions were [12,13].

$$-\kappa \frac{\partial T}{\partial y} = q_w = D_1 \left(\frac{x}{l} \right)^{\delta_T} \sqrt{\frac{a}{\nu}}, \text{aty} = 0, \quad (30)$$

$$T = T_\infty, \text{asy} \rightarrow \infty,$$

here κ defines the thermal conductivity, D_1 represents constant, δ_T is power index.

Then incorporating in to temperature equation exhibits

$$(1 + Nr) \frac{d^2 \varphi(\eta)}{d\eta^2} + Pr f(\eta) \frac{d\varphi(\eta)}{d\eta} - Pr \delta_T \frac{df(\eta)}{d\eta} \varphi(\eta) = 0, \quad (31)$$

with the boundary restrictions as

$$\frac{d\varphi(0)}{d\eta} = -1, \varphi(\infty) = 0. \quad (32)$$

Incorporating the velocity functions yields

$$(1 + Nr) \frac{d^2 \varphi(\eta)}{d\eta^2} + Pr \left(S + \frac{d}{\beta} (1 - e^{-\beta \eta}) \right) \frac{d\varphi(\eta)}{d\eta} - Pr \delta_T (d e^{-\beta \eta}) \varphi(\eta) = 0, \quad (33)$$

Introducing novel variable, $\Omega = \frac{Pr d}{\beta^2 (1 + Nr)} e^{-\beta \eta}$, and incorporating it into Eq. (33) gives

$$\Omega \frac{d^2 \varphi(\eta)}{d\Omega^2} + (1 - b + \Omega) \frac{d\varphi(\eta)}{d\Omega} - \delta_T \varphi(\eta) = 0, \quad (34)$$

with boundary restrictions as

$$\frac{d\varphi\left(\frac{Prd}{(1+Nr)\beta^2}\right)}{d\Omega} = \frac{(1+Nr)\beta}{Prd}, \varphi(0) = 0, \quad (35)$$

by using the above series solution, the solution obtains

$$\varphi(\eta) = \left[\frac{-\frac{\beta(b+1)(1+Nr)}{Prd} e^{-\beta b\eta} \times \text{Hypergeometric}\left(b-\delta_T, b+1, -\frac{Prd}{(1+Nr)\beta^2} e^{-\beta\eta}\right)}{(b-\delta_T) \text{Hypergeometric}\left(b-\delta_T+1, b+2, -\frac{Prd}{(1+Nr)\beta^2}\right)} - \frac{b\beta^2(1+Nr)}{Prd} (b+1) \text{Hypergeometric}\left(b-\delta_T, b+1, -\frac{Prd}{(1+Nr)\beta^2}\right) \right] \quad (36)$$

and also

$$\varphi(0) = \left[\frac{-\frac{\beta(b+1)(1+Nr)}{Prd} \times \text{Hypergeometric}\left(b-\delta_T, b+1, -\frac{Prd}{(1+Nr)\beta^2}\right)}{(b-\delta_T) \text{Hypergeometric}\left(b-\delta_T+1, b+2, -\frac{Prd}{(1+Nr)\beta^2}\right)} - \frac{b\beta^2(1+Nr)}{Prd} (b+1) \text{Hypergeometric}\left(b-\delta_T, b+1, -\frac{Prd}{(1+Nr)\beta^2}\right) \right] \quad (37)$$

For the algebraic decay solution, we use the velocity transport equation incorporated as

$$(1+Nr) \frac{d^2\varphi(\eta)}{d\eta^2} + \left(\frac{6Pr}{\eta+\sqrt{6}}\right) \frac{d\varphi(\eta)}{d\eta} + \frac{\delta_T Pr 6}{(\eta+\sqrt{6})^2} \varphi(\eta) = 0. \quad (38)$$

with the same BCs as in Eq. (38). The solution obtained by algebraic decay defined as $\varphi(\eta) = E(\eta+\sqrt{6})^r$.

Incorporating Eq. (39) with power law index defines

$$(1+Nr)r^2 + (6Pr - (1+Nr))r + 6Pr\delta_T = 0. \quad (39)$$

Then it is obtained

$$r = \frac{(1+Nr-6Pr) \pm \zeta}{2(1+Nr)}. \quad (40)$$

To obtain value of r , it requires

$$(6Pr - (1+Nr))^2 - 24Pr\delta_T(1+Nr) \geq 0, \quad (41)$$

which is same as

$$\delta_T \leq \frac{(6Pr - (1+Nr))^2}{24Pr(1+Nr)}. \quad (42)$$

Based on (35), $E = -\frac{1}{r(6(r-1)/2)}$. if r is negative, it satisfies $y \rightarrow \infty$.

The energy gradient defined as $\varphi(0) = -\frac{\sqrt{6}}{r}$.

By using second branch solution obtains

$$(1+Nr-6Pr) + \zeta < 0, \quad (43)$$

for the Lower solution branch, it is obtained that

$$(1+Nr-6Pr) - \zeta < 0 \quad (44)$$

5. Analytical solutions for concentration equation for PSC case

Analytical solutions for the concentration equation under the Prescribed Surface Concentration (PSC) condition provide exact expres-

sions for the distribution of a solute when the concentration at the boundary is fixed. These solutions are important because they allow precise prediction of mass transfer rates and concentration profiles without relying solely on numerical methods For the PSC case, such as boundary conditions [20,21,23].

$$C = C_w = C_\infty + A_c \left(\frac{x}{l}\right)^{\delta_c} \text{ at } y = 0, \quad (46)$$

$$C = C_\infty, \text{ as } y \rightarrow \infty.$$

Here, δ_c is the power index, C_w and C_∞ is the concentration, using the similarity for concentration to obtain Eq. (5) as

$$\frac{d^2\phi(\eta)}{d\eta^2} + Scf(\eta) \frac{d\phi(\eta)}{d\eta} - Sc\delta_c \frac{d\phi(\eta)}{d\eta} \phi(\eta) = 0, \quad (47)$$

with corresponding boundary restrictions

$$\phi(0) = 1, \phi(\infty) = 0, \quad (48)$$

incorporating the solutions of velocity equation obtains

$$\frac{d^2\phi(\eta)}{d\eta^2} + Sc\left(S + \frac{d}{\beta}(1 - e^{-\beta\eta})\right) \frac{d\phi(\eta)}{d\eta} - Sc\delta_c (de^{-\beta\eta})\phi(\eta) = 0, \quad (49)$$

To solve the above equation, we introduce the novel variable $\Theta = \frac{Scd}{\beta^2} e^{-\beta\eta}$, incorporating the variable Eq. (15) obtains

$$\Theta \frac{d^2\phi}{d\Theta^2} + (1 - b_c + \Theta) \frac{d\phi}{d\Theta} - \delta_c \phi(\Theta) = 0, \quad (50)$$

with the boundary restrictions Eq. (16) as

$$\phi\left(\frac{dSc}{\beta^2}\right) = 1, \phi(0) = 0, \quad (51)$$

here $b_c = \frac{Sc(S\beta+d)}{\beta^2}$, and δ_c is the power index

Using the above series solution, the concentration solution as a function of Θ is

$$\phi(\Theta) = \left(\frac{\Theta}{\Theta_0}\right)^{b_c} \frac{\text{Hypergeometric}(b_c - \delta_c, b_c + 1, -\Theta)}{\text{Hypergeometric}(b_c - \delta_c, b_c + 1, -\Theta_0)}, \quad (52)$$

here, $\Theta_0 = \frac{Scd}{\beta^2}$. The solution as a function of η reads

$$\phi(\eta) = e^{-\beta b_c \eta} \frac{\text{Hypergeometric}\left(b_c - \delta_c, b_c + 1, -\frac{Scd}{\beta^2} e^{-\beta\eta}\right)}{\text{Hypergeometric}\left(b_c - \delta_c, b_c + 1, -\frac{Scd}{\beta^2}\right)}, \quad (53)$$

Mass transfer gradient defined as

$$\phi_\eta(0) = -\beta b_c + \frac{Scd}{\beta} \frac{b_c - \delta_c}{b_c + 1} \frac{\text{Hypergeometric}\left(b_c - \delta_c + 1, b_c + 2, -\frac{Scd}{\beta^2}\right)}{\text{Hypergeometric}\left(b_c - \delta_c, b_c + 1, -\frac{Scd}{\beta^2}\right)}, \quad (54)$$

For the algebraically decaying solution above equation calculated as

$$\frac{d^2\phi(\eta)}{d\eta^2} + Sc\left(\frac{6}{\eta+\sqrt{6}}\right) \frac{d\phi(\eta)}{d\eta} + \delta_c Sc \frac{6}{(\eta+\sqrt{6})^2} \phi(\eta) = 0. \quad (55)$$

with the same BCs as equation. The solution gives an algebraically decaying function as $\phi(\eta) = D_c((\eta+\sqrt{6}))^r$.

Incorporate it into Eq. (21) obtains

$$r^2 + (6Sc - 1)r + 6Sc\delta_c = 0. \quad (56)$$

The above equation formulated as

$$r = \frac{1 - 6Sc \pm \Xi c}{2}, \quad (57)$$

to obtain the value r , it requires

$$(6Sc - 1)^2 - 24Sc\delta_c \geq 0, \quad (58)$$

which is same to the following

$$\delta_c \leq \frac{(6Sc - 1)^2}{24Sc}. \quad (59)$$

Using wall B. Cs, we obtain $D = (6)^{-r/2}$. If $y \rightarrow \infty$, then r must be negative. The energy gradient was formulated as follows:

$$\frac{d\phi(0)}{d\eta} = \frac{r}{\sqrt{6}}. \quad (60)$$

For the upper branch solution

$$1 - 6Sc + \Xi c < 0. \quad (61)$$

For the Lower solution branch, it is obtained that

$$1 - 6Sc - \Xi c < 0, \quad (62)$$

6. Analytical solutions for concentration equation for PMF case

Analytical solutions for the concentration equation under the Prescribed Mass Flux (PMF) condition provide exact expressions for the solute distribution when the rate of solutal transfer at the surface is specified. These solutions are important because they enable the accurate determination of concentration gradients and mass transfer rates without relying solely on numerical approximation. The corresponding boundary restrictions were [22–24].

$$-D_B \frac{\partial C}{\partial y} = m_w = C_1 \left(\frac{x}{l} \right)^{\beta_c} \sqrt{\frac{a}{v}}, \text{ at } y = 0, \quad (63)$$

$$C = C_\infty, \text{ as } y \rightarrow \infty,$$

C_1 represents constant, δ_c is power index.

Then incorporating in to concentration equation exhibits

$$\frac{d^2\chi(\eta)}{d\eta^2} + Scf(\eta) \frac{d\chi(\eta)}{d\eta} - Sc\delta_c \frac{df(\eta)}{d\eta} \chi(\eta) = 0, \quad (64)$$

with the boundary constraints as

$$\frac{d\chi(0)}{d\eta} = -1, \chi(\infty) = 0. \quad (65)$$

Incorporating the velocity functions yields

$$\frac{d^2\chi(\eta)}{d\eta^2} + Sc \left(S + \frac{d}{\beta} (1 - e^{-\beta\eta}) \right) \frac{d\chi(\eta)}{d\eta} - Sc\delta_c (de^{-\beta\eta}) \chi(\eta) = 0, \quad (66)$$

Introducing a novel variable, $\Theta = \frac{Scd}{\beta^2} e^{-\beta\eta}$, and incorporating it into Eq. (66) gives

$$\Theta \frac{d^2\chi(\eta)}{d\Theta^2} + (1 - b_c + \Theta) \frac{d\chi(\eta)}{d\Theta} - \delta_c \chi(\eta) = 0, \quad (67)$$

with boundary conditions as

$$\frac{d\chi}{d\Theta} \left(\frac{Scd}{\beta^2} \right) = \frac{\beta}{Scd} \chi(0) = 0. \quad (68)$$

By using the above series solution, the solution obtains

$$\chi(\eta) = \left[\frac{-\frac{\beta(b_c+1)}{Scd} e^{-\beta\eta} \times \text{Hypergeometric} \left(b_c - \delta_c, b_c + 1, -\frac{Scd}{\beta^2} e^{-\beta\eta} \right)}{(b_c - \delta_c) \text{Hypergeometric} \left(b_c - \delta_c + 1, b_c + 2, -\frac{Scd}{\beta^2} \right)} - \frac{b_c \beta^2}{Scd} (b_c + 1) \text{Hypergeometric} \left(b_c - \delta_c, b_c + 1, -\frac{Scd}{\beta^2} \right) \right] \quad (69)$$

and also

$$\chi(0) = \left[\frac{-\frac{\beta(b_c+1)}{Scd} \times \text{Hypergeometric} \left(b_c - \delta_c, b_c + 1, -\frac{Scd}{\beta^2} \right)}{(b_c - \delta_c) \text{Hypergeometric} \left(b_c - \delta_c + 1, b_c + 2, -\frac{Scd}{\beta^2} \right)} - \frac{b_c \beta^2}{Scd} (b_c + 1) \text{Hypergeometric} \left(b_c - \delta_c, b_c + 1, -\frac{Scd}{\beta^2} \right) \right]$$

for the algebraic decay solution, we use the velocity transport equation incorporated as

$$\frac{d^2\chi(\eta)}{d\eta^2} + Sc \left(\frac{6}{\eta + \sqrt{6}} \right) \frac{d\chi(\eta)}{d\eta} + \delta_c Sc \frac{6}{(\eta + \sqrt{6})^2} \chi(\eta) = 0. \quad (70)$$

Using the same BCs as those in Eq. (65). The solution obtained by algebraic decay is defined as $\chi(\eta) = E_c(\eta + \sqrt{6})^r$.

Incorporating Eq. (70) with power law index defines

$$r^2 + (6Sc - 1)r + 6Sc\delta_c = 0. \quad (71)$$

Then it is obtained

$$r = \frac{1 - 6Sc \pm \Xi c}{2}, \quad (72)$$

where $\Xi c = \sqrt{(6Sc - 1)^2 - 24Sc\delta_c}$.

To obtain value of r , it requires

$$(6Sc - 1)^2 - 24Sc\delta_c \geq 0, \quad (73)$$

which is same as

$$\delta_c \leq \frac{(6Sc - 1)^2}{24Sc}. \quad (74)$$

Based on the BCs, gain $E_c = \frac{-6(r-1)/2}{r}$, if r is negative, it satisfies $y \rightarrow \infty$.

The energy gradient defined as $\chi(\eta) = -\frac{(\eta + \sqrt{6})^r}{r6^{(r-1)/2}}$.

By using second branch solution obtains

$$1 - 6Sc + \Xi c < 0, \quad (75)$$

for the Lower solution branch, it is obtained that

$$1 - 6Sc - \Xi c < 0. \quad (76)$$

7. Results and discussion

The aim of the current analyses is to examine the thermal and solutal transfer impact on Newtonian fluid flow across permeable expanding/shrinking surfaces, and the impact of solar radiation, magnetic field, and Schmidt number on fluid flow has been extracted. Governing Navier-Stokes partial differential equations are formulated into ordinary differential equations via similarity variables, and closed-form multiple analytic solutions are obtained. The power law heat and mass flux

conditions were examined using the Kummer function and hyper geometric series solution. Thermal and solutal transfer problems for the algebraically decaying flow across the shrinking solution were also investigated. It was found that the energy and mass transfer with mass injection were significantly different from the mass suction values.

Several physical parameters were studied using the following plots:

Fig. 2(a) and (b) represents the contour plots for temperature and Fig. 2(c) and (d) represents the contour plots for concentration; temperature graphs are compared with the peer work [18], graphs plotted for the shrinking surface and obtained for both the first- and second-

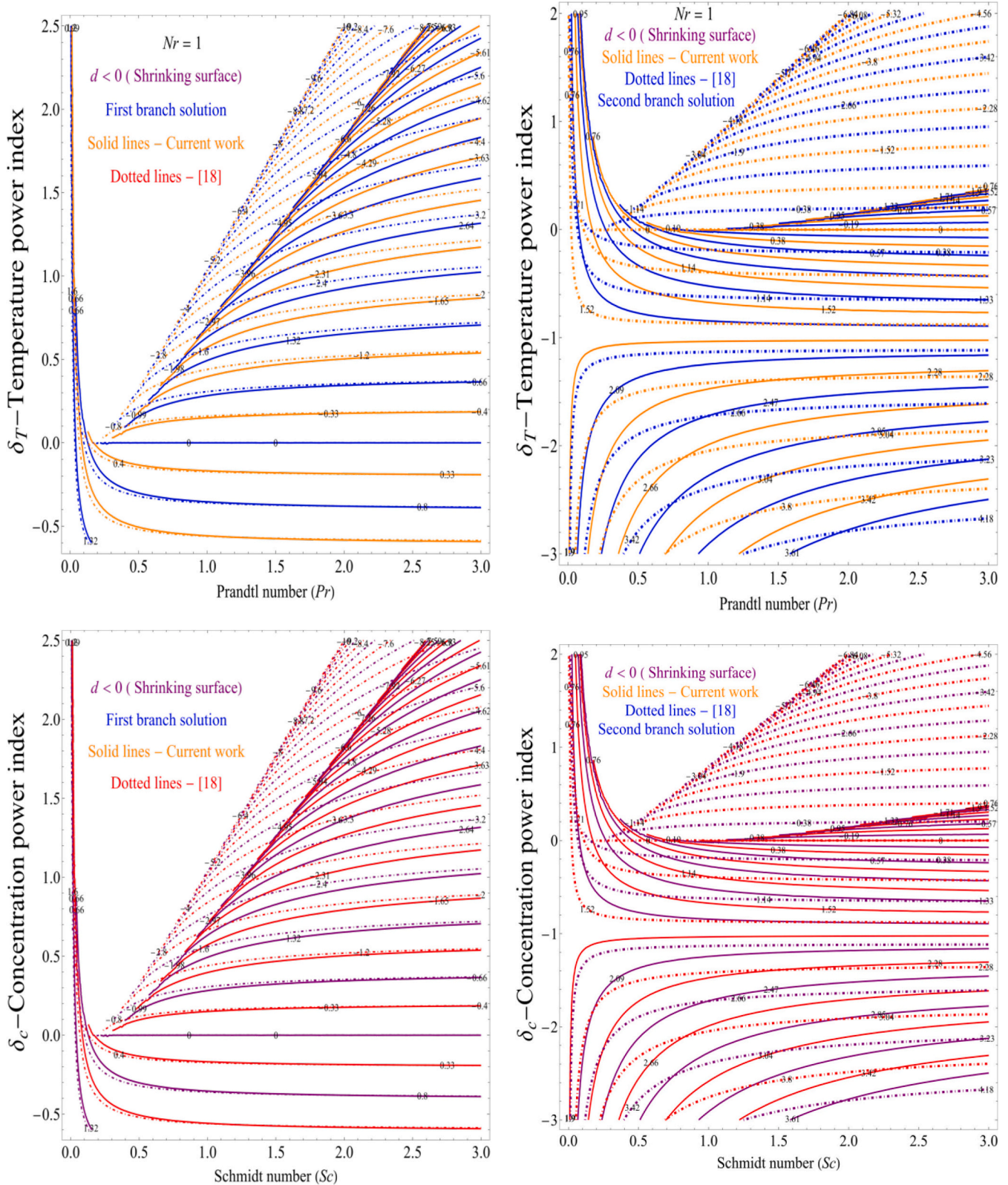
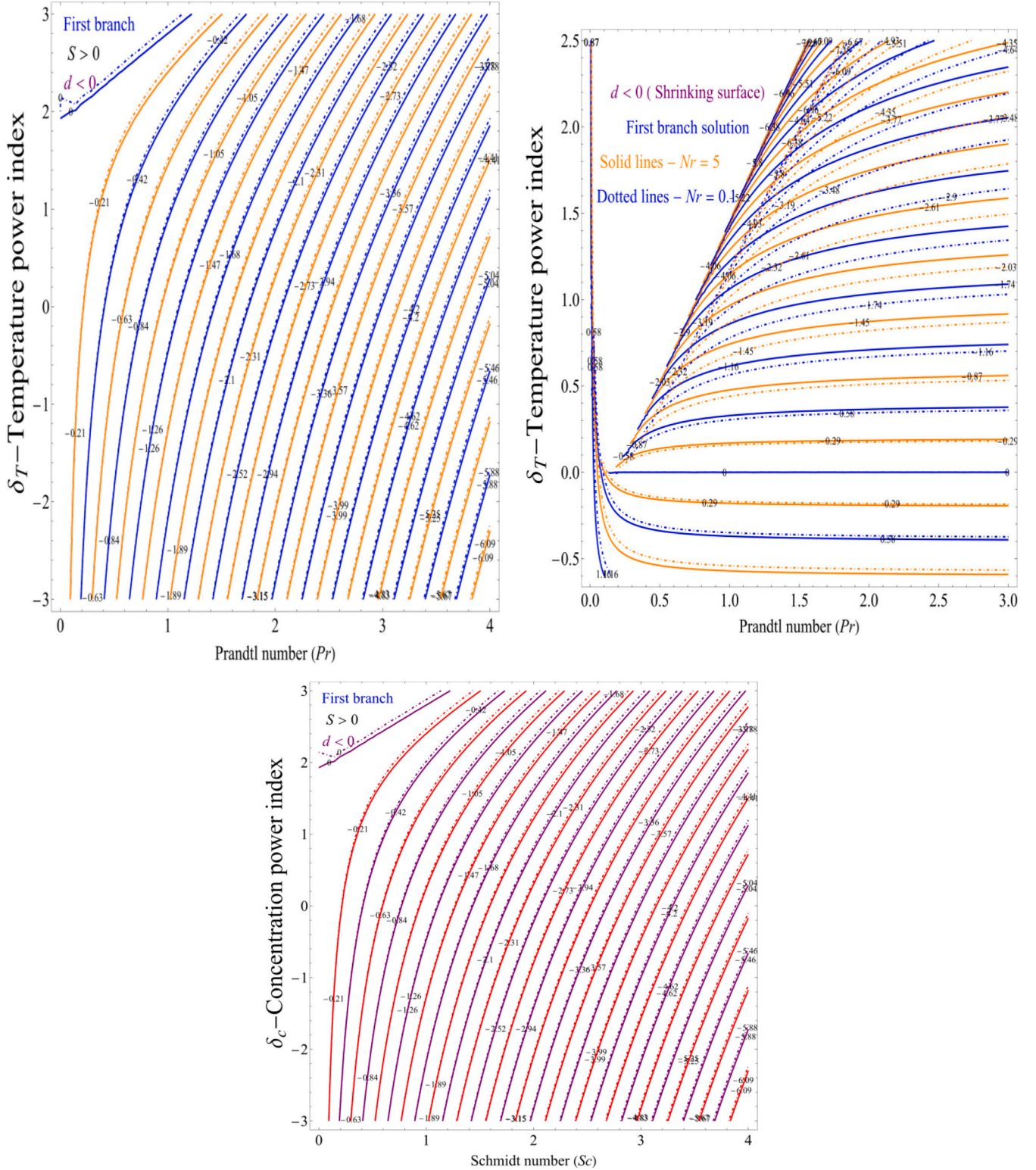


Fig. 2. (a): δ_T plot for stretching surface. (b) δ_T plot for shrinking surface. (c) δ_c plot for first branch solution. (d) δ_c plot for second branch solution.

branch solutions. In the first branch, the velocity gradients initially rise steeply near the wall and gradually level off, showing strong boundary-layer adherence and stable flow development. The comparison between the solid lines (current results) and dotted lines from the literature demonstrates close agreement, confirming the accuracy of the present formulation. In contrast, the second branch exhibits a distinctly different trend, where the profiles either overshoot or decay more slowly,

indicating the onset of reversed flow or weaker boundary layer attachment, a characteristic commonly associated with shrinking surfaces owing to the adverse pressure environment.

Fig. 3(a) and (b) portrays the contour plots for energy and Fig. 3(c) portrays the contour plots for concentration, graphs plotted for upper branch solution and plotted for various values of porosity term, in Fig. 3 (a), the flow characteristic is shown for a unique set of parameters,



where the curves demonstrate that the momentum decreases more rapidly with an upsurge in the shrinking rate, confirming that the concentration precisely weakens fluid movement away from the sheet. Fig. 3(b) compares two various wall deformation terms, where the solid lines correspond to greater values and the dotted lines to lesser ones. The

outcomes clearly show that upsurging the surface term enhances the flow strength and decays velocity, leading to a greater boundary layer, whereas a lesser value causes a rapid fall in momentum, reflecting a greater shrinking effect. Fig. 3(c) portrays the impact of thermal radiation, and the observed similarity among the systems suggests that

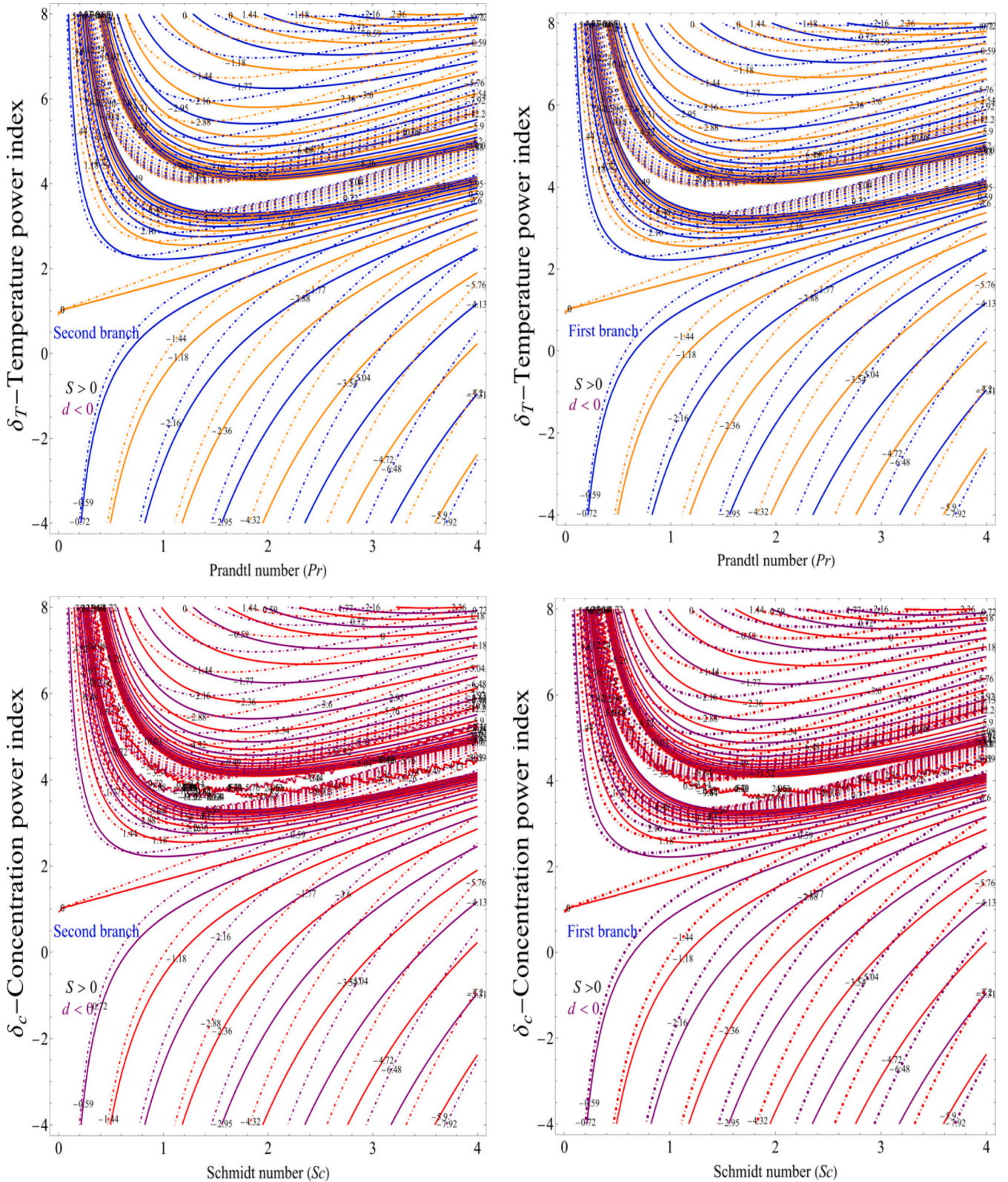


Fig. 4. (a) δ_T plot for second branch. (b): δ_T plot for first branch solution at $d < 0$. (c): δ_c plot for second branch at $d < 0$. (d): δ_c plot for first branch solution at $d < 0$.

radiation produce to stabilizing the movement by decaying resistance and smoothing the velocity gradient across the boundary layer.

Fig. 4(a) and (b) shows the contour graphs for energy and Fig. 4(c) and (d) represents the contour plots for mass transfer; graphs plotted for both branch solutions and plotted for different numbers of thermal radiation. The comparison between the current results and those from the literature was in close agreement, confirming the accuracy of the current formulation. In contrast, the second branch provides a distinctly different phenomenon, where the profiles either boost or decay gradually, indicating the onset of reversed movement or weaker boundary layer attachment, a behavior commonly associated with shrinking sheets owing to the adverse pressure environment.

Fig. 5 (a) shows the impact of the porosity resistance term on the velocity, and Fig. 5 (b) and (c) represents the impact of the porosity term on the PST and PHF cases. Upsurging the porosity term decreases the velocity and increases the temperature in both the PST and PHF cases. Increasing the porous-resistance parameter enhances the resistance offered by the porous medium and consequently suppresses the fluid velocity. The reduction in fluid motion weakens convective transport and modifies the thermal boundary layer, resulting in higher temperature levels in both the PST and PHF cases. Thus, the observed velocity reduction is physically attributed to the increased resistance imposed by

the porous medium, rather than to a direct increase in the physical porosity or void fraction of the medium.

Fig. 6(a) and (b) shows the magnetic field and stretching term impact on velocity, increasing the magnetic field, and reducing the stretching term. The magnetic field and stretching term play major roles in governing the characteristics of fluid flow across a surface. The magnetic field produces a resistive force, known as the Lorentz force, which is opposite to the direction of movement and slows down the fluid, thereby decaying the momentum and thinning the boundary layer. This damping impact increases with an increase in the magnetic field strength, making the movement more stable but less mobile. Meanwhile, the stretching term controls how rapidly the surface stretches, directly impacting the momentum distribution within the boundary layer. A higher stretching rate increases the fluid movement near the wall, enhances the velocity gradient, and boosts the flow away from the surface.

Fig. 7(a) and (b) shows the influence of the temperature power index on both the PST and PHF temperature profiles. Upgrading the power index term increases temperature. The temperature power index indicates how the sheet temperature or heat flux varies along the expanding or shrinking sheet, and it plays a precise role in controlling the energy behavior of fluid movement. When parameter increases, the temperature difference between the sheet and the ambient fluid

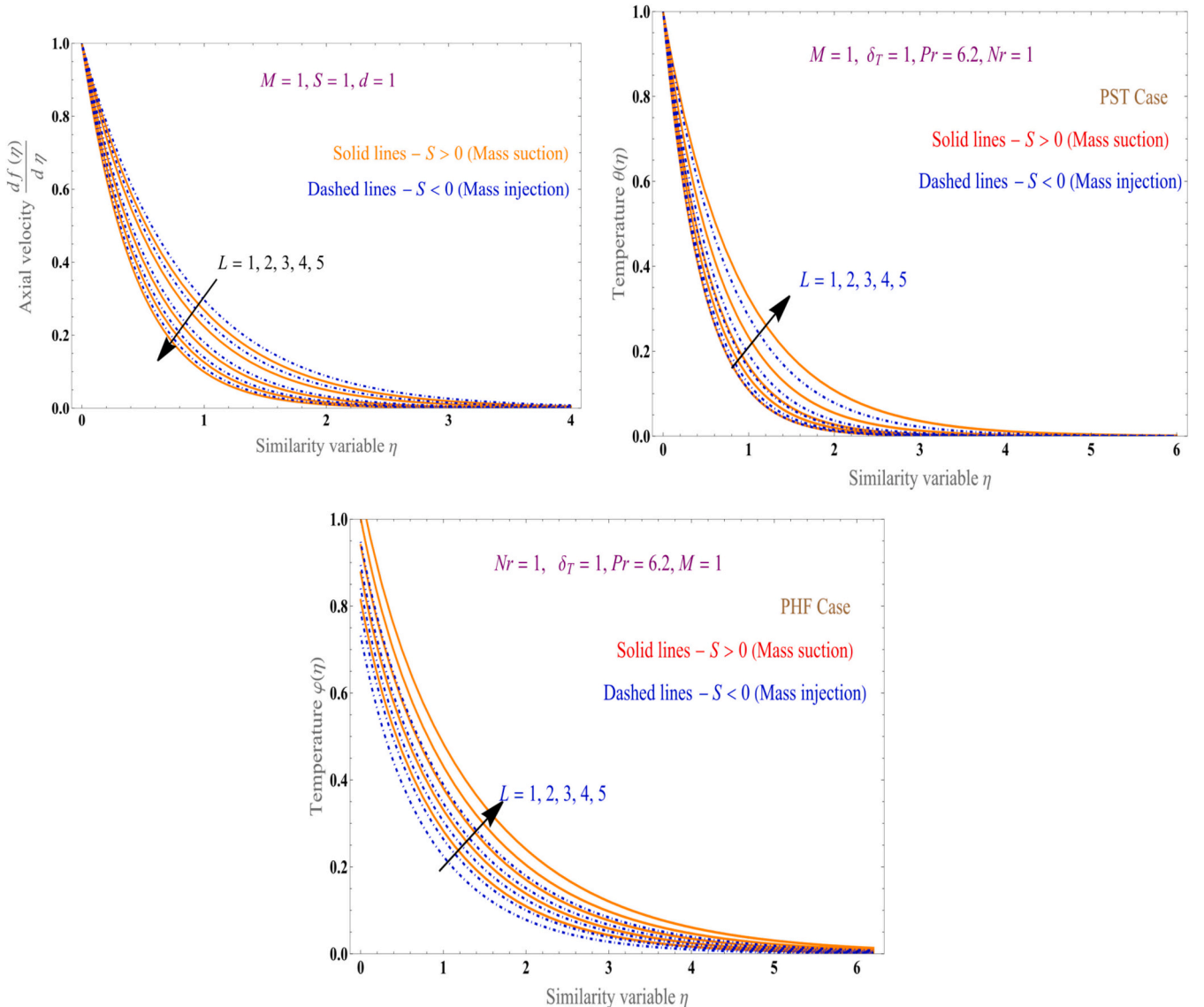


Fig. 5. (a): Porosity term impact on velocity. (b): Porosity term impact on PST temperature. (c): Porosity term impact on PHF temperature.

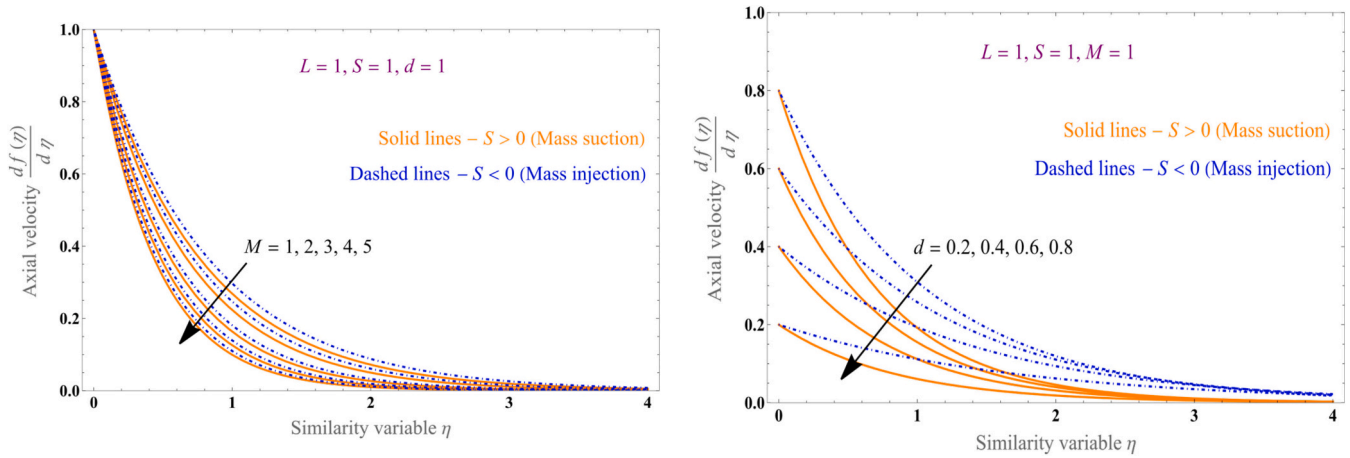


Fig. 6. (a) Magnetic term impact on velocity. (b): Stretching term impact on velocity.

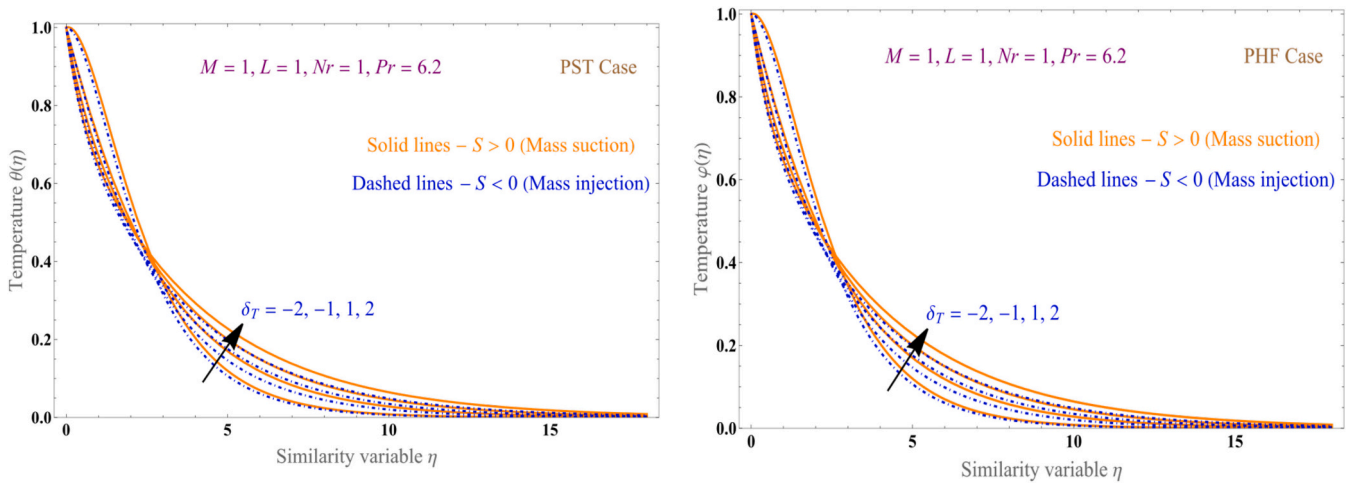


Fig. 7. (a): Temperature power index impact on PST temperature. (b): Temperature power index impact on PHF temperature.

increases, which increases the heat exchange and modifies the thickness of the energy boundary layer. In the case of the prescribed heat flux, a greater temperature power index modifies the surface heating rate and, tends to result in stronger thermal gradients and more efficient energy

transport within the fluid.

Fig. 8(a) and (b) shows the influence of thermal radiation on both the PST and PHF case energy profiles. Increasing the thermal radiation increased the temperature. Thermal radiation plays a major role in

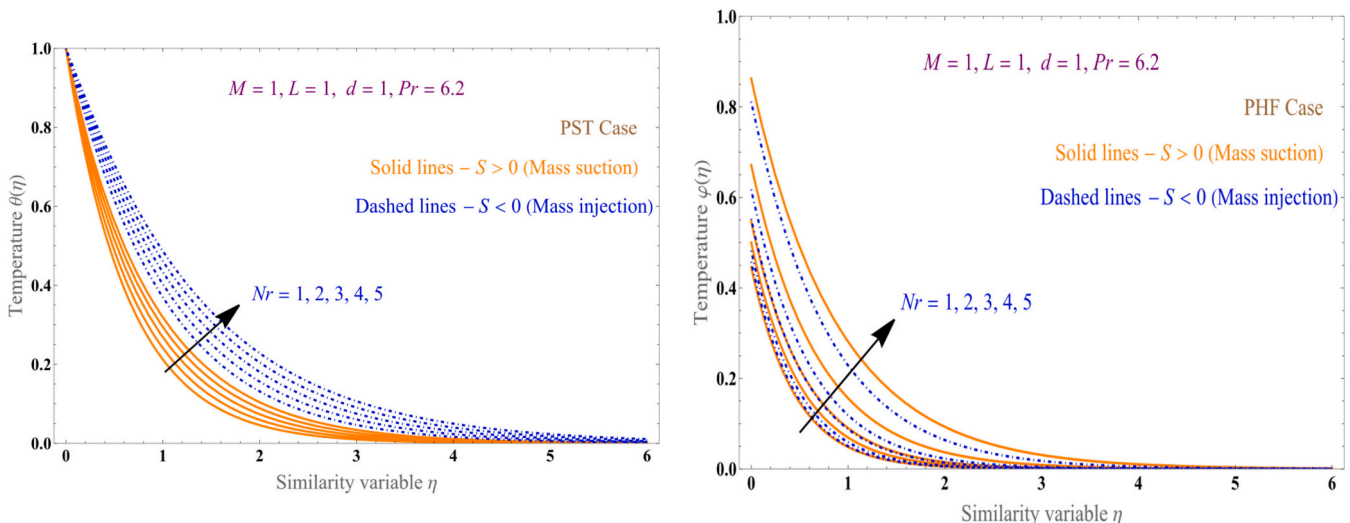


Fig. 8. (a): Thermal radiation impact on PST temperature. (b): Thermal radiation influence on PHF temperature.

studying the heat transfer behavior of fluid flow, especially when the sheet is subjected to prescribed temperature conditions. It defines the emission and absorption of temperature in the form of electromagnetic waves. When thermal radiation is enhanced, the energy exchange from the sheet to the fluid becomes more significant, leading to enhanced fluid energy and a reduction in the thermal gradient near the wall. Under the prescribed heat flux cases, stronger radiation increases the overall heat transfer rate, whereas in the prescribed surface energy scenarios, it causes the energy profiles to become lower and more uniform across the boundary layer.

Fig. 9(a) and (b) shows the influence of Schmidt number on both the PMF and PSC concentration profiles. Upgrading the Schmidt number decreases the concentration. The Schmidt number is a key parameter that expresses the ratio of momentum diffusivity to mass diffusivity, and it precisely affects the mass distribution in fluid movement. A greater Schmidt number produces lower mass diffusivity, meaning that the species diffusion rate is less than the velocity transfer. As a result, the concentration decreases and the concentration gradient near the sheet increased. In contrast, a lower Schmidt number increases mass diffusion, provides a thicker concentration, and results in a more gradual variation in concentration. Under the prescribed mass flux cases, variations in the Schmidt number directly affect the rate at which masses are transferred from the sheet into the fluid, whereas for the prescribed sheet concentration, it determines how quickly the concentration reaches the ambient value. The Schmidt number play an important role in processes such as drying, gas absorption, pollutant dispersion, and biological fluid transport.

Fig. 10(a) and (b) shows the influence of concentration power index on the concentration profile. Upgrading the term increases concentration. The concentration power index characterizes how the sheet concentration varies along the fluid direction and plays a major role in shaping the mass distribution within the fluid. An upsurge in the concentration power index generally modifies the variation in sheet concentration, leading to a greater concentration gradient near the wall. Under the prescribed mass flux cases, a greater concentration power index increases the rate at which mass diffuses into or out of the sheet, whereas for the prescribed surface concentration, it modifies the concentration and impacts how quickly the mass spreads within the fluid.

Fig. 11 (a) and (b) represents the temperature and concentration profiles and the temperature profile, respectively, compared with earlier results by examining how profiles change with the similarity variable. In both graphs, the solid lines representing the current study display a sharper initial peak and maintain higher temperature and concentration values near the wall, especially for the PST and PSC cases, compared

with to the dotted lines corresponding to prior work. The difference was most pronounced close to the surface, reflecting the influence of the boundary conditions of the current model. Dual branches are visible in the upper branch, with a steeper decline near the wall and a lower branch with a gentler descent suggesting multiple solution regimes for each case. As the similarity variable increases, all profiles eventually converge to zero; however, the current work shows a more extended influence further from the wall.

Fig. 12(a)–(d) shows the heat transfer for various values of mass suction, and the graphs show how the mass suction and temperature power index influence the flow and energy transfer for both prescribed conditions. When the corresponding term upsurges sharp transitions and sudden changes, the accuracy as index crosses certain thresholds, portraying bifurcation in solution characteristics. These changes are enhanced in the PST case, where the profiles produce strong nonlinearity between the enhancing and decaying branches. Mass suction further enhances the patterns, shifting points and modifying the curves where precise transition occurs.

Fig. 13(a) and (b) shows the mass transfer profiles for various mass suction values and, graphs plotted for both PSC and PMF conditions. As the term increases, the concentration portions for the prescribed mass concentration and prescribed mass flux conditions exhibit abrupt transitions and distinct turning points. The PMC profiles reflect a greater sensitivity, with more frequent jumps and a complex pattern of enhancement and decay compared to the smoother trends seen in the PMF scenario. The presence of mass transpiration changes these behaviours by changing the position and magnitude of these transitions, which becomes evident as the lines respond more strongly to changes. Together, these observations indicates that the interplay between the mass power index and boundary conditions provides multiple solution branches and enhanced nonlinear characteristics.

8. Conclusion

The aim of the current analyses is to examine the thermal and solutal transfer impact on the Newtonian fluid flow across permeable expanding/shrinking surfaces, and the impact of solar radiation, magnetic field and Schmidt number on fluid flow has been extracted. Governing Navier-Stokes partial differential equations are formulated into ordinary differential equations via similarity variables, and closed-form multiple analytic solutions are obtained. The power law heat and mass flux conditions examined using the Kummer function and hyper geometric series solution. Thermal and solutal transfer problems for the algebraically decaying flow across the shrinking solution were also investigated.

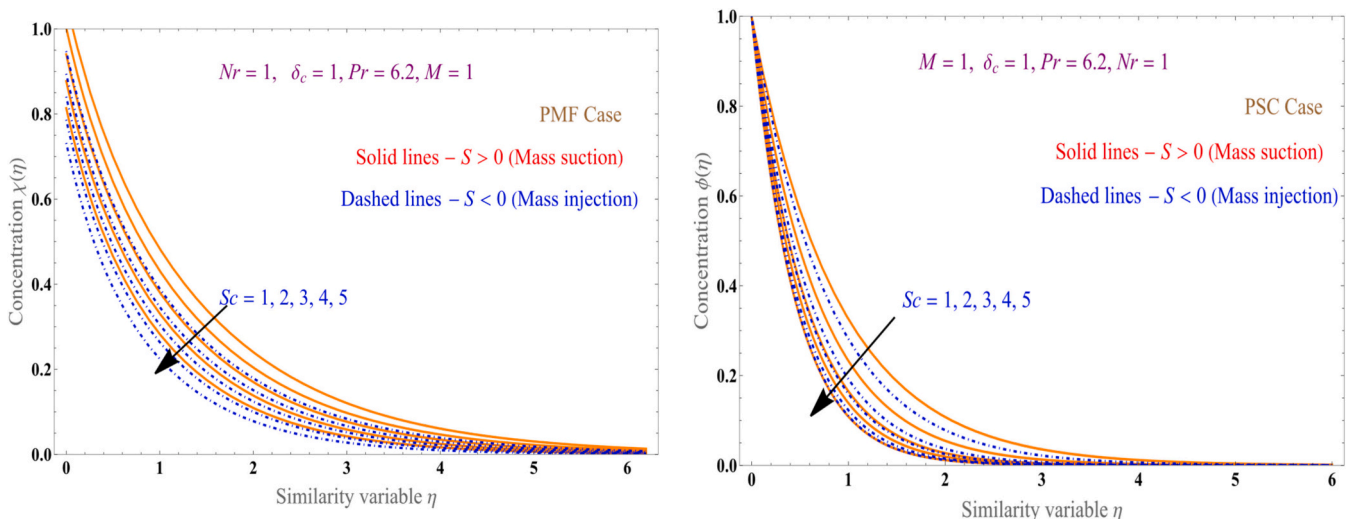


Fig. 9. (a): Schmidt number impact on PMF concentration. (b): Schmidt number impact on PSC concentration.

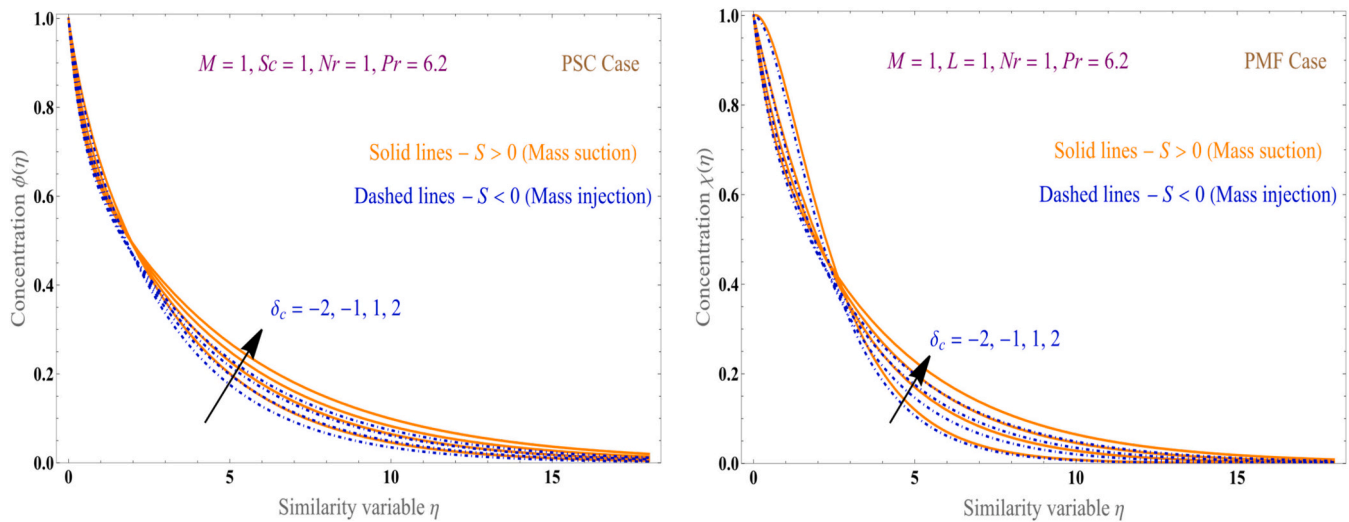


Fig. 10. (a): Concentration power index impact on PSC concentration. (b): Concentration power index influence on PMF concentration.

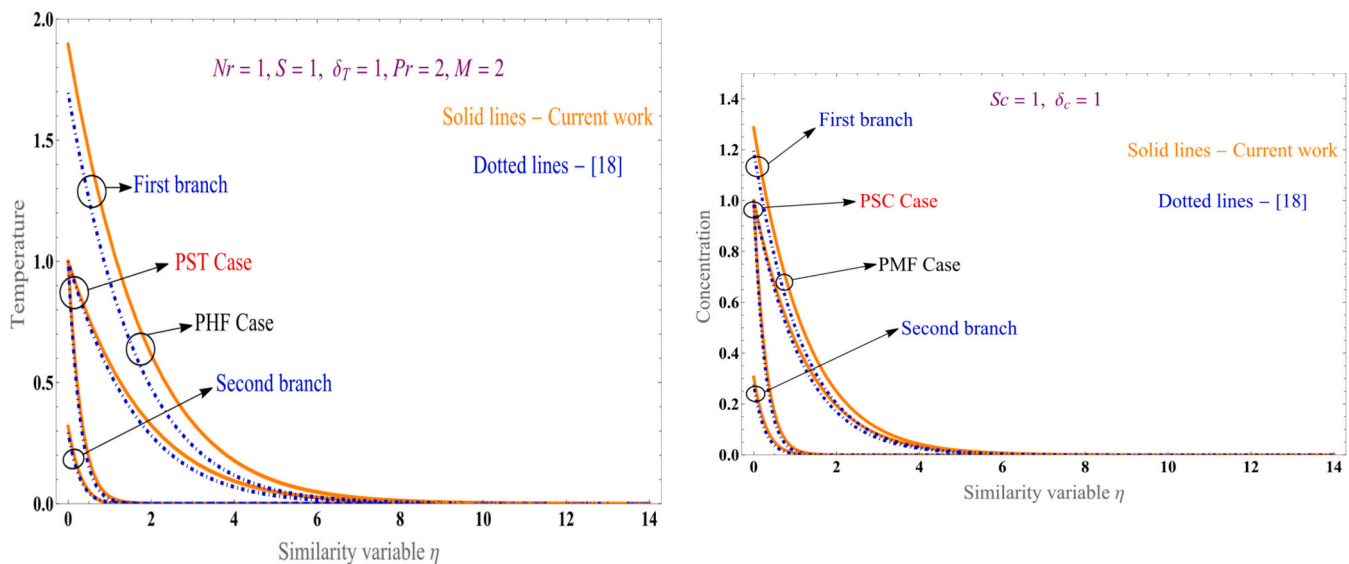


Fig. 11. (a): Temperature of the Algebraic decay solutions. (b): Concentration of the Algebraic decay solutions.

It was found that energy and mass transfer with mass injection were significantly different from the mass suction values, and the significant findings are listed below:

- Enhancing the magnetic field and stretching parameter reduces the velocity.
- Increasing the porous-resistance parameter suppresses the velocity and enhances the temperature profiles by strengthening the resistance to fluid motion within the porous medium.
- Enhancing the temperature power index raises the temperature profile.
- Raising the thermal radiation increases the temperature in both the PST and PHF cases.
- Upsurging the Schmidt number decays the concentration in both the PSC and PMF cases.
- Upsurging the concentration power law index raises the mass transfer profile.

8.1. Limitations and future scope

Current research is limited to laminar, Newtonian, and two-dimensional fluid movements. Solar radiation is formulated using the Rosseland approximation which is only valid for optically thick media, chemical reactions and heat dissipation have been neglected. In the future the present work can be extended to unsteady, non-Newtonian and three-dimensional flows to increase practical relevance, adding more radiation effects, chemical reactions, and variable fluid properties to provide a deeper understanding of thermal and solutal transfer.

CRediT authorship contribution statement

S.M. Sachhin: Writing – review & editing, Writing – original draft, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **U.S. Mahabaleswar:** Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Formal analysis. **S.R. Pradeepkumar:** Visualization, Validation, Supervision, Software, Resources, Methodology. **L.M. Pérez:** Supervision, Software, Project administration, Methodology,

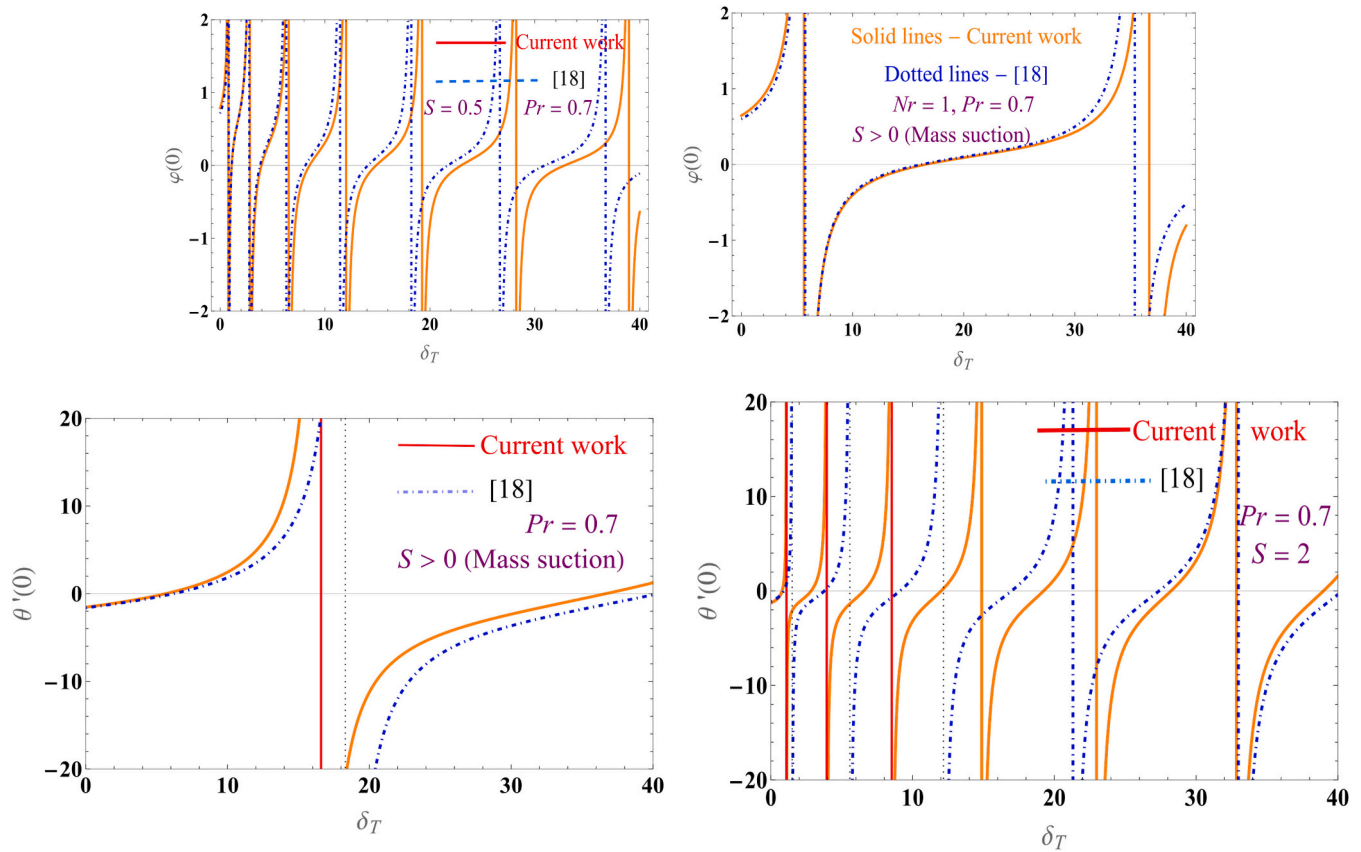


Fig. 12. (a): Results of $\varphi(0)$ for $S = 0.5$. (b): Results of $\varphi(0)$ for $S = 2$. (c): Results of $\theta'(0)$ for $S = 0.5$. (d): Results of $\theta'(0)$ for $S = 2$.

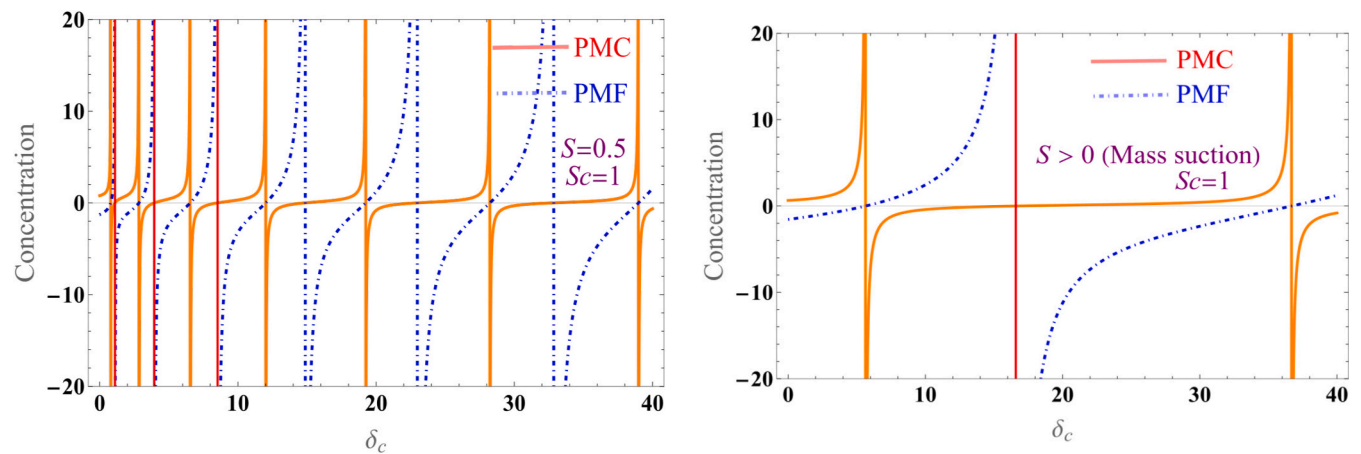


Fig. 13. (a): Results of concentration profile for $S = 0.5$. (b): Results of mass transfer profile for $S > 0$.

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Ethical approval

Not applicable.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

Data will be made available on request.

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