



Stability and sensitivity analysis of entropy generation of 3D EMHD Casson ternary hybrid nanofluid flow over an inclined porous surface: Response surface methodology

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ABSTRACT

This study investigates the electro-magnetohydrodynamic (EMHD) flow of a Casson-based ternary hybrid nanofluid over an inclined porous plate, incorporating heat and mass transfer, entropy generation, and stability analysis. The influence of key parameters, including the electric field parameter (E_1), Hartmann parameter (M), velocity slip parameter (λ), Casson parameter (β), Darcy resistance, and exponential heat source, is analyzed using Response Surface Methodology (RSM). Sensitivity analysis reveals that E_1 reduces the Nusselt number (Nu) by 13.6% and increases the Bejan number (Be) by 9.6%, indicating higher entropy generation. Meanwhile, β enhances Nu by 2.1% and the Sherwood number (Sh) by 1.8%, improving heat and mass transfer. Increasing λ reduces skin friction (C_f) by 14.2% and decreases Be by 3.4%, optimizing thermodynamic efficiency. Statistical validation via ANOVA confirms the model's reliability, with high coefficients of determination ($R^2 > 0.99$), ensuring strong predictive accuracy. Graphical interpretations of contour and surface plots highlight significant parameter interactions. Higher Gr_T and Gr_C boost velocity, while increasing M reduces it via the Lorentz force. Temperature drops with M but rises with Ec due to viscous dissipation. Concentration decreases with K_1 and Sc , reducing solute diffusivity. These insights contribute to optimizing heat and mass transfer in thermal management, biomedical transport, and industrial processing.

1. Introduction

RSM is a statistical and mathematical framework used to develop, improve, and optimize processes where multiple input variables influence the response of interest. It employs regression analysis and ANOVA to model the response with a quadratic polynomial, enabling a deeper understanding of factor-response interactions. In fluid dynamics, RSM plays a crucial role in optimizing heat transfer, hydrodynamic properties, and EMHD flows. Researchers use it to model complex fluid behavior and enhance system performance by systematically evaluating input parameters. RSM is widely applied in engineering, manufacturing, and scientific research to improve the efficiency of systems and processes. Box and Wilson first introduced RSM in 1951 [1], proposing polynomial regression models and using designed experiments to optimize processes through sequential experimentation and the steepest ascent method. The study by Kumar et al. [2] explores RSM-based

optimization of electromagnetohydrodynamic (EMHD) nanofluid flow, accounting for Joule heating and exponential heat sources. Khanduri et al. [3] conducted a sensitivity analysis of electroosmotic MHD flow in a stenosed artery using RSM for parametric optimization. Gandhi et al. [4] extended RSM to analyze entropy generation in EMHD mixed-convective flows, incorporating higher-order chemical reactions and heat-transfer mechanisms. Pyari et al. [5] compared RSM and sensitivity analysis for radiative hybrid nanofluid flow over a spinning disk, emphasizing the effects of heat sources and optimization techniques. Ali et al. [6] investigated non-Newtonian fluid dynamics in reverse roll coating, employing the Sisko fluid model, perturbation methods, and sensitivity analysis using response surface methodology. The study revealed the influence of non-Newtonian parameters and the velocity ratio on coating thickness, power input, and roll-separating forces. Kumar et al. [7] have more recently developed and applied deep learning and RSM-based

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Nomenclature

x, y, z	Cartesian coordinates
$\vec{u}, \vec{v}, \vec{w}$	Velocity vectors
$\vec{u}_s, \vec{v}_s$	Outer velocities in x, y directions
$\vec{T}$	Temperature
$\vec{g}$	Gravitational acceleration
$\vec{C}$	Concentration
ρ_H	Density of hybrid nanofluid
ν_H	Kinematic viscosity
D_T	Thermophoretic diffusion coefficient
K_r^2	Chemical reaction rate constant
σ	Electrical conductivity
w_0	Suction/injection velocity
Λ	Mean free path
β_T	Thermal expansion coefficient
E_1	Electric field parameter
λ	Slip velocity parameter
Gr_T, Gr_C	Thermal and solutal Grashof numbers
Nu	Nusselt number (heat transfer rate)

Be	Bejan number (entropy generation)
u, v, w	Velocity components in x, y, z directions
K^*	Permeability of porous medium
$h(\eta), k(\eta)$	Axial velocity components in z -direction
$\tilde{Q}$	Heat generation/absorption coefficient
a	Stretching constant
B	Uniform magnetic field strength
μ_H	Dynamic viscosity of hybrid nanofluid
k_H	Thermal conductivity
D_B	Brownian diffusion coefficient
Sc	Schmidt number
K_n	Knudsen number
C_p	Specific heat capacity at constant pressure
Ec	Eckert number (viscous dissipation effect)
β_C	Concentration expansion coefficient
M	Hartmann number (magnetic field effect)
s	Mass transport parameter (suction/injection)
C_f	Skin friction coefficient
Sh	Sherwood number (mass transfer rate)
Φ	Nanoparticle volume fraction

modeling techniques to accurately predict heat transfer rates and optimize flow problems involving nanofluids. Palencia et al. [8] predicted the heat and mass transfer characteristics in magnetized tetra-hybrid nanofluid systems for biomedical applications through an ANFIS-PSO-based approach. Raju et al. [9] have used RSM-based sensitivity and optimization to identify heat and mass transfer characteristics for magnetized hybrid nanofluid systems with different types of nanoparticles. Yang et al. [10] have found that the stress-dependent creep-active site transition is a primary mechanism to control the nonlinear tensile creep behavior of ultra-high-performance concrete (UHPC). Zheng and Xu [11] formulated advanced numerical methods to solve the nonlinear poroelastic equations in the porous media with enhanced accuracy. The selected base fluid and nanoparticles used in the ternary hybrid nanofluid are illustrated in Fig. 1, with their physical characteristics.

EMHD explores the interaction of electrically conducting fluids with externally applied electric and magnetic fields. The Lorentz force, generated by the combination of induced electric currents and the applied magnetic field, plays a crucial role in regulating fluid motion, heat transfer, and mass transport. EMHD finds applications in plasma physics, geophysics, microfluidics, and advanced thermal management systems, where precise control over fluid flow is essential. The behavior of non-Newtonian nanofluids under electric and magnetic fields has been the focus of several recent studies. Sharma et al. [12] analyzed entropy generation and thermal radiation in EMHD Jeffrey nanofluid flow, highlighting applications in solar energy systems. Gandhi et al. [13] conducted computer simulations of EMHD Casson nanofluid flow through stenosed, permeable arteries, providing insights into biomedical applications. Furthermore, Kumar et al. [14] utilized artificial neural networks to optimize Jeffrey hybrid nanofluid flow and improve solar thermal collector efficiency. These works collectively emphasize the importance of Joule heating, entropy generation, and Casson fluid dynamics in understanding EMHD effects in various engineering and biomedical applications. Gandhi et al. [15] theoretically modeled EMHD pulsatile nanofluid flow in a bifurcated stenosed artery. The model uses the Casson fluid model for blood, considers temperature-dependent viscosity, and incorporates EMHD effects. Zeeshan et al. [16] examined time-dependent MHD flow with induced magnetic fields, thermal radiation, and chemical reactions, highlighting the effects of stability analysis and heat sources on velocity and temperature. Kopp et al. [17] analyzed MHD flow and heat transfer of a ternary hybrid ferrofluid over a porous sheet, incorporating Brownian motion, thermophoresis, and radiation

effects to enhance thermal performance. Sneha et al. [18] examined the inclined MHD flow of a hybrid nanofluid over a stretching/shrinking sheet, highlighting the effects of radiation, mass transpiration, and a heat source/sink on the thermal and flow behavior. Anusha et al. [19] investigated MHD flow and heat transfer of Jeffrey fluid containing carbon nanotubes (SWCNTs and MWCNTs) over a stretching/shrinking surface, considering thermal radiation, heat source/sink, and Navier's slip. The study highlights CNTs' role in enhancing thermal conductivity and their potential in biomedical applications.

EMHD effects in non-Newtonian Casson fluids introduce complexities due to viscosity variations, yield stress, and electromagnetic interactions. Honey, a commonly studied Casson fluid, exhibits unique rheology that influences flow under electric and magnetic fields. The addition of nanoparticles such as Ag , Cu , and TiO_2 enhances thermal conductivity, electrical properties, and mass diffusion, thereby significantly affecting heat and mass transfer. These combined effects modify velocity profiles, shear stress, and energy transport, all of which are crucial for optimizing thermal management, industrial processing, and biomedical applications. Gandhi and Sharma [20] modeled pulsatile blood flow in a stenotic artery, using the Casson fluid model, hematocrit-dependent viscosity, and a magnetic field to analyze magneto-hemodynamic effects with hybrid nanoparticles. Kumar et al. [21] studied Casson hybrid nanofluid flow over a curved Riga surface, considering slip, Soret/Dufour effects, catalytic reaction, and nanofluid properties, using boundary layer assumptions. Patil et al. [22] studied the Soret and Dufour effects on Casson nanofluid flow in a vertical channel with thermal radiation, considering entropy generation. The study employed Buongiorno's model, incorporating Brownian motion and thermophoresis, and used the differential transform method and MATLAB's bvp4c solver for numerical analysis. The findings highlight that the Casson, Dufour, and Soret parameters enhance velocity and temperature while minimizing entropy generation. Madiwal and Naduvanamani [23] analyzed a Casson hybrid nanofluid ($MoS_2 + ZnO$)/engine oil over a stretching sheet, solving the transformed ODEs using bvp4c. Results showed enhanced thermal efficiency and altered velocity-temperature profiles. Using nanofluids in the context of thermal systems, Mahariq et al. [24] provided thermal enhancement abilities for ternary hybrid nanofluids exposed to heat sources, magnetic fields, and non-Newtonian flow. Additionally, Mahariq et al. [25] compared tri-hybrid nanofluids, considering the impact of nanoparticle shape and magnetic field; they concluded that these systems exhibit greater heat-transfer potential.

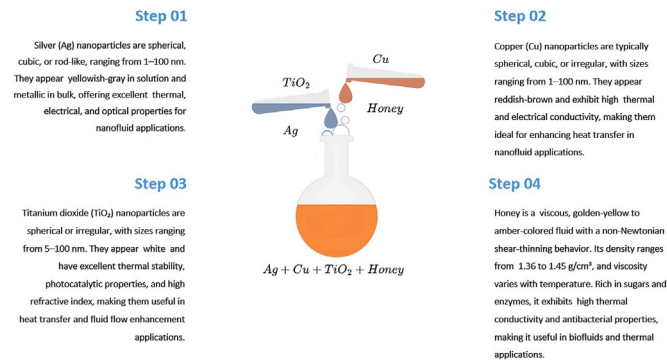


Fig. 1. Schematic representation of Ag–Cu–TiO₂/honey ternary hybrid nanofluid preparation.

Finally, Zhao et al. [26] investigated the effect of baffled fluid flows, finding they induce disruptions of flows to enhance interface mixing and improve the rate of mass transfer within thermal-fluid systems.

The flow of EMHD Casson fluid over an inclined plane (ϕ) is governed by intricate interactions between hydrodynamic, thermal, and chemical effects. Free-stream velocity propels the fluid, while electromagnetic forces and gravity dictate its stability and momentum. An exponential heat source intensifies thermal energy distribution, counterbalanced by viscous dissipation, which generates internal heating through shear effects. Concurrently, a first-order chemical reaction modifies species concentration, influencing diffusion and reactive transport. These coupled effects play a crucial role in optimizing heat and mass transfer in advanced thermal systems, chemical reactors, and biomedical applications, including targeted drug delivery and biofluid transport in lab-on-chip devices. Additionally, such analyses contribute to the enhancement of cooling technologies in microfluidics, aerospace engineering, and energy storage systems. Sharma and Gandhi [27] studied unsteady MHD flow over a stretching surface in a porous medium, considering Joule heating, non-uniform heat source/sink, and velocity/thermal slip effects. Tripathi and Sharma [28] studied heat and mass transfer in MHD arterial blood flow through an inclined stenosed artery, considering variable viscosity and chemical reaction, using the homotopy perturbation method. Sharma et al. [29] studied unsteady flow in a porous medium with thermophoresis and Brownian motion, considering Arrhenius activation and microorganisms over an inclined stretching sheet, using similarity analysis. Yaseen et al. [30] examined hybrid nanofluid flow with assisting and opposing effects over a moving permeable surface, analyzing the effects of a heat source/sink, thermal radiation, velocity slip, and the Biot number on heat transfer. Shamshuddin and Thumma [31] investigated dissipative micropolar fluid flow over an inclined porous plate using the finite element method, highlighting the effects of heat source/sink, chemical reactions, and inclination on transport properties.

Entropy generation quantifies the irreversible energy losses in a system due to thermal gradients, viscous dissipation, electromagnetic effects, and chemical reactions. In EMHD, Casson fluid flow over an inclined plane, free-stream velocity governs motion, while an exponential heat source and viscous dissipation enhance thermal energy, leading to higher entropy production. Joule heating from electromagnetic forces and species diffusion due to first-order chemical reactions further contribute to entropy generation. Controlling these effects is crucial for optimizing energy efficiency in thermal management, biomedical transport, and industrial fluid systems. Kumawat et al. [32] studied MHD blood flow through a curved, permeable artery, modeling two-phase flow with temperature-dependent viscosity and employing a finite-difference method. Sharma et al. [33] analyzed MHD mixed convective flow over a stretching surface, considering higher-order chemical reactions, activation energy, thermophoresis, and Brownian motion.

Khanduri and Sharma [34] analyzed entropy generation in MHD flow with temperature-dependent viscosity and thermal conductivity, solving the governing equations using BVP4C in MATLAB for optimization studies. Mishra et al. [35] investigated the optimization of entropy generation in cilia-driven MHD ternary hybrid nanofluid flow with Arrhenius activation energy and an induced magnetic field, solving the model numerically using BVP5C. Kumar and Gandhi [36] conducted an entropy-driven optimization study on radiative Jeffrey tetrahybrid nanofluid flow through a stenosed bifurcated artery, incorporating Hall effects, Joule heating, and viscous dissipation. Rafique et al. [37] analyzed entropy generation in hybrid nanofluids with Joule heating and viscous dissipation, demonstrating significant effects on heat transfer and fluid flow parameters. Sohail et al. [38] explored entropy generation in MHD Casson fluid flow over a stretching surface, emphasizing the effects of Joule heating, radiation, and variable thermal conductivity on heat and mass transport.

Recent investigations on EMHD and hybrid nanofluids mainly consider simple Newtonian base liquids or ordinary binary mixtures, using mono-variate parameter studies that miss dual-solution flow-reverse constraints. This research aims to develop a unified model to discuss EMHD fluid flow of a non-Newtonian Casson-based ternary hybrid nanofluid (Ag–Cu–TiO₂ dispersed in a Honey base) over an inclined porous plate, including Darcy's law, velocity slip, viscous dissipation, and chemical reaction. To our knowledge, this is the first study in which, we: (i) propose an enhanced ternary hybrid nanofluid formulation based on Ag, Cu and TiO₂ nanoparticles within a High Viscous Honey-Casson viscous fluid over an inclined surface; (ii) develop and establish stable dual solution branches by extracting under opposed suction and injection effects; (iii) utilize decoupled RSM multi-stage approach for an independent and robust optimizing strategy of contrast flow parameters in the to magnetic damping forces and the active electric field accelerations.

2. Mathematical modeling

The investigation explores the steady, laminar, incompressible, three-dimensional boundary-layer flow of a Casson-based ternary hybrid nanofluid on an inclined porous plate under EMHD effects. The ternary hybrid nanofluid, prepared with Ag, Cu, and TiO₂ nanoparticles in a honey-based Casson fluid, is presumed homogeneous and stable without agglomeration or sedimentation of particles. The Darcy model is used to represent the porous medium's resistance, and the first-order velocity-slip condition is imposed to characterize momentum slip at the surface. An inclined plate at an angle ϕ gives rise to the buoyant effects by utilizing the Boussinesq approximation, which contributes to the varying nature of flow in both x and y directions. Additionally, an exponentially changing heat source, viscous dissipation, and the first-order chemical reaction have been taken into account to examine the heat and mass transfer processes. Uniform electric and magnetic fields are used to account for EMHD effects. The stretching/shrinking surface lies in the x - y plane, whereas the flow develops along the z -axis. Two types of wall mass-flux conditions (suction, $w_0 > 0$, and injection, $w_0 < 0$) are considered to examine their effects on flow stability and transport behavior. The presented assumptions yield a manageable mathematical model, although the solutions are confined to idealized boundary-layer phenomena, uniform thermophysical properties, and within the considered range of parameter variations. The presented findings are pertinent for thermal management, biomedical transportation, and industrial production. Fig. 2 is a schematic diagram illustrating the physical model, showing the inclined porous plate, the coordinate system, the flow direction, the magnetic and electric fields, and the exponential heat source [39,40].

Continuity Equation

$$\frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} + \frac{\partial \tilde{w}}{\partial z} = 0 \quad (1)$$

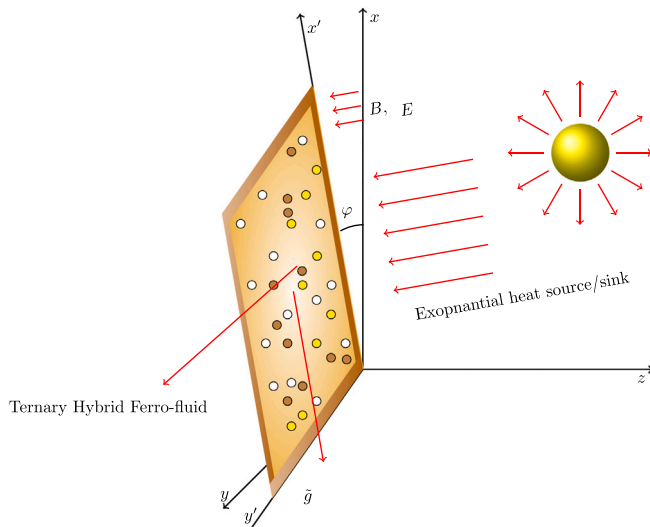


Fig. 2. Schematic diagram for the developed model.

Momentum Equation

$$\begin{aligned} \tilde{u} \frac{\partial \tilde{u}}{\partial x} + \tilde{v} \frac{\partial \tilde{u}}{\partial y} + \tilde{w} \frac{\partial \tilde{u}}{\partial z} &= \tilde{u}_s \frac{\partial \tilde{u}_s}{\partial x} + \nu_H (1 + \frac{1}{\beta}) \frac{\partial^2 \tilde{u}}{\partial z^2} \\ &\quad - \frac{\nu_H}{K^*} (1 + \frac{1}{\beta}) (\tilde{u} - \tilde{u}_s)(x+y)^{-2/3} \\ &\quad + \tilde{g} \beta_{\tilde{T}} (\tilde{T} - \tilde{T}_\infty) \cos \varphi + \tilde{g} \beta_{\tilde{C}} (\tilde{C} - \tilde{C}_\infty) \cos \varphi \\ &\quad + \frac{B E \sigma}{\rho_H} (x+y)^{-1/3} - \frac{B^2 \sigma}{\rho_H} (\tilde{u} - \tilde{u}_s)(x+y)^{-2/3} \end{aligned} \quad (2)$$

$$\begin{aligned}
\tilde{u} \frac{\partial \tilde{v}}{\partial x} + \tilde{v} \frac{\partial \tilde{v}}{\partial y} + \tilde{w} \frac{\partial \tilde{v}}{\partial z} &= \tilde{v}_s \frac{\partial \tilde{v}_s}{\partial y} + v_H \left(1 + \frac{1}{\beta}\right) \frac{\partial^2 \tilde{v}}{\partial z^2} \\
&\quad - \frac{v_H}{K^*} \left(1 + \frac{1}{\beta}\right) (\tilde{v} - \tilde{v}_s)(x+y)^{-2/3} + \tilde{g} \beta_{\tilde{T}} (\tilde{T} - \tilde{T}_\infty) \sin \varphi \\
&\quad + \tilde{g} \beta_{\tilde{C}} (\tilde{C} - \tilde{C}_\infty) \sin \varphi + \frac{B E \sigma}{\rho_H} (x+y)^{-1/3} \\
&\quad - \frac{B^2 \sigma}{\rho_H} (\tilde{v} - \tilde{v}_s)(x+y)^{-2/3}
\end{aligned} \tag{3}$$

$$\begin{aligned} \ddot{u} \frac{\partial \ddot{w}}{\partial x} + \ddot{v} \frac{\partial \ddot{w}}{\partial y} + \ddot{w} \frac{\partial \ddot{w}}{\partial z} &= \ddot{w}_s \frac{\partial \ddot{w}_s}{\partial z} + \nu_H \left(1 + \frac{1}{\beta}\right) \frac{\partial^2 \ddot{w}}{\partial z^2} \\ &\quad - \frac{\nu_H}{K_*} \left(1 + \frac{1}{\beta}\right) (\ddot{w} - \ddot{w}_s)(x+y)^{-2/3} \end{aligned} \quad (4)$$

Temperature Equation

$$\begin{aligned} \tilde{u} \frac{\partial \tilde{T}}{\partial x} + \tilde{v} \frac{\partial \tilde{T}}{\partial y} + \tilde{w} \frac{\partial \tilde{T}}{\partial z} &= \frac{k_H}{(\rho C_p)_H} \frac{\partial^2 \tilde{T}}{\partial z^2} \\ &+ \frac{\tilde{Q}}{(\rho C_p)_H} e^{-n \sqrt{\frac{a}{v_B}} z(x+y)^{1/3}} (\tilde{T} - T_\infty)(x+y)^{-2/3} \\ &+ \frac{\mu_H}{(\rho C_p)_H} \left(1 + \frac{1}{\beta}\right) \left[\left(\frac{\partial \tilde{u}}{\partial z}\right)^2 + \left(\frac{\partial \tilde{v}}{\partial z}\right)^2 \right] \\ &+ \frac{\sigma}{(\rho C_p)_H} \left[E - B \left[(\tilde{u} - \tilde{u}_s) + (\tilde{v} - \tilde{v}_s) \right] (x+y)^{-1/3} \right]^2 \\ &+ \tau \left[\frac{D_T}{\tilde{T}_\infty} \left(\frac{\partial \tilde{T}}{\partial z}\right)^2 + D_B \left(\frac{\partial \tilde{T}}{\partial z}\right) \left(\frac{\partial \tilde{C}}{\partial z}\right) \right] \end{aligned} \quad (5)$$

Concentration Equation

$$\tilde{u} \frac{\partial \tilde{C}}{\partial x} + \tilde{v} \frac{\partial \tilde{C}}{\partial y} + \tilde{w} \frac{\partial \tilde{C}}{\partial z} = D_B \left[\frac{\partial^2 \tilde{C}}{\partial z^2} \right] + \frac{D_T}{\tilde{T}_\infty} \left[\frac{\partial^2 \tilde{T}}{\partial z^2} \right] - K_r^2 (\tilde{C} - \tilde{C}_\infty)(x+y)^{-2/3} \quad (6)$$

With Boundary Conditions

$$\left\{ \begin{array}{l} \tilde{u} = \tilde{u}_w(x, y) = \xi \tilde{U} + \tilde{U}_{slip}, \quad \tilde{v} = \tilde{v}_w(x, y) = \xi \tilde{V} + \tilde{V}_{slip}, \quad \tilde{w} = \tilde{w}_0, \\ \tilde{T} = \tilde{T}_w, \quad \tilde{C} = \tilde{C}_w, \quad \text{at } z = 0, \\ \tilde{u} = \tilde{u}_s(x, y) \rightarrow a(x + y)^{1/3}, \quad \tilde{v} = \tilde{v}_s(x, y) \rightarrow a(x + y)^{1/3}, \quad \tilde{w} = \tilde{w}_e(z) \rightarrow -2az, \\ \tilde{T} \rightarrow \tilde{T}_\infty, \quad \tilde{C} \rightarrow \tilde{C}_\infty, \text{ as } z \rightarrow \infty. \end{array} \right. \quad (7)$$

where $\tilde{U}_{slip}$ and $\tilde{V}_{slip}$ are defined as

$$\left\{ \begin{array}{l} \tilde{U}_{slip} = \frac{2}{3} \left(\frac{3-at^3}{a} - \frac{3}{2} \left(\frac{1-l^2}{K_n} \right) \right) \Lambda \frac{\partial \tilde{u}}{\partial z}(x+y)^{1/3} \quad \text{or} \quad \tilde{U}_{slip} = M_1 \frac{\partial \tilde{u}}{\partial z}(x+y)^{1/3}, \\ \tilde{V}_{slip} = \frac{2}{3} \left(\frac{3-at^3}{\alpha} - \frac{3}{2} \left(\frac{1-l^2}{K_n} \right) \right) \Lambda \frac{\partial \tilde{v}}{\partial z}(x+y)^{1/3} \quad \text{or} \quad \tilde{V}_{slip} = M_2 \frac{\partial \tilde{v}}{\partial z}(x+y)^{1/3} \end{array} \right. \quad (8)$$

Introducing the following similarity transformations:

$$\left\{ \begin{aligned} \tilde{u} &= a(f' + \lambda h)(x+y)^{1/3}, \quad \tilde{v} = a(g' + \lambda_1 k)(x+y)^{1/3}, \\ \tilde{w} &= -\sqrt{av_f} \left[\frac{2}{3}(f+g) - \frac{1}{3}\eta(f' + \lambda h + g' + \lambda_1 k) \right] (x+y)^{-1/3}, \\ \theta &= \frac{\tilde{T} - \tilde{T}_\infty}{\tilde{T}_w - \tilde{T}_\infty} (x+y)^{-2/3}, \quad \phi = \frac{\tilde{C} - \tilde{C}_\infty}{\tilde{C}_w - \tilde{C}_\infty} (x+y)^{-2/3}, \quad \eta = \sqrt{\frac{a}{\nu_f}} z(x+y)^{-1/3}. \end{aligned} \right. \quad (9)$$

The surface normal velocity w_0 defines mass transport as $\frac{2}{3}(f(0) + g(0)) = s$, where $s = -\sqrt{av_f}$. Setting $f(0) = \frac{3}{2}s$ and $g(0) = 0$, we examine suction ($s > 0$) and injection ($s < 0$). Dimensionless parameters for slip velocities are governed by λ and λ_1 .

After implication, the similarity transformation from Eq. (9) on Eqs. (2)–(7), we obtain

$$\begin{aligned} & \left(\frac{\mu_H}{\rho_H} \right) \left(\frac{\rho_B}{\mu_B} \right) \left(1 + \frac{1}{\beta} \right) f''' + \frac{2}{3} (f + g) f'' - \frac{1}{3} (f' + g') f' \\ & - \beta^* \left(1 + \frac{1}{\beta} \right) \left(\frac{\mu_H}{\rho_H} \right) \left(\frac{\rho_B}{\mu_B} \right) (f' - 1) + \frac{1}{3} f' \lambda_1 k + (Gr)_{\bar{T}} \theta \cos \varphi \\ & + (Gr)_{\bar{C}} \phi \cos \varphi + \frac{1}{3} - M(f' - 1) + M E_1 = 0 \end{aligned} \quad (10)$$

$$\begin{aligned} & \left(\frac{\mu_H}{\rho_H} \right) \left(\frac{\rho_B}{\mu_B} \right) \left(1 + \frac{1}{\beta} \right) h'' + \frac{2}{3} (f + g) h' - \frac{1}{3} (f' + g') h - \frac{1}{3} h (\lambda h + \lambda_1 k) \\ & - \frac{1}{3} f' h - \beta^* \left(1 + \frac{1}{\beta} \right) \left(\frac{\mu_H}{\rho_H} \right) \left(\frac{\rho_B}{\mu_B} \right) h - M h = 0 \end{aligned} \quad (11)$$

$$\begin{aligned} & \left(\frac{\mu_H}{\rho_H} \right) \left(\frac{\rho_B}{\mu_B} \right) \left(1 + \frac{1}{\beta} \right) g''' + \frac{2}{3} (f + g) g'' - \frac{1}{3} (f' + g') g' \\ & - \beta^* \left(1 + \frac{1}{\beta} \right) \left(\frac{\mu_H}{\rho_H} \right) \left(\frac{\rho_B}{\mu_B} \right) (g' - 1) + \frac{1}{3} g' \lambda h + (Gr)_T \theta \sin \varphi \\ & + (Gr)_C \phi \sin \varphi + \frac{1}{3} - M(g' - 1) + M E_1 = 0 \end{aligned} \quad (12)$$

$$\begin{aligned} & \left(\frac{\mu_H}{\rho_H} \right) \left(\frac{\rho_B}{\mu_B} \right) \left(1 + \frac{1}{\beta} \right) k'' + \frac{2}{3} (f + g) k' - \frac{1}{3} (f' + g') k \\ & - \frac{1}{3} k (\lambda h + \lambda_1 k) - \frac{1}{3} g' k - \beta^* \left(1 + \frac{1}{\beta} \right) \left(\frac{\mu_H}{\rho_H} \right) \left(\frac{\rho_B}{\mu_B} \right) k - M k = 0 \quad (13) \end{aligned}$$

Table 1
Thermophysical properties [2,14,41,42].

Properties	Base fluid (Honey)	Cu	Ag	TiO ₂
C_p	4200	385	235	692
ρ	1400	8960	10,500	4230
κ	0.5	401	429	8.4

$$\frac{1}{Pr} \frac{k_H}{k_B} \frac{(\rho C_p)_B}{(\rho C_p)_H} \theta'' + \left(\frac{(\rho C_p)_B}{(\rho C_p)_H} \right) Q e^{-m\theta} + E_c \left(1 + \frac{1}{\beta} \right) \left(\frac{\nu_H}{\nu_B} \right) \left[(f'' + \lambda h')^2 + (g'' + \lambda_1 k')^2 \right] + M E_c [E_1 - (f' + \lambda h + g' + \lambda_1 k - 2)]^2 + \tau [N_t \theta'^2 + N_b \theta' \phi'] - \frac{2}{3} (f' + g' + \lambda h + \lambda_1 k) \theta + \frac{2}{3} (f + g) \theta' \quad (14)$$

$$\phi'' + \frac{2}{3} S_c (f + g) \phi' - \frac{2}{3} S_c (f' + g' + \lambda h + \lambda_1 k) \phi + S_c S_r \theta'' - S_c K_1 \phi = 0 \quad (15)$$

with

$$\begin{cases} f(0) = \frac{3}{2}s, & g(0) = 0, & f'(0) = A_1 f''(0), & g'(0) = B_1 g''(0), \\ h(0) = A_1 h'(0) + \xi, & k(0) = B_1 k'(0) + \xi, & f'(\infty) \rightarrow 1, & g'(\infty) \rightarrow 1, \\ h(\infty) \rightarrow 0, & k(\infty) \rightarrow 0, & \theta(0) = 1, & \theta(\infty) \rightarrow 0, & \phi(0) = 1, & \phi(\infty) \rightarrow 0. \end{cases} \quad (16)$$

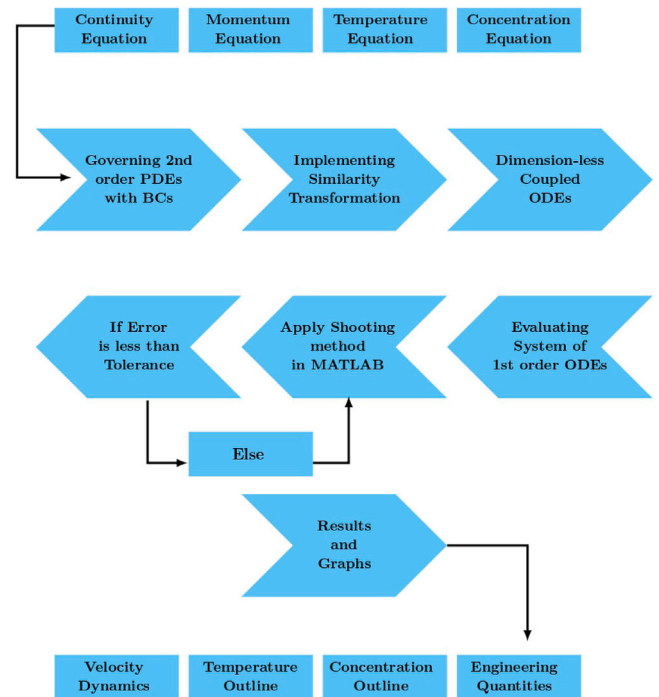
The governing equations involve the following dimensionless parameters:

$$\begin{aligned} M &= \frac{\sigma B^2}{a \rho_H}, & Pr &= \frac{(\rho C_p)_B \nu_B}{k_B}, & Ec &= \frac{a^2}{(C_p)_H (\tilde{T}_w - \tilde{T}_\infty)}, \\ Q &= \frac{\tilde{Q}}{a(\rho C_p)_B}, & A_1 &= M_1 \sqrt{\frac{a}{\nu_B}}, & B_1 &= M_2 \sqrt{\frac{a}{\nu_B}}, & E_1 &= \frac{\sigma B E}{a^2 \rho_H}, \\ \nu_H &= \frac{\mu_H}{\rho_H}, & Sr &= \frac{D_T \tilde{T}_0}{\nu_B \tilde{C}_0 \tilde{T}_\infty}, & \beta^* &= \frac{\nu_B}{a K^*}, & (Gr)_T &= \frac{\beta_T \tilde{g} \tilde{C}_0 \tilde{T}_0}{a^2}, \\ (Gr)_C &= \frac{\beta_C \tilde{g} d \tilde{C}_0}{a^2}, & Sc &= \frac{\nu_B}{D_B}, & \tau &= \frac{(\rho C_p)_p}{(\rho C_p)_H}, & N_t &= \frac{D_T (\tilde{T}_w - \tilde{T}_\infty)}{\tilde{T}_\infty \nu_B}, \\ N_b &= \frac{D_B (\tilde{C}_w - \tilde{C}_\infty)}{\nu_B}, & \alpha_1 &= \frac{\tilde{T}_w - \tilde{T}_\infty}{\tilde{T}_\infty (x + y)^{\frac{1}{3}}}, & \alpha_2 &= \frac{\tilde{C}_w - \tilde{C}_\infty}{\tilde{C}_\infty (x + y)^{\frac{1}{3}}}, \\ c \tilde{T}_0 &= (\tilde{T}_w - \tilde{T}_\infty)(x + y)^{-\frac{1}{3}}, & d \tilde{C}_0 &= (\tilde{C}_w - \tilde{C}_\infty)(x + y)^{-\frac{1}{3}}. \end{aligned}$$

Table 1 presents the thermophysical properties of the base fluid (honey) and the nanoparticles (Cu, Ag, TiO) used in the study. Table 2 provides the equations for calculating the transport properties of the ternary hybrid nanofluid, defining functions of the base fluid and nanoparticle volume fractions. Finally, Fig. 3 is a flowchart that outlines the mathematical modeling process, from the governing equations to the numerical solution in MATLAB and the generation of results and graphs.

Table 2
Transport properties of ternary hybrid nanofluids.

Property	Hybrid ternary nanofluid
Heat Capacity	$(C_p)_H = \phi_{Cu}(C_p)_{Cu} + \phi_{Ag}(C_p)_{Ag} + \phi_{TiO_2}(C_p)_{TiO_2} + (1 - \phi_{Cu} - \phi_{Ag} - \phi_{TiO_2})(C_p)_B$
Density	$\rho_H = \phi_{Cu}\rho_{Cu} + \phi_{Ag}\rho_{Ag} + \phi_{TiO_2}\rho_{TiO_2} + (1 - \phi_{Cu} - \phi_{Ag} - \phi_{TiO_2})\rho_B$
Viscosity	$\mu_H = \mu_B (1 - (\phi_{Cu} + \phi_{Ag} + \phi_{TiO_2}))^{-2.5}$
Thermal conductivity	$k_H = k_B \frac{k_p + 2k_B - 2(\phi_{Cu} + \phi_{Ag} + \phi_{TiO_2})(k_B - k_p)}{k_p + 2k_B + (\phi_{Cu} + \phi_{Ag} + \phi_{TiO_2})(k_B - k_p)}$ $k_p = \frac{(\phi_{Cu}k_{Cu} + \phi_{Ag}k_{Ag} + \phi_{TiO_2}k_{TiO_2})}{(\phi_{Cu} + \phi_{Ag} + \phi_{TiO_2})}$

**Fig. 3.** Schematic map for mathematical modeling using a flowchart.

3. Stability analysis

To assess the stability of the dual solutions, UBS (Solution first) and LBS (Solution second), a temporal stability analysis is conducted. A time-dependent perturbation variable τ is introduced into the governing equations, and by setting $\tau = 0$, the steady-flow functions transform into $f_0(\eta)$, $g_0(\eta)$, $h_0(\eta)$, and $k_0(\eta)$. These transformations enable the analysis of the initial growth or decay of the dual solutions through an eigenvalue problem.

To verify the stability, the perturbed functions are substituted into the governing equations, leading to the following eigenvalue problem [39]:

$$\begin{aligned} & \left(\frac{\mu_H}{\rho_H} \right) \left(\frac{\rho_B}{\mu_B} \right) \left(1 + \frac{1}{\beta} \right) F_0''' + \frac{2}{3} (F_0 + G_0) f_0'' + \frac{2}{3} (f_0 + g_0) F_0'' \\ & - \frac{2}{3} f_0' F_0' - \left[\beta^* \left(\frac{\mu_H}{\rho_H} \right) \left(\frac{\rho_B}{\mu_B} \right) \left(1 + \frac{1}{\beta} \right) - \frac{1}{3} \lambda_1 k_0 + \frac{1}{3} g_0' + M - \gamma \right] F_0' \\ & - \frac{1}{3} (G_0' - \lambda_1 K_0) f_0' + (Gr)_T \cos \varphi \Theta_0 + (Gr)_C \cos \varphi \Phi_0 = 0 \end{aligned} \quad (17)$$

$$\begin{aligned} & \left(\frac{\mu_H}{\rho_H} \right) \left(\frac{\rho_B}{\mu_B} \right) \left(1 + \frac{1}{\beta} \right) G_0''' + \frac{2}{3} (F_0 + G_0) g_0'' + \frac{2}{3} (f_0 + g_0) G_0'' \\ & - \frac{2}{3} g_0' G_0' - \left[\beta^* \left(\frac{\mu_H}{\rho_H} \right) \left(\frac{\rho_B}{\mu_B} \right) \left(1 + \frac{1}{\beta} \right) - \frac{1}{3} \lambda h_0 + \frac{1}{3} f_0' + M - \gamma \right] G_0' \\ & - \frac{1}{3} (F_0' - \lambda H_0) g_0' + (Gr)_T \sin \varphi \Theta_0 + (Gr)_C \sin \varphi \Phi_0 = 0 \end{aligned} \quad (18)$$

$$\begin{aligned} & \left(\frac{\mu_H}{\rho_H} \right) \left(\frac{\rho_B}{\mu_B} \right) \left(1 + \frac{1}{\beta} \right) H_0''' + \frac{2}{3} (F_0 + G_0) h_0'' + \frac{2}{3} (f_0 + g_0) H_0'' \\ & - \left[\beta^* \left(\frac{\mu_H}{\rho_H} \right) \left(\frac{\rho_B}{\mu_B} \right) \left(1 + \frac{1}{\beta} \right) + \frac{2}{3} \lambda h_0 + \frac{1}{3} \lambda_1 k_0 + M - \gamma \right] H_0' \\ & - \frac{1}{3} (2f_0' + g_0') H_0 - \frac{1}{3} (2F_0' + G_0' + \lambda_1 K_0) h_0 = 0 \end{aligned} \quad (19)$$

Table 3

Smallest eigen value γ at selected values of M for $E_1 = 1$.

M	Solution first	Solution second
0.1	1.2947	-0.2716
0.2	1.3008	-0.2428
0.3	1.3038	-0.0746
0.4	1.3113	-1.8984
0.5	2.8727	-4.3153

$$\begin{aligned} & \left(\frac{\mu_H}{\rho_H} \right) \left(\frac{\rho_B}{\mu_B} \right) \left(1 + \frac{1}{\beta} \right) K_0'' + \frac{2}{3} (F_0 + G_0) k_0' + \frac{2}{3} (f_0 + g_0) K_0' \\ & - \left[\beta^* \left(\frac{\mu_{nf}}{\rho_{nf}} \right) \left(\frac{\rho_f}{\mu_f} \right) \left(1 + \frac{1}{\beta} \right) + \frac{2}{3} \lambda_1 k_0 + \frac{1}{3} \lambda h_0 + M - \gamma \right] K_0 \\ & - \frac{1}{3} (2g_0' + f_0') K_0 - \frac{1}{3} (2G_0' + F_0' + \lambda H_0) k_0 = 0 \end{aligned} \quad (20)$$

$$\begin{aligned} & \frac{1}{Pr} \frac{k_H}{k_B} \left(\frac{(\rho C_p)_B}{(\rho C_p)_H} \right) \Theta_0'' + \frac{2}{3} (f_0 + g_0) \Theta_0' \\ & + \frac{2}{3} (F_0 + G_0) \theta_0' + \left(Q e^{-m\eta} \frac{(\rho c_p)_B}{(\rho c_p)_H} - \gamma \right) \Theta_0 \\ & + \tau [2N_t \theta_0' \Theta_0' + N_b (\phi_0' \Theta_0' + \theta_0' \Phi_0')] \\ & - \frac{2}{3} (F_0' + G_0' + \lambda H_0 + \lambda_1 K_0) \theta_0 - \frac{2}{3} (f_0' + g_0' + \lambda h_0 + \lambda_1 k_0) \Theta_0 \\ & + 2M E_c (F_0' + \lambda H_0 + G_0' + \lambda_1 K_0) (f_0' + \lambda h_0 + g_0' + \lambda_1 k_0) \\ & + 2E_c (1 + \frac{1}{\beta}) \left(\frac{\nu_H}{\nu_B} \right) [(F_0'' + \lambda H_0') (f_0'' + \lambda h_0') \\ & + (G_0'' + \lambda_1 K_0') (g_0'' + \lambda_1 k_0')] = 0 \end{aligned} \quad (21)$$

$$\begin{aligned} & \Phi_0'' + \frac{2}{3} S_c (F_0 + G_0) \phi_0' + \frac{2}{3} S_c (f_0 + g_0) \Phi_0' - \frac{2}{3} S_c (F_0' + G_0' + \lambda H_0 + \lambda_1 K_0) \phi_0 \\ & - \frac{2}{3} S_c (f_0' + g_0' + \lambda h_0 + \lambda_1 k_0) \Phi_0 + S_c S_r \Theta_0'' - S_c K_1 \Phi_0 + \gamma \Phi_0 = 0 \end{aligned} \quad (22)$$

with

$$\begin{cases} F_0(0) = 0, & G_0(0) = 0, & F_0'(0) = A_1 F_0''(0), & G_0'(0) = B_1 G_0''(0), \\ H(0) = A_1 H_0'(0), & K(0) = B_1 K_0'(0), \\ F_0'(\infty) \rightarrow 0, & G_0'(\infty) \rightarrow 0, & H_0(\infty) \rightarrow 0, & K_0(\infty) \rightarrow 0, & \Theta_0(0) = 0, \\ \Theta_0(\infty) \rightarrow 0, & \Phi_0(0) = 0, & \Phi_0(\infty) \rightarrow 0. \end{cases} \quad (23)$$

The smallest eigenvalue, denoted as γ , determines the stability of a solution. Specifically, a positive $\gamma > 0$ indicates stability, while a negative $\gamma < 0$ signifies instability. The stability analysis focuses on porosity, incompressible, and steady flow solutions, refining them to obtain the least eigenvalue, γ_1 , using parameters A_1 , B_1 , and s . To track the trajectory of the smallest eigenvalue, one boundary condition for $F_0(\eta)$, $G_0(\eta)$, $H_0(\eta)$, or $K_0(\eta)$ is relaxed. The analysis imposes the condition $G_0'(\eta) \rightarrow 1$ as $\eta \rightarrow \infty$ for a given γ . This requires solving the linear Eqs. (27)–(33) with the modified condition $G_0''(0) = 1$. Table 3 presents the smallest eigenvalues γ at selected values of magnetic parameter M , which are critical in determining the stability of dual solutions.

4. Entropy

Entropy generation quantifies system irreversibilities, highlighting the inherent limitations of complete heat transfer. Its dimensional form,

influenced by multiple factors, is expressed as: [37,43]

$$\begin{aligned} S_G = & \frac{k_H}{\tilde{T}_\infty^2} \left(\frac{\partial \tilde{T}}{\partial z} \right)^2 (x+y)^{-\frac{2}{3}} + \frac{\mu_H}{\tilde{T}_\infty} \left(1 + \frac{1}{\beta} \right) \left[\left(\frac{\partial \tilde{u}}{\partial z} \right)^2 + \left(\frac{\partial \tilde{v}}{\partial z} \right)^2 \right] \\ & + \frac{\mu_H}{K_p \tilde{T}_\infty} \left(1 + \frac{1}{\beta} \right) [(\tilde{u} - \tilde{u}_s)^2 + (\tilde{v} - \tilde{v}_s)^2] (x+y)^{-\frac{2}{3}} \\ & + \left[\frac{R^* D_B}{\tilde{T}_\infty} \left(\frac{\partial \tilde{T}}{\partial z} \right) \left(\frac{\partial \tilde{C}}{\partial z} \right) + \frac{R^* D_B}{\tilde{C}_\infty} \left(\frac{\partial \tilde{C}}{\partial z} \right)^2 \right] (x+y)^{-\frac{2}{3}} \\ & + \frac{\sigma}{\tilde{T}_\infty} \left[E - B ((\tilde{u} - \tilde{u}_s) + (\tilde{v} - \tilde{v}_s)) (x+y)^{-\frac{1}{3}} \right]^2. \end{aligned} \quad (24)$$

The entropy generation number is defined as,

$$N_S = \frac{S_G}{S_0'''}, \quad S_0''' = \frac{(\tilde{T}_w - \tilde{T}_\infty)}{\tilde{T}_\infty} k_B \left(\frac{a}{\nu_B} \right). \quad (25)$$

The dimensionless entropy number can be obtained by using Eq. (17) in Eq. (18)

$$\begin{aligned} N_S = & \frac{k_H}{k_B} \alpha_1 \theta'^2 + B_r \left(\frac{\mu_H}{\mu_B} \right) \left(1 + \frac{1}{\beta} \right) (f'' + \lambda h')^2 \\ & + K_1^p B_r \left(\frac{\mu_H}{\mu_B} \right) \left(1 + \frac{1}{\beta} \right) [(f' + \lambda h - 1)^2 + (g' + \lambda_1 k - 1)^2] \\ & + \beta_1^* \left[\theta' \phi' + \phi'^2 \left(\frac{\alpha_2}{\alpha_1} \right) \right] \\ & + \left(\frac{\rho_H}{\rho_B} \right) M B_r [E_1 - ((f' + \lambda h - 1) + (g' + \lambda_1 k - 1))]^2. \end{aligned} \quad (26)$$

The Bejan number (Be) determines the ratio of heat transport to the total entropy generation of any system given by:

$$Be = \left(\frac{\frac{k_H}{k_B} \alpha_1}{N_S} \right) \theta'^2 \quad (27)$$

The Bejan number typically ranges from $[0, 1]$. When $Be = 0$, fluid friction dominates system irreversibility. When $Be = 1$, heat transfer is the primary source of irreversibility. At $Be = \frac{1}{2}$, both fluid friction and heat transfer contribute equally to system irreversibility.

5. Numerical procedure

The numerical procedure for evaluating the solutions of the coupled non-linear ordinary differential Eqs. (10)–(15), subject to the boundary conditions (16), is implemented via the `bvp4c` method in MATLAB. The numerical solution to the equations is obtained using a finite number of nodes on the domain $\eta \in [0, 3]$, initially set to 1000. High resolution and stringent convergence are ensured by defining a very small tolerance `RelTol` of 10^{-6} . Two different initial guess solutions for the BVP solver are introduced through `bvpinit` for identifying the two branches of solutions: `yinit1` = $[1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0]^T$ for the first branch and `yinit2` = $[-3, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]^T$ to separate the second branch.

$$\begin{aligned} f &= y_1, & f' &= y_2, & f'' &= y_3, \\ g &= y_4, & g' &= y_5, & g'' &= y_6, \\ h &= y_7, & h' &= y_8, & k &= y_9, & k' &= y_{10}, \\ \theta &= y_{11}, & \theta' &= y_{12}, & \phi &= y_{13}, & \phi' &= y_{14}. \end{aligned} \quad (28)$$

$$\begin{aligned}
y'_1 &= y_2 \\
y'_2 &= y_3 \\
y'_3 &= \left(\frac{\mu_B}{\rho_B}\right) \left(\frac{\rho_H}{\mu_H}\right) \frac{1}{(1+\frac{1}{\beta})} \left[\frac{1}{3}(y_2 + y_5)y_2 - \frac{2}{3}(y_1 + y_4)y_3 - \frac{1}{3}\lambda_1 y_2 y_9 \right. \\
&\quad \left. - \frac{1}{3} + M(y_2 - 1) - ME_1 - Gr_T \cos \varphi y_{11} \right. \\
&\quad \left. - Gr_C \cos \varphi y_{13} + \beta^*(1 + \frac{1}{\beta}) \left(\frac{\mu_H}{\rho_H}\right) \left(\frac{\rho_B}{\mu_B}\right) (y_2 - 1) \right] \\
y'_4 &= y_5 \\
y'_5 &= y_6 \\
y'_6 &= \left(\frac{\mu_B}{\rho_B}\right) \left(\frac{\rho_H}{\mu_H}\right) \frac{1}{(1+\frac{1}{\beta})} \left[\frac{1}{3}(y_2 + y_5)y_5 - \frac{2}{3}(y_1 + y_4)y_6 - \frac{1}{3}\lambda_1 y_7 y_5 \right. \\
&\quad \left. - \frac{1}{3} + M(y_2 - 1) - ME_1 - Gr_T \sin \varphi y_{11} \right. \\
&\quad \left. - Gr_C \sin \varphi y_{13} + \beta^*(1 + \frac{1}{\beta}) \left(\frac{\mu_H}{\rho_H}\right) \left(\frac{\rho_B}{\mu_B}\right) (y_5 - 1) \right] \\
y'_7 &= y_8 \\
y'_8 &= \left(\frac{\mu_B}{\rho_B}\right) \left(\frac{\rho_H}{\mu_H}\right) \frac{1}{(1+\frac{1}{\beta})} \left[\frac{1}{3}(y_2 + y_5)y_7 - \frac{2}{3}(y_1 + y_4)y_8 \right. \\
&\quad \left. + \frac{1}{3}(\lambda_1 y_7 + \lambda_1 y_9)y_7 + \frac{1}{3}y_2 y_7 + \beta^*(1 + \frac{1}{\beta}) \left(\frac{\mu_H}{\rho_H}\right) \left(\frac{\rho_B}{\mu_B}\right) y_7 \right] \\
&\quad + 2y_2 y_7 + M y_7 \\
y'_9 &= y_{10} \\
y'_{10} &= \left(\frac{\mu_B}{\rho_B}\right) \left(\frac{\rho_H}{\mu_H}\right) \frac{1}{(1+\frac{1}{\beta})} \left[\frac{1}{3}(y_2 + y_5)y_9 - \frac{2}{3}(y_1 + y_4)y_{10} \right. \\
&\quad \left. + \frac{1}{3}(\lambda_1 y_8 + \lambda_1 y_9)y_9 + \frac{1}{3}y_5 y_9 + \beta^*(1 + \frac{1}{\beta}) \left(\frac{\mu_H}{\rho_H}\right) \left(\frac{\rho_B}{\mu_B}\right) y_9 \right] \\
&\quad + 2y_5 y_9 + M y_9 \\
y'_{11} &= y_{12} \\
y'_{12} &= Pr \left(\frac{k_B}{k_H}\right) \left(\frac{\rho C_p}{\rho C_p}\right) \left[\frac{2}{3}(y_2 + y_5 + \lambda_1 y_7 + \lambda_1 y_9)y_{11} - \frac{2}{3}(y_1 + y_4)y_{12} \right. \\
&\quad \left. - MEc[E_1 - (y_2 + \lambda_1 y_7 + y_5 + \lambda_1 y_9 - 2)]^2 - \tau(Nt y_{12}^2 + Nb y_{12} y_{14}) \right. \\
&\quad \left. - Ec(1 + \frac{1}{\beta}) \left(\frac{\nu_H}{\nu_B}\right) [(y_3 + \lambda_1 y_8)^2 + (y_6 + \lambda_1 y_{10})^2] \right. \\
&\quad \left. - Qe^{-m} \left(\frac{\rho C_p}{\rho C_p}\right) y_{11} \right] \\
y'_{13} &= y_{14} \\
y'_{14} &= \left[\frac{2}{3}Sc(y_2 + y_5 + \lambda_1 y_7 + \lambda_1 y_9)y_{13} - \frac{2}{3}Sc(y_1 + y_4)y_{14} \right. \\
&\quad \left. - ScSr y_{12} + ScK_1 y_{13} \right] \\
&\quad (29) \\
N_S &= \left(\frac{k_{nf}}{k_f}\right) \alpha_1 y_{12}^2 + Br(1 + \frac{1}{\beta}) \left(\frac{\mu_H}{\mu_B}\right) (y_3 + \lambda_1 y_8)^2 \\
&\quad + \left(\frac{\mu_H}{\mu_B}\right) (1 + \frac{1}{\beta}) K_1^p Br [(y_2 + \lambda_1 y_7 - 1)^2 + (y_5 + \lambda_1 y_9 - 1)^2] \\
&\quad + \beta_1^* \left[y_{12} y_{14} + y_{14}^2 \left(\frac{\alpha_2}{\alpha_1}\right) \right] \\
&\quad + \left(\frac{\rho_H}{\rho_B}\right) M Br [E_1 - ((y_2 + \lambda_1 y_7 - 1) + (y_5 + \lambda_1 y_9 - 1))]^2 \\
&\quad (30)
\end{aligned}$$

Table 4Mesh Independence test for the present numerical solution at $E_1 = 1$.

Initial mesh points	Final nodes	$f''(0)$	$g''(0)$	$-\theta'(0)$	$-\phi'(0)$
250	358	0.79794	1.20990	5.38410	0.45605
500	571	0.79794	1.20990	5.38410	0.45605
750	803	0.79794	1.20990	5.38410	0.45605
1000	1000	0.79794	1.20990	5.38410	0.45605
1250	1250	0.79794	1.20990	5.38410	0.45605
1500	1500	0.79794	1.20990	5.38410	0.45605

$$\begin{cases}
yl(1) = \frac{3}{5}s, & yl(4) = 0, & yr(2) = 1, & yr(5) = 1, & yr(7) = 0, \\
yr(9) = 0, & yl(11) = 1, & yr(11) = 0, & yl(13) = 1, \\
yr(13) = 0, & yl(2) = A_1 \cdot yl(3), & yl(5) = B_1 \cdot yl(6), \\
yl(7) = A_1 \cdot yl(8) + \xi, & yl(9) = B_1 \cdot yl(10) + \xi.
\end{cases} \quad (31)$$

Mesh independence and convergence tests are conducted by increasing the number of the initial grid points from 250 to 1500 for a fixed relative error tolerance of 10^{-6} in the MATLAB bvp4c solver over the finite region of the physical domain $\eta \in [0, 3]$, and by allowing the adaptive grid refinement procedure to enforce the convergence tolerance of 10^{-6} . Table 4 shows that $f''(0)$, $g''(0)$, $-\theta'(0)$, and $-\phi'(0)$ calculated values are the same up to 5 decimal digits. The maximum percent difference was found to be negligible, so it can be assumed that the used grid could yield an independent and well-conditioned result. Hence, we selected a grid of 1000 points as the initial grid.

6. Engineering quantities

The dimensional form of engineering interest quantities, the skin friction (Cf and Cg), the Nusselt number (Nu), and the Sherwood number (Sh) are defined as:

$$\begin{cases} Cf = \frac{\tau_{zx}}{\rho_B a^2 (x+y)^{\frac{2}{3}}}, & \text{where } \tau_{zx} = \mu_H \left(\frac{\partial \tilde{u}}{\partial z}\right)_{z=0} \\ Cg = \frac{\tau_{zy}}{\rho_B a^2 (x+y)^{\frac{2}{3}}}, & \text{where } \tau_{zy} = \mu_H \left(\frac{\partial \tilde{v}}{\partial z}\right)_{z=0} \end{cases} \quad (32)$$

$$Nu = \frac{(x+y)^{\frac{1}{3}} q_w}{k_B (\tilde{T}_w - \tilde{T}_\infty)}, \text{ where } q_w = -k_H \left(\frac{\partial \tilde{T}}{\partial z}\right)_{z=0} + O_T \quad (33)$$

$$Sh = \frac{m_w (x+y)^{\frac{1}{3}}}{\rho_B D_B (\tilde{C}_w - \tilde{C}_\infty)}, \text{ where } m_w = \rho_H D_B \left(\frac{\partial \tilde{C}}{\partial z}\right)_{z=0} \quad (34)$$

After the implication of similarity transformation, the non dimensional forms are:

$$\begin{cases} Cf = \frac{\mu_H}{\mu_B} Re^{-0.5} (f'' + \lambda h')_{\eta=0}, \\ Cg = \frac{\mu_H}{\mu_B} Re^{-0.5} (g'' + \lambda_1 k')_{\eta=0}. \end{cases} \quad (35)$$

$$Nu = -\frac{k_H}{k_B} Re^{0.5} \theta'_{\eta=0}, \quad (36)$$

$$Sh = -\frac{\rho_H}{\rho_B} Re^{0.5} \phi'_{\eta=0}. \quad (37)$$

where, $Re^{0.5} = \sqrt{\frac{a}{\nu_B}} (x+y)^{\frac{2}{3}}$ indicates the Reynolds number.

7. Results and graphs

This section analyzes the variations in velocity, temperature, and concentration profiles with respect to different physical parameters such as Grashof number, Hartmann number, electric field parameter, slip parameter, etc. The influence of these parameters is studied through graphical representations, highlighting their physical significance in

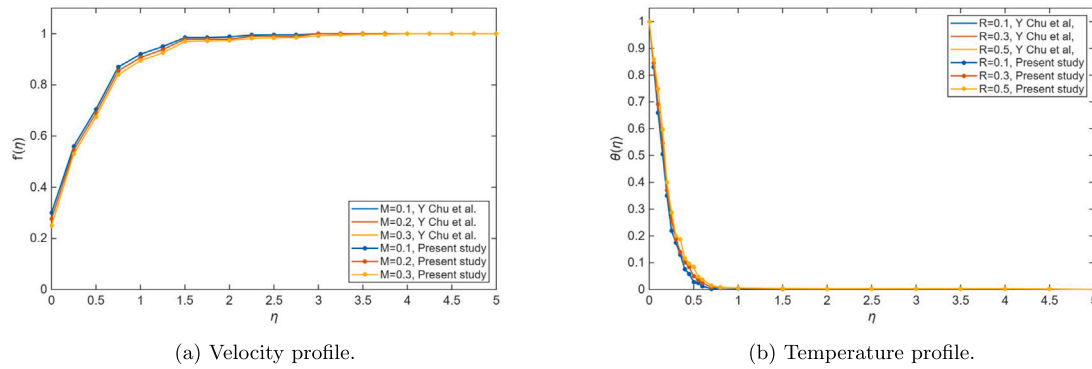


Fig. 4. Validation of the present numerical results with the published results of Y. Chu et al. [39] for the velocity and temperature profiles.

Table 5

Contrasting $f''(0)$ with the outcomes presented by Wang [44].

$A_1 = B_1$		Present Result	Wang [44]	Error
0	$f''(0)$	1.311933	1.311938	0.000005
0.25	$f''(0)$	0.866884	0.866880	0.000004
0.5	$f''(0)$	0.617311	0.617300	0.000011

modifying fluid flow behavior. The results based on the current numerical technique were validated using the comparison of velocity and temperature profiles with Y. Chu et al, [39] in Fig. 4, and Table 5 shows a numerical comparison of our evaluated ($f''(0)$) and the corresponding values from Wang et al. [44], which supports the correctness of our solution.

Fig. 5(a) illustrates the effect of the thermal Grashof number on the velocity profile. The velocity increases as Gr_T rises, indicating enhanced buoyancy-driven flow. Physically, Gr_T represents the ratio of buoyancy forces to viscous forces. A higher Gr_T enhances the temperature-induced buoyancy effect, accelerating fluid motion due to density variations in the heated fluid. Similarly, Fig. 5(b) demonstrates the influence of the solutal Grashof number on $f'(\eta)$. The velocity increases as Gr_C rises, signifying stronger buoyancy forces induced by concentration gradients. Since Gr_C represents the effect of solutal buoyancy relative to viscous resistance, a higher value promotes increased fluid motion driven by concentration differences. Fig. 5(c) presents the impact of the electric field parameter E_1 on velocity distribution. An increase in E_1 results in a reduction in velocity due to the presence of the electromagnetic resistance force. The interaction between the electric and magnetic fields generates the Lorentz force, which opposes the flow motion and exerts a resistive effect on the fluid, slowing its velocity. This phenomenon is particularly significant in EMHD flows, where electromagnetic forces alter fluid dynamics. Since your analysis is based on s (mass transport parameter related to suction/injection), the explanation should focus on how suction ($s > 0$) and injection ($s < 0$) affect tangential velocity. Fig. 5(d) and (e) examine the impact of the mass transport parameter s on tangential velocity. When $s > 0$ (suction), the fluid is drawn toward the surface, which reduces the tangential velocity due to increased resistance from the inward flow. As a result, the boundary layer thins, and the velocity profile shifts closer to the wall. Conversely, for $s < 0$ (injection), the fluid is pushed away from the surface, enhancing the tangential velocity due to the outward momentum transfer. This causes boundary layer thickening and higher velocity farther from the wall. The effects of s play a crucial role in controlling fluid transport in porous systems, cooling applications, and biomedical flows. Fig. 5(f) examines the effect of the Hartmann number on velocity. The velocity decreases with increasing M , highlighting the resistive nature of the magnetic field. The application of a magnetic field induces a Lorentz force that opposes the motion, increasing the flow resistance and reducing velocity. The suppression of velocity due to the magnetic field is a characteristic feature

of EMHD flows, which is crucial in controlling fluid transport in electromagnetic systems. Fig. 6(a) and (b) illustrate the effect of the s on axial velocity. For $s > 0$ (suction), increasing s reduces axial velocity because the inward fluid movement towards the surface limits longitudinal momentum, suppressing axial flow. In contrast, for $s < 0$ (injection), the axial velocity increases as the injected fluid enhances momentum transfer, pushing the flow more effectively in the axial direction. These effects are critical in flow control, porous media applications, and optimizing transport in industrial and biomedical systems. Fig. 6(c) shows the impact of the Hartmann number on temperature distribution. As M increases, the temperature profile rises due to increased electromagnetic energy dissipation. The opposing Lorentz force increases thermal energy retention within the fluid, leading to higher temperatures. This effect is important in applications where thermal control through electromagnetic manipulation is required. Fig. 6(d) examines the effect of the Eckert number on temperature distribution. As Ec increases, the temperature rises due to enhanced viscous dissipation. The Ec represents the ratio of kinetic energy to boundary layer enthalpy. A higher Ec signifies greater conversion of mechanical energy into thermal energy due to frictional effects within the fluid. This conversion contributes to increased internal energy, raising the temperature near heated surfaces. Regarding concentration profiles, Fig. 6(e) illustrates the effect of the chemical reaction rate parameter. As K_1 increases, the concentration decreases due to intensified chemical reactions consuming the diffusing species. A higher reaction rate implies faster depletion of the chemical species, reducing its concentration within the fluid. This is particularly relevant in chemical processing and catalysis, where reaction kinetics dictate species distribution. Finally, Fig. 6(f) highlights the influence of the Schmidt number on concentration profiles. A higher Sc leads to reduced concentration due to lower solute diffusivity. The Sc is the ratio of momentum diffusivity to mass diffusivity. When Sc is large, mass diffusion occurs at a slower rate, limiting solute dispersion. This results in steeper concentration gradients and reduced overall solute distribution within the fluid.

8. Optimization

RSM is a powerful approach for modeling complex output functions. RSM was selected because it reduces the number of computational runs while capturing both individual and interaction effects of E_1 , λ , and β . Unlike one-factor-at-a-time or trial-and-error methods, RSM provides regression equations, ANOVA-based validation, and response surface plots, making the optimization process more reliable, efficient, and physically interpretable. In this study, we employ a second-order polynomial model incorporating linear, interaction, and squared terms, following the framework proposed by Box and Wilson. RSM is applied to optimize and assess the influence of key parameters on flow and heat transfer in an EMHD Casson-based ternary hybrid nanofluid over an inclined porous plate. The analysis focuses on the effects of the E_1 , λ , and β .

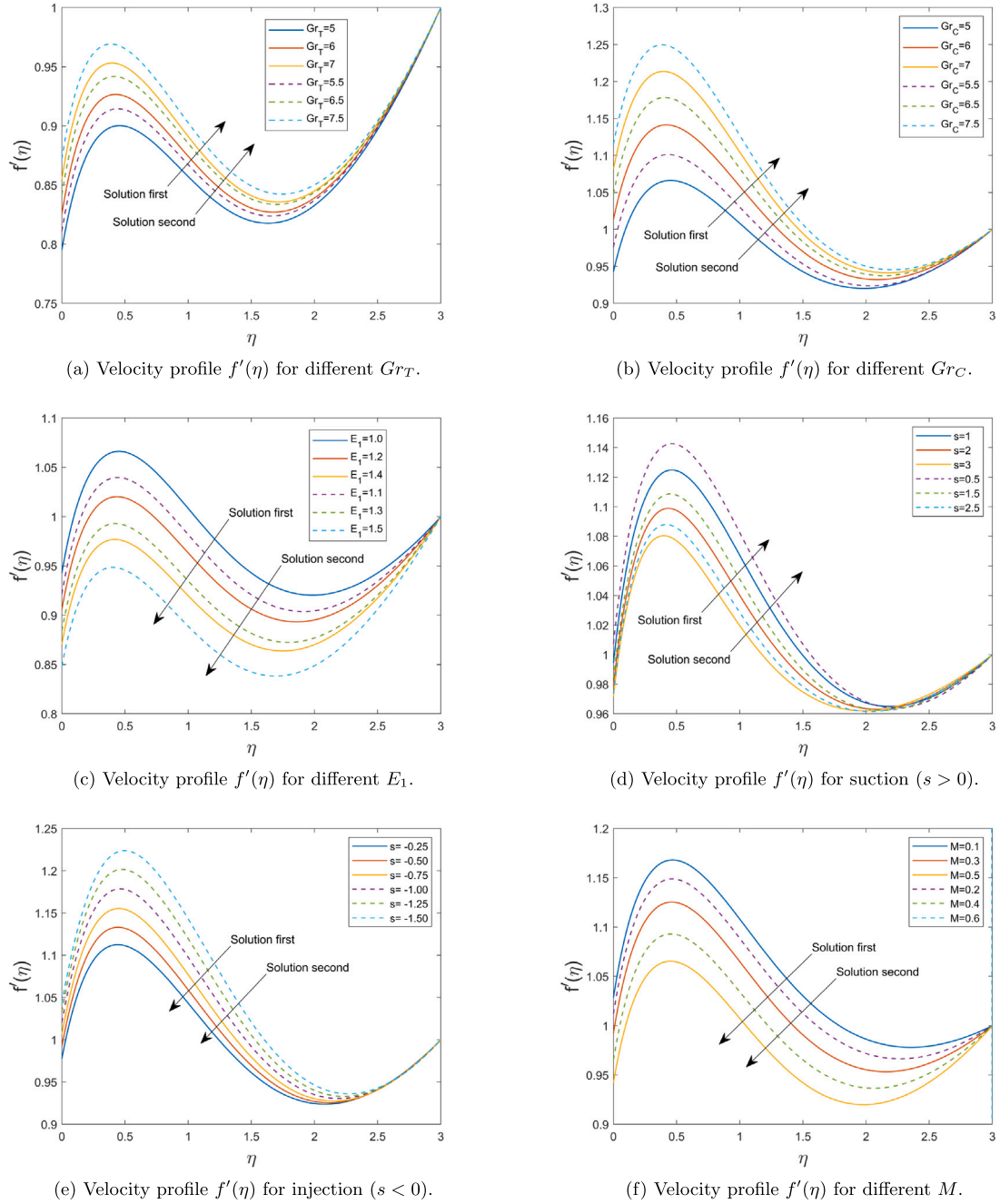


Fig. 5. Influence of governing parameters on the velocity profile $f'(\eta)$.

- **Identification:** The response variables, namely Nu , C_f , Sh , and Be , are influenced by E_1 , λ , and β .
- **Decision-Making:** A face-centered composite design (CCF) is used to evaluate factor interactions, with ANOVA ensuring model validity.
- **Optimization:** RSM optimizes parameter values to enhance heat transfer and control entropy generation.
- **Validation:** Optimized results are verified against numerical simulations for accuracy and reliability.

The general form of the model is given as:

$$R = \delta_0 + \sum_{i=1}^n \delta_i x_i + \sum_{i=1}^n \delta_{ii} x_i^2 + \sum_{i=1}^n \sum_{j=i+1}^n \delta_{ij} x_i x_j \quad (38)$$

where δ_0 is the intercept, δ_i represents the linear regression coefficients, δ_{ii} denotes the quadratic regression coefficients, and δ_{ij} accounts for interaction effects between parameters. To obtain reliable response predictions, the CCF design is implemented, conducting $2a + 2b + c$ experiments, where a is the number of factors, b is the number of central points, and c represents replicates for statistical accuracy. Fig. 7 provides a detailed roadmap for conducting sensitivity analysis, outlining the steps and key considerations for evaluating the influence of design variables on the Nusselt number, skin friction coefficient, Sherwood number and Bejan number.

8.1. Response surface analysis

The RSM is employed to optimize and analyze the influence of governing parameters on key fluid dynamics and heat transfer

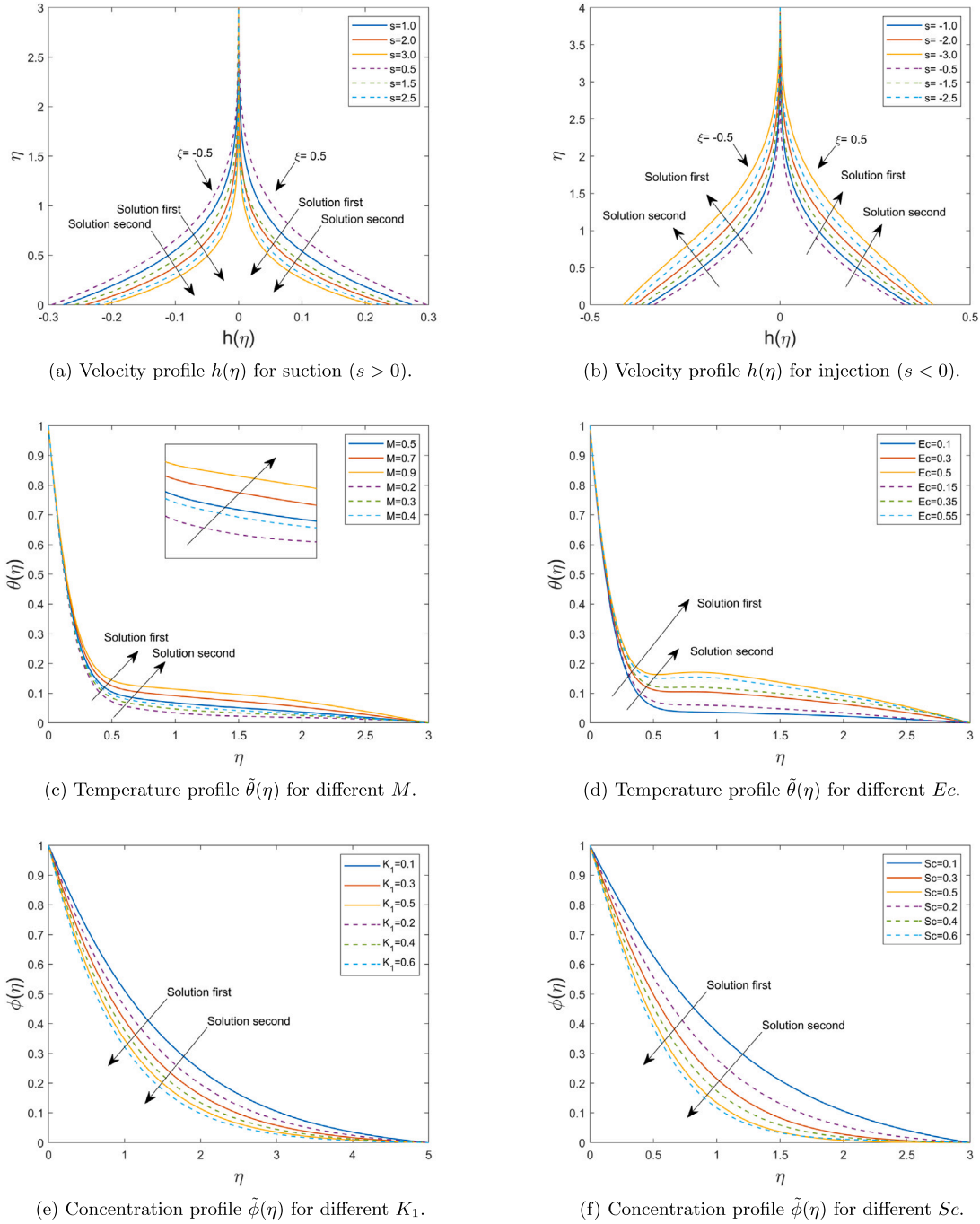


Fig. 6. Influence of governing parameters on the axial velocity, temperature, and concentration profiles.

characteristics in an EMHD Casson-based ternary hybrid nanofluid system. The four primary response variables under investigation are the Nu , Cf , Sh , and Be . The general second-order polynomial model for each response is given as:

$$R = \delta_0 + \delta_1 E_1 + \delta_2 \lambda + \delta_3 \beta + \delta_4 E_1^2 + \delta_5 \lambda^2 + \delta_6 \beta^2 + \delta_7 E_1 \lambda + \delta_8 \lambda \beta + \delta_9 E_1 \beta \quad (39)$$

where R represents Nu , Cf , Sh , or Be , δ_0 is the intercept, δ_i are linear coefficients, δ_{ii} represent quadratic effects, and δ_{ij} capture the interaction effects between parameters.

8.2. Model Validation

The accuracy and significance of the developed models were evaluated using ANOVA, with high F-values confirming statistical

significance. The predicted R^2 and adjusted R^2 values for all responses indicate a strong correlation between the predicted and actual values. The final response equations in terms of coded factors are as follows:

$$Nu = 1.58435 - 0.13608 E_1 + 0.01145 \lambda + 0.02148 \beta - 0.0086375 E_1 \lambda + 0.0179875 E_1 \beta - 0.0008125 \lambda \beta - 0.0791273 E_1^2 - 0.0209773 \lambda^2 - 0.00952727 \beta^2. \quad (40)$$

$$Cf = 1.69408 + 0.10547 E_1 - 1.42191 \lambda - 0.12537 \beta + 0.0107 E_1 \lambda - 0.009875 E_1 \beta + 0.233375 \lambda \beta + 0.254488 E_1^2 - 0.120892 \lambda^2 + 0.120288 \beta^2. \quad (41)$$

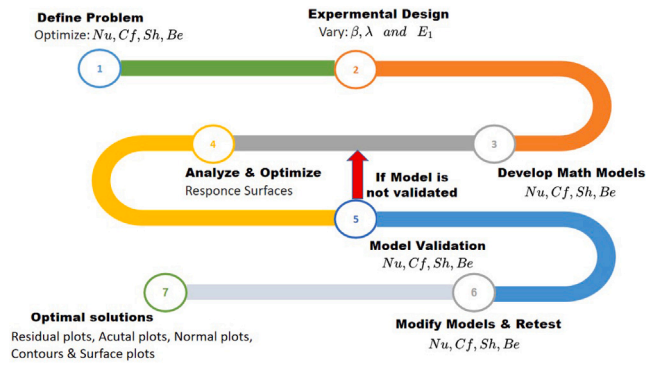


Fig. 7. Roadmap for performing sensitivity analysis.

$$Sh = 0.2139 - 0.00052E_1 + 0.00118\lambda - 0.00011\beta + 8.75E^{-05}E_1\lambda - 0.000338E_1\beta - 0.000138\lambda\beta + 0.001564E_1^2 + 0.000164\lambda^2 + 1.364E^{-05}\beta^2. \quad (42)$$

$$Be = 0.5581 - 0.0964E_1 - 0.0344\lambda + 0.0086\beta - 0.0064E_1\lambda + 0.0051E_1\beta - 0.0004\lambda\beta - 0.0281E_1^2 - 0.0117\lambda^2 - 0.0037\beta^2. \quad (43)$$

The predicted R^2 values for Nu , C_f , Sh , and Be are 0.9926, 0.9885, 0.9857, and 0.9944, respectively, aligning closely with their adjusted R^2 values, confirming model reliability. The Adequate Precision for all responses exceeds the threshold of 4, ensuring a strong signal-to-noise ratio. The coded equations normalize factor levels (+1 for high, -1 for low) and provide insights into parameter influence on heat transfer, momentum transport, and entropy generation.

These results validate the effectiveness of the RSM-based model in predicting and optimizing heat and mass transfer characteristics within the EMHD Casson-based ternary hybrid nanofluid system.

8.3. Graphical analysis

This analysis utilizes residual plots and response surface visualizations to evaluate model performance and parameter influence. The Residuals vs. Predicted plots (Fig. 8(a), (d), (g) and (j)) and Residuals vs. Run plots (Fig. 8(b), (e), (h) and (k)) assess model fit by examining residual scatter. A random distribution around zero indicates a good fit, whereas deviations suggest potential inaccuracies or biases. Data points within the red lines (typically ± 4.14579) are generally acceptable, while outliers may signal model issues. The Predicted vs. Actual plots (Fig. 8(c), (f), (i) and (l)) demonstrate a strong linear correlation when data points closely align with the diagonal, indicating high predictive accuracy. The Normal Probability Plot of Residuals (Fig. 9(a), (b), (c) and (g)) assesses whether the model's residuals follow a normal distribution. Data points closely following the red line suggest normally distributed residuals, whereas deviations especially at the extremes may indicate non-normality and potential model issues. Residuals vs. E_1 Plot (Fig. 9(d), (e), (f) and (h)) examines residual distribution against parameter E_1 to detect any systematic patterns. A random scatter around zero suggests the model accurately captures the effect of E_1 , whereas visible patterns may indicate model inadequacies or parameter-related biases. Outliers beyond the red lines (± 4.14579) warrant further investigation. 3D surface plots (Fig. 10(a)–(l)) provide a visual representation of the response variable's magnitude across the E_1 - λ , E_1 - β , and λ - β parameter spaces, where the color gradient reflects response intensity. These plots collectively emphasize the complex parameter interactions and their influence on heat transfer and system behavior. Contour plots (Fig. 11(a)–(l)) illustrate the interaction between parameter pairs: E_1 and λ , E_1 and β , and λ and β with respect to the Nusselt number,

skin-friction coefficient, Sherwood number, and Bejan number. Red regions indicate higher Nu , C_f , Sh , and Be values, while blue regions signify lower values, highlighting the impact of these parameters on heat transfer, friction, mass transport, and thermodynamic irreversibility.

The graphical analysis of the Nu , C_f , Sh , and Be provides deeper insights into the influence of governing parameters, including E_1 , λ , and β . The contour and surface plots demonstrate the variations in response due to these independent parameters. From a thermodynamic viewpoint, the Bejan number identifies the dominant irreversibility mechanism in the system. Higher Be values indicate heat-transfer-dominated irreversibility, whereas lower Be values suggest stronger contributions from fluid-friction, electromagnetic, and mass-transfer irreversibilities. Thus, the combined interpretation of Nu , C_f , Sh , and Be helps locate parameter regions with improved transport performance and reduced useful energy loss.

- The plots in Figs. 10(a)–(c) and 11(a)–(c) indicate that increasing E_1 reduces heat transfer, while higher β enhances it. The interaction between λ and β has minimal impact on Nu . However, the maximum Nu occurs at low E_1 and high β , suggesting that strong EMHD effects hinder heat transfer efficiency.
- The plots in Figs. 10(d)–(f) and 11(d)–(f) show that increasing E_1 raises shear stress, whereas higher λ reduces it. The interaction between E_1 and β is nonlinear, with peak C_f observed at high E_1 and moderate λ . A greater λ lowers shear resistance, enhancing flow smoothness.
- The plots in Figs. 10(g)–(i) and 11(g)–(i) indicate that increasing E_1 slightly reduces mass transfer, while β significantly enhances the Sh . The maximum Sh occurs at high β and moderate E_1 , highlighting the role of the Casson parameter in improving species diffusion.
- The plots in Figs. 10(j)–(l) and 11(j)–(l) show that the Be increases with E_1 and decreases with higher λ . This indicates that stronger electric field effects increase heat-transfer irreversibility relative to total irreversibility, while velocity slip reduces wall resistance and helps suppress frictional losses. Since Be represents the ratio of heat-transfer irreversibility to total irreversibility, high Be regions correspond to thermally dominated entropy production, whereas lower Be regions indicate stronger fluid-friction, electromagnetic, and diffusion-related irreversibilities. A moderate β contributes to entropy production, with peak Be occurring at high E_1 and low λ . Therefore, controlling E_1 and λ is important for improving thermodynamic efficiency and reducing useful energy loss in EMHD transport systems.

8.4. Statistical analysis and validation

Statistical analysis plays a crucial role in quantifying the influence of various parameters on response functions within complex systems. The ANOVA method quantifies the influence of E_1 , λ , and β on the response functions. Table 6 presents the computed response functions Nu , C_f , Sh , and Be based on the E_1 , λ , and β . These response functions, derived from MATLAB simulations, analyze how variations in the independent parameters influence heat transfer, shear stress, mass transfer, and entropy generation across 20 simulation runs. To assess the statistical significance of these relationships, Tables 7–10 provide ANOVA results for Nu , C_f , Sh , and Be , respectively. The coefficient of determination (R^2), calculated as:

$$R^2 = 1 - \frac{\text{Sum of Squares of Residuals}}{\text{Total Sum of Squares}},$$

The R^2 measures how well the model explains the variability in the data, with values close to unity indicating strong model performance. For Nu , the high R^2 value and significant F-value confirm the influence of E_1 , λ , and β on heat transfer. The adjusted R^2 remains consistent, demonstrating minimal noise interference. Similarly, the ANOVA table

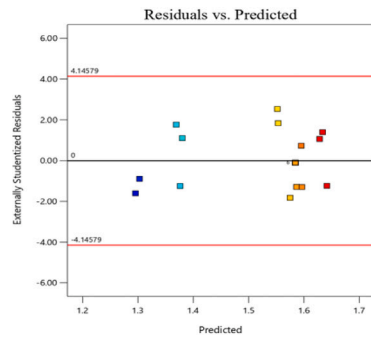
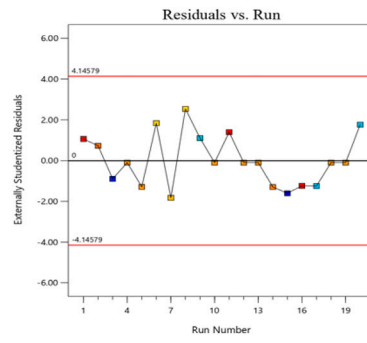
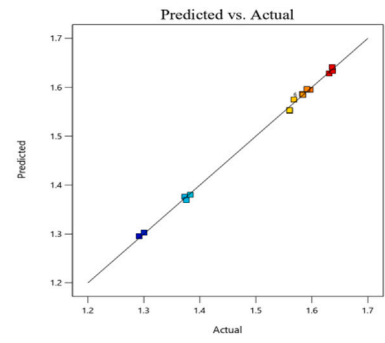
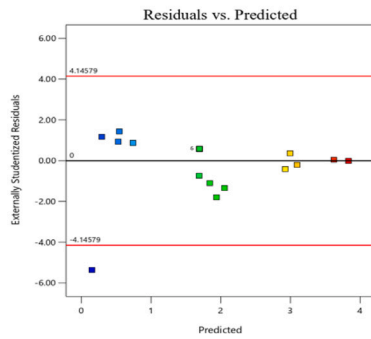
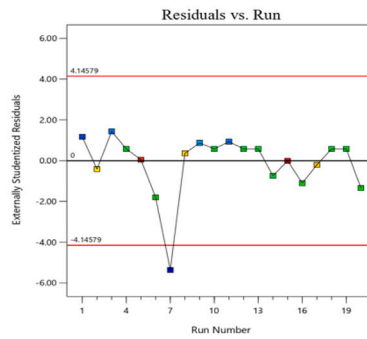
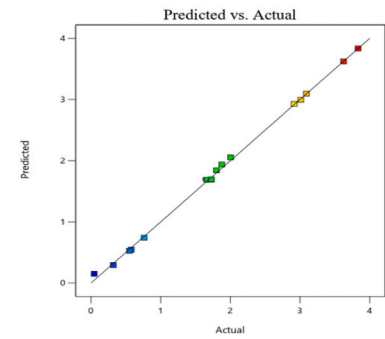
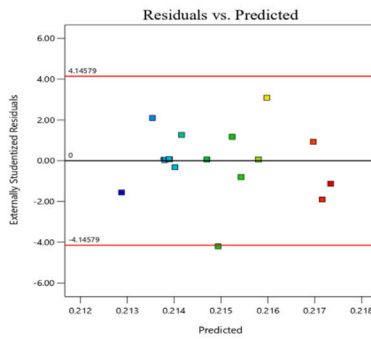
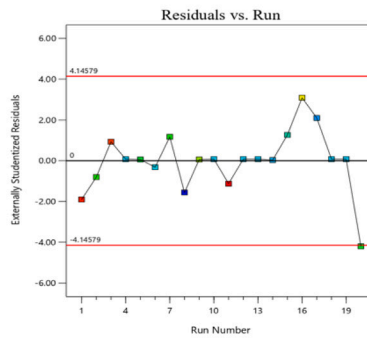
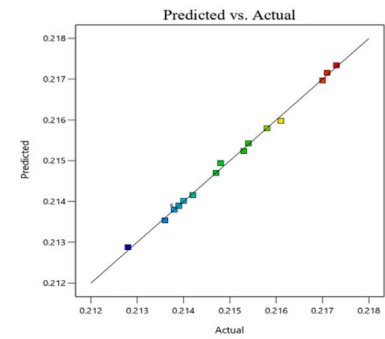
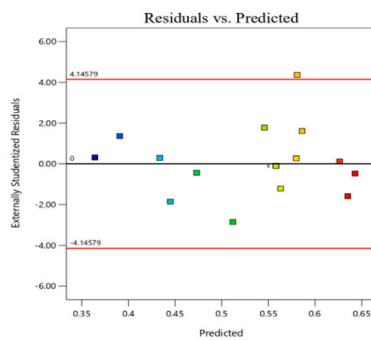
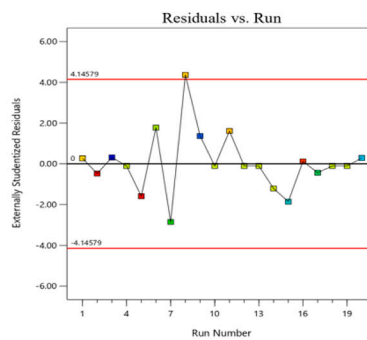
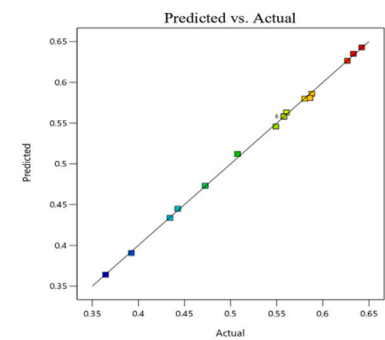
(a) $Nu(a)$ (b) $Nu(b)$ (c) $Nu(c)$ (d) $Cf(a)$ (e) $Cf(b)$ (f) $Cf(c)$ (g) $Sh(a)$ (h) $Sh(b)$ (i) $Sh(c)$ (j) $Be(a)$ (k) $Be(b)$ (l) $Be(c)$

Fig. 8. Residuals vs. Predicted, Residuals vs. Run, and Predicted vs. Actual plots for Nu , Cf , Sh , and Be , illustrating model accuracy, consistency, and predictive reliability.

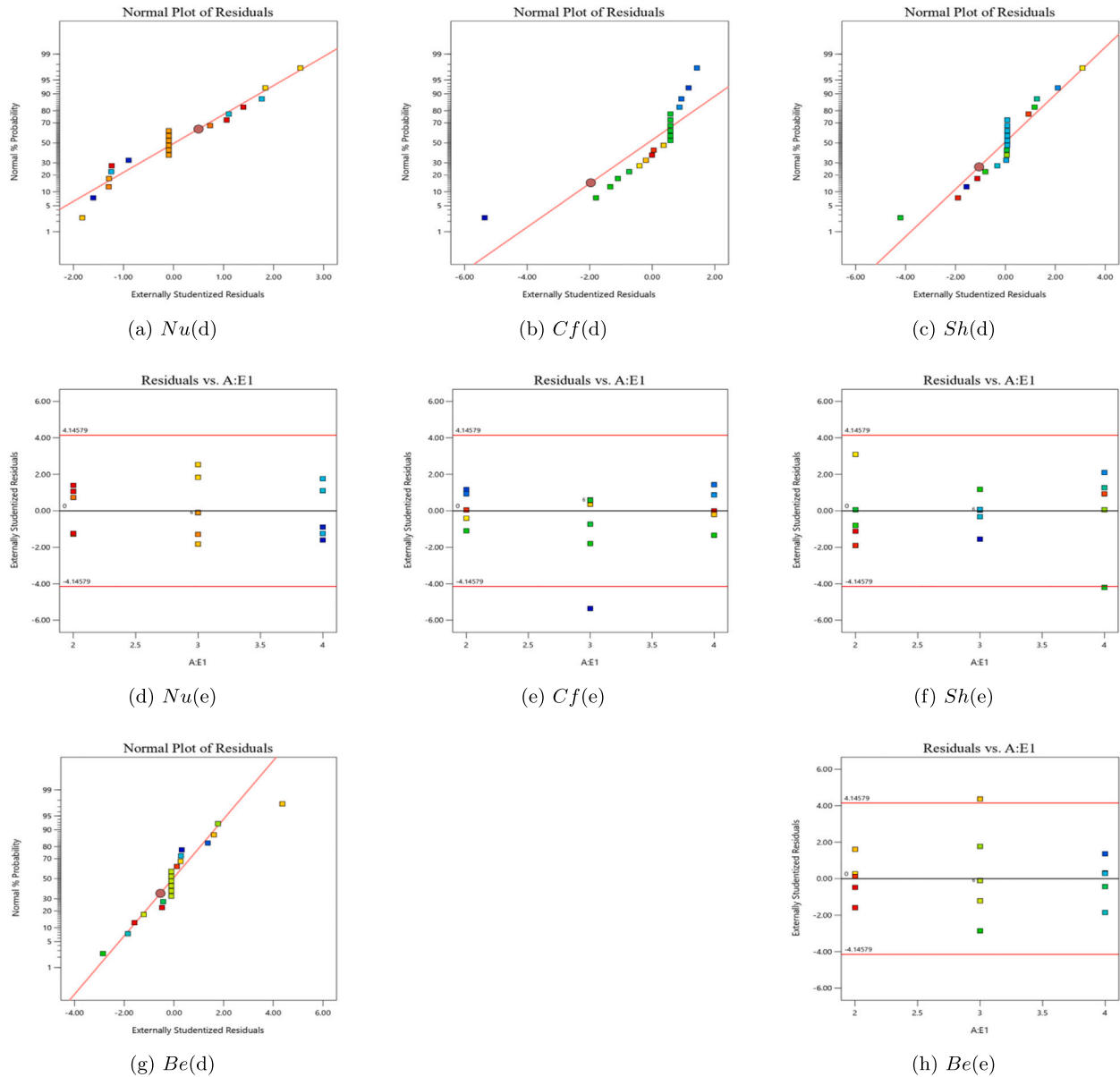


Fig. 9. Normal probability plot of residuals and Residuals vs. E_1 for Nu , Cf , Sh , and Be , assessing the model's error distribution and the influence of the electric field parameter.

for Cf shows high R^2 and low p-values (< 0.05), validating the substantial impact of the independent parameters on shear stress. For Sh , the statistical analysis confirms the strong contribution of E_1 and λ to mass transfer, supported by high F-values and low p-values. The Be also exhibits significant parameter dependence, as indicated by a high F-value and statistically significant model terms. Additionally, the predicted R^2 values closely align with the adjusted R^2 values, confirming the model's reliability for new observations. Terms with p-values less than 0.05 are statistically significant, while those exceeding this threshold may be excluded. Furthermore, the Adequate Precision values surpass recommended thresholds, indicating a strong signal-to-noise ratio. Overall, the ANOVA results validate that the regression models for Nu , Cf , Sh , and Be effectively capture the influence of E_1 , λ , and β on heat transfer, skin friction, mass transfer, and entropy generation, respectively. The statistical analysis confirms high model significance ($p < 0.0001$), with $R^2 > 0.99$, ensuring precise predictions. The key findings from the statistical analysis are:

- The sensitivity analysis reveals that E_1 has a strong negative impact on the Nu (-0.13608), reducing the heat transfer rate. In contrast, the β (0.02148) enhances Nu , improving thermal performance. The interaction term $E_1\beta$ (0.01799) indicates that increasing β mitigates the negative influence of E_1 .
- The (λ) has the most significant negative impact on the Cf (-1.42191), indicating that increasing λ reduces shear stress. Meanwhile, E_1 (0.10547) increases Cf , suggesting that stronger electromagnetic effects enhance wall shear stress. The interaction between λ and β (0.2334) plays a crucial role, as confirmed by its statistical significance ($p < 0.0001$).
- Mass transfer is primarily influenced by λ (0.00118), followed by E_1 (-0.00052). A higher β slightly reduces the Sh (-0.00011). The interaction term $E_1\beta$ (-0.00034) suggests a minor effect on species diffusion.
- Entropy generation due to heat transfer is strongly influenced by E_1 (-0.0964), indicating that increasing E_1 leads to greater irreversibilities. The slip parameter (λ) (-0.0344) reduces entropy

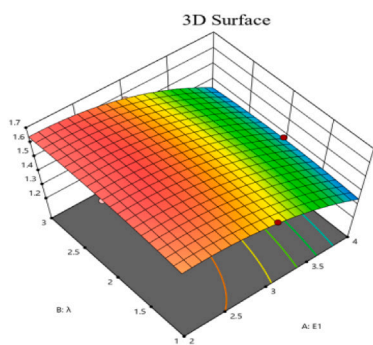
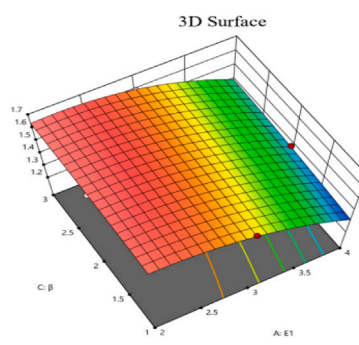
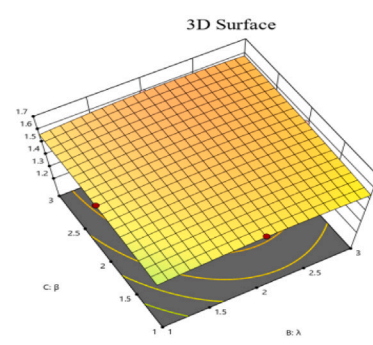
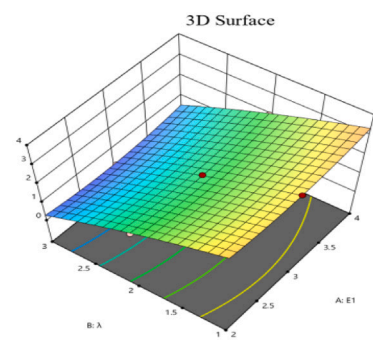
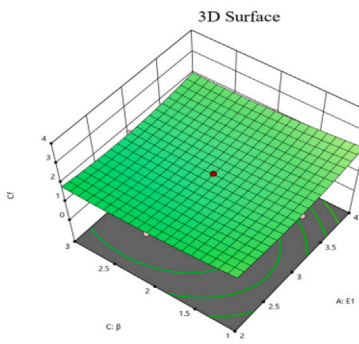
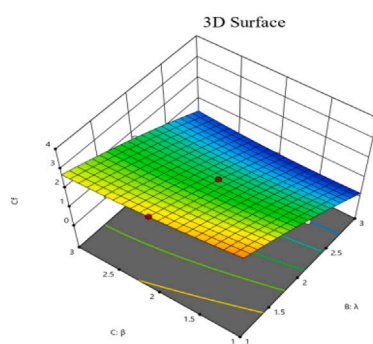
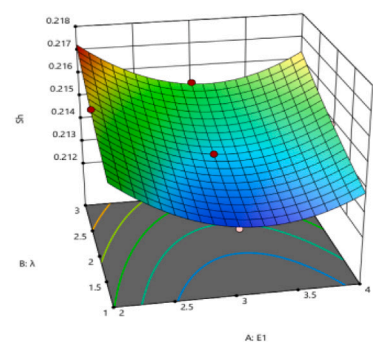
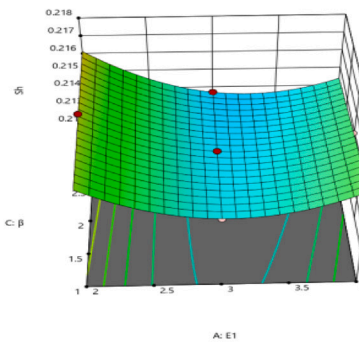
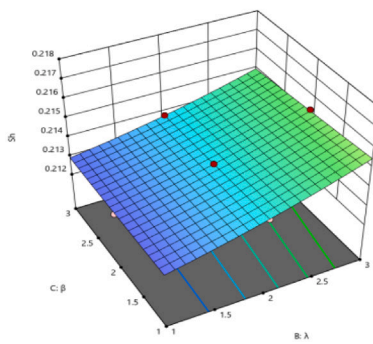
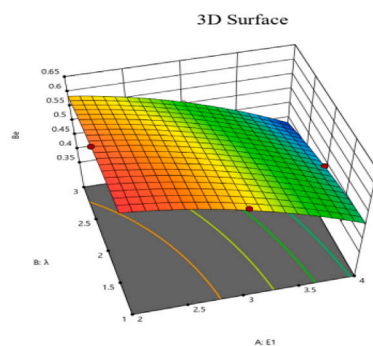
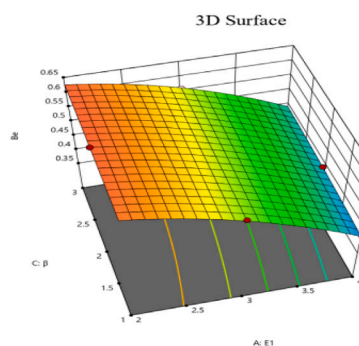
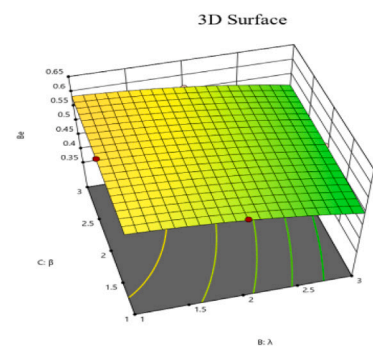

(a) $Nu(a)$

(b) $Nu(b)$

(c) $Nu(c)$

(d) $Cf(a)$

(e) $Cf(b)$

(f) $Cf(c)$

(g) $Sh(a)$

(h) $Sh(b)$

(i) $Sh(c)$

(j) $Be(a)$

(k) $Be(b)$

(l) $Be(c)$

Fig. 10. Surface plots depicting the influence of λ , β , and E_1 on Nu , Cf , Sh , and Be , highlighting their impact on heat transfer, friction, and mass transport characteristics.

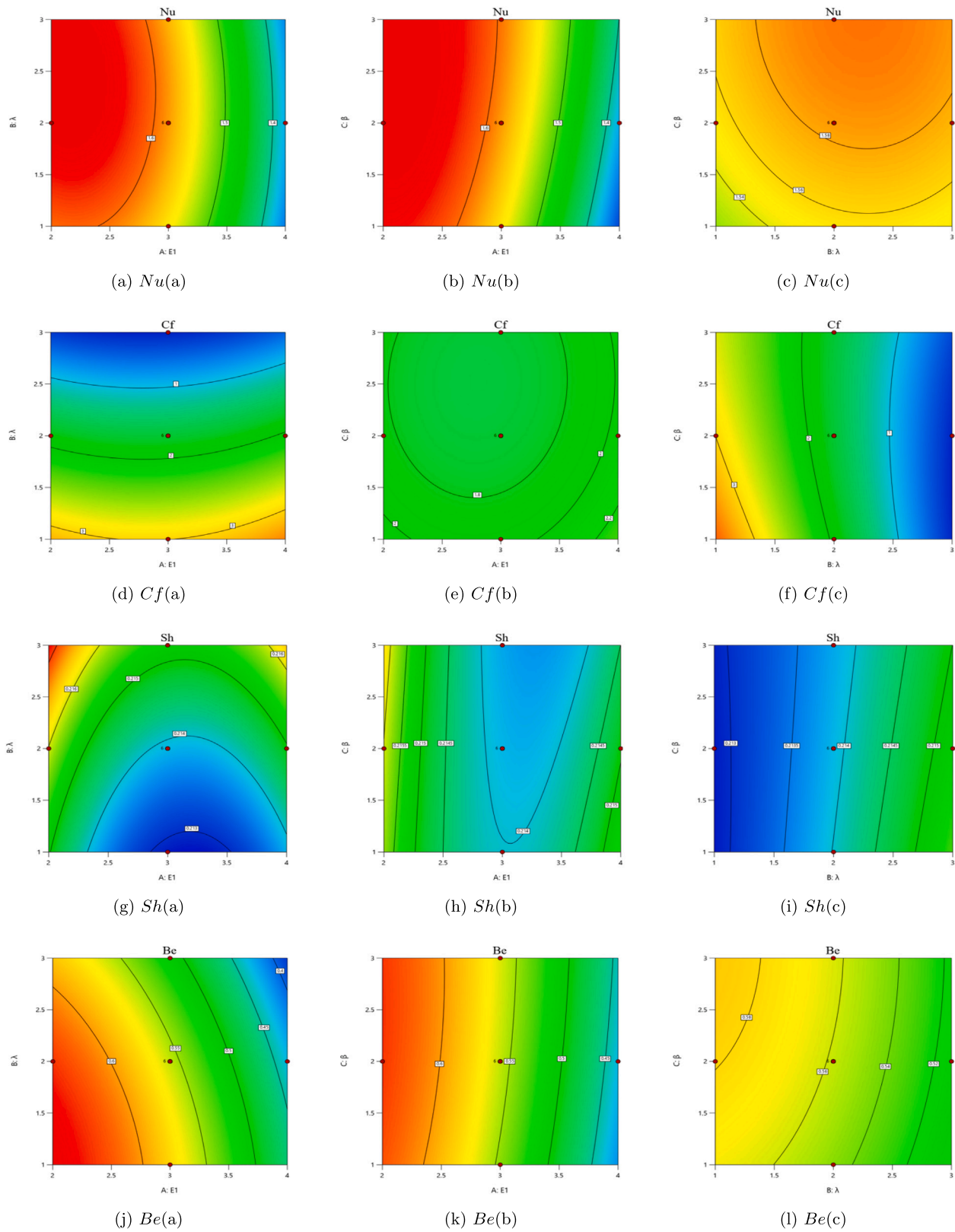


Fig. 11. Contour plots representing the impact of λ , β , and E_1 on Nu , Cf , Sh , and Be , demonstrating variations in heat transfer, friction, and mass transport.

Table 6
Experimental Results.

Std	Run	Factor 1 A: E_1	Factor 2 B: λ	Factor 3 C: β	Response 1 Nu	Response 2 C_f	Response 3 Sh	Response 4 Be
1	5	2	1	1	1.5831	3.6252	0.2147	0.6331
2	15	4	1	1	1.2916	3.8331	0.2142	0.4427
3	1	2	3	1	1.631	0.3202	0.2171	0.5802
4	3	4	3	1	1.3004	0.5776	0.217	0.3645
5	2	2	1	3	1.597	2.9157	0.2154	0.6422
6	17	4	1	3	1.3729	3.0908	0.2136	0.4725
7	11	2	3	3	1.6371	0.5509	0.2173	0.588
8	9	4	3	3	1.383	0.7621	0.2158	0.3924
9	16	2	2	2	1.6363	1.8011	0.2161	0.6266
10	20	4	2	2	1.3758	2.0042	0.2148	0.4342
11	8	3	1	2	1.5604	3.0098	0.2128	0.5861
12	7	3	3	2	1.568	0.0447	0.2153	0.5075
13	6	3	2	1	1.5602	1.877	0.214	0.5491
14	14	3	2	3	1.5911	1.6599	0.2138	0.5606
15	4	3	2	2	1.5838	1.7247	0.2139	0.5578
16	18	3	2	2	1.5838	1.7247	0.2139	0.5578
17	13	3	2	2	1.5838	1.7247	0.2139	0.5578
18	19	3	2	2	1.5838	1.7247	0.2139	0.5578
19	10	3	2	2	1.5838	1.7247	0.2139	0.5578
20	12	3	2	2	1.5838	1.7247	0.2139	0.5578

Table 7
ANOVA Table for Nu.

Source	Sum of Squares	df	Mean Square	F-value	p-value	
Model	0.2439	9	0.0271	800.01	< 0.0001	significant
A- E_1	0.1852	1	0.1852	5465.83	< 0.0001	
B- λ	0.0013	1	0.0013	38.70	< 0.0001	
C- β	0.0046	1	0.0046	136.19	< 0.0001	
AB	0.0006	1	0.0006	17.62	0.0018	
AC	0.0026	1	0.0026	76.40	< 0.0001	
BC	5.281E-06	1	5.281E-06	0.1559	0.7013	
A ²	0.0172	1	0.0172	508.22	< 0.0001	
B ²	0.0012	1	0.0012	35.72	0.0001	
C ²	0.0002	1	0.0002	7.37	0.0218	
Residual	0.0003	10	0.0000			
Lack of Fit	0.0003	5	0.0001			
Pure Error	0.0000	5	0.0000			
Cor Total	0.2443	19				

Table 8
ANOVA Table for Cf.

Source	Sum of Squares	df	Mean Square	F-value	p-value	
Model	21.31	9	2.37	808.26	< 0.0001	significant
A- E_1	0.1112	1	0.1112	37.98	0.0001	
B- λ	20.22	1	20.22	6903.22	< 0.0001	
C- β	0.1572	1	0.1572	53.67	< 0.0001	
AB	0.0009	1	0.0009	0.3127	0.5883	
AC	0.0008	1	0.0008	0.2664	0.6170	
BC	0.4357	1	0.4357	148.77	< 0.0001	
A ²	0.1781	1	0.1781	60.81	< 0.0001	
B ²	0.0402	1	0.0402	13.72	0.0041	
C ²	0.0398	1	0.0398	13.59	0.0042	
Residual	0.0293	10	0.0029			
Lack of Fit	0.0293	5	0.0059			
Pure Error	0.0000	5	0.0000			
Core Total	21.33	19				

generation, improving system efficiency. The interaction effect $E_1 \lambda$ (-0.0064) suggests that moderate E_1 combined with slip conditions can optimize thermodynamic performance.

9. Sensitivity analysis

Sensitivity analysis is a crucial tool for evaluating the influence of key parameters on system responses. In this study, the impact of the E_1 , λ , and β on heat transfer, skin friction, mass transport, and entropy

Table 9
ANOVA Table for Sh.

Source	Sum of Squares	df	Mean Square	F-value	p-value	
Model	0.0000	9	3.545E-06	626.25	< 0.0001	significant
A- E_1	2.704E-06	1	2.704E-06	477.62	< 0.0001	
B- λ	0.0000	1	0.0000	2459.48	< 0.0001	
C- β	1.210E-07	1	1.210E-07	21.37	0.0009	
AB	6.125E-08	1	6.125E-08	10.82	0.0082	
AC	9.113E-07	1	9.113E-07	160.96	< 0.0001	
BC	1.513E-07	1	1.513E-07	26.72	0.0004	
A ²	6.724E-06	1	6.724E-06	1187.64	< 0.0001	
B ²	7.364E-08	1	7.364E-08	13.01	0.0048	
C ²	5.114E-10	1	5.114E-10	0.0903	0.7699	
Residual	5.661E-08	10	5.661E-09			
Lack of Fit	5.661E-08	5	1.132E-08			
Pure Error	0.0000	5	0.0000			
Cor Total	0.0000	19				

Table 10
ANOVA Table for Be.

Source	Sum of Squares	df	Mean Square	F-value	p-value	
Model	0.1136	9	0.0126	1532.47	< 0.0001	significant
A- E_1	0.0929	1	0.0929	11,280.65	< 0.0001	
B- λ	0.0118	1	0.0118	1437.07	< 0.0001	
C- β	0.0007	1	0.0007	90.03	< 0.0001	
AB	0.0003	1	0.0003	39.79	< 0.0001	
AC	0.0002	1	0.0002	25.27	0.0005	
BC	1.280E-06	1	1.280E-06	0.1554	0.7017	
A ²	0.0022	1	0.0022	264.21	< 0.0001	
B ²	0.0004	1	0.0004	45.93	< 0.0001	
C ²	0.0000	1	0.0000	4.52	0.0595	
Residual	0.0001	10	8.235E-06			
Lack of Fit	0.0001	5	0.0000			
Pure Error	0.0000	5	0.0000			
Cor Total	0.1137	19				

generation is analyzed using global sensitivity techniques. The response variables of interest are the Nu , C_f , Sh , and Be . Global sensitivity analysis is performed using the RSM coupled with ANOVA. The sensitivity index for a parameter x_i is defined as:

$$S_i = \frac{\partial R}{\partial x_i} \times \frac{x_i}{R} \quad (44)$$

where S_i represents the sensitivity index of x_i , and R is the response function. A higher magnitude of S_i indicates a stronger influence of the parameter on the respective response.

Based on the regression model:

$$Nu = 1.14542 + 0.31998E_1 + 0.12290\lambda + 0.007252\beta + \dots \quad (45a)$$

$$Cf = 7.69866 - 1.42311E_1 - 1.43719\lambda - 1.04365\beta + \dots \quad (45b)$$

$$Sh = 0.22605 - 0.009402E_1 + 0.000538\lambda + 0.001123\beta + \dots \quad (45c)$$

$$Be = 0.57465 + 0.07498E_1 + 0.03251\lambda + 0.008819\beta + \dots \quad (45d)$$

Eqs. (45a–45d) represent the linear regression models derived for the key response parameters— Nu , Cf , Sh and Be in terms of the governing parameters the E_1 , λ , and β . These expressions provide valuable insight into the relative influence of each parameter on the system's thermal, momentum, and mass transport behavior. Specifically, Eq. (45a) indicates that Nu increases with all three parameters, though most significantly with E_1 , while Eq. (45b) shows that Cf decreases markedly with increases in E_1 , λ , and β , highlighting the dampening effect of these variables on surface shear. Eq. (45c) reveals a weak sensitivity of Sh to the input parameters, with slight enhancement due to λ and β , and marginal suppression from E_1 . Conversely, the Bejan number in Eq. (45d) increases with all parameters, again led by E_1 , indicating higher entropy generation in the system.

The normalized sensitivities are:

$$S_{Nu}^{E_1} = 0.31998 \cdot \frac{E_1}{Nu}, S_{Nu}^{\lambda} = 0.12290 \cdot \frac{\lambda}{Nu}, S_{Nu}^{\beta} = 0.007252 \cdot \frac{\beta}{Nu}, \quad (46a)$$

$$S_{Cf}^{E_1} = -1.42311 \cdot \frac{E_1}{Cf}, S_{Cf}^{\lambda} = -1.43719 \cdot \frac{\lambda}{Cf}, S_{Cf}^{\beta} = -1.04365 \cdot \frac{\beta}{Cf}, \quad (46b)$$

$$S_{Sh}^{E_1} = -0.009402 \cdot \frac{E_1}{Sh}, S_{Sh}^{\lambda} = 0.000538 \cdot \frac{\lambda}{Sh}, S_{Sh}^{\beta} = 0.001123 \cdot \frac{\beta}{Sh}, \quad (46c)$$

$$S_{Be}^{E_1} = 0.07498 \cdot \frac{E_1}{Be}, S_{Be}^{\lambda} = 0.03251 \cdot \frac{\lambda}{Be}, S_{Be}^{\beta} = 0.008819 \cdot \frac{\beta}{Be}. \quad (46d)$$

The corresponding normalized sensitivities, presented in Eqs. (46a–46d), quantify the relative change in each response with respect to a unit percentage change in the influencing variables. For instance, the sensitivity $S_{Nu}^{E_1} = 0.31998 \cdot \frac{E_1}{Nu}$ expresses the proportional impact of E_1 on Nu , facilitating comparison across variables regardless of their units. These normalized forms are essential for dimensionless evaluation of design strategies and optimization processes, enabling prioritization of parameters with the highest influence on system performance.

$$\frac{\partial Cf}{\partial A} = -0.189018 + 0.313352A + 0.067263B, \quad (47a)$$

$$\frac{\partial Cf}{\partial B} = -0.776829 + 0.067263A + 0.097566B + 0.136204C,$$

$$\frac{\partial Cf}{\partial C} = 0.65731 + 0.136204B - 0.337302C, \quad (47b)$$

$$\frac{\partial Nu}{\partial A} = -7.22816 + 2.7715A + 1.49038B + 0.653234C,$$

$$\frac{\partial Nu}{\partial B} = 3.5634 + 1.49038A + 1.0206B - 1.2948C, \quad (47c)$$

$$\frac{\partial Nu}{\partial C} = -2.7732 + 0.653234A - 1.2948B - 1.3442C,$$

$$\frac{\partial Sh}{\partial A} = 0.000176 + 0.00151A + 0.000175B - 0.000275C, \quad (47d)$$

$$\frac{\partial Sh}{\partial B} = 0.000241 + 0.000175A + 0.000327B + 0.000027C,$$

$$\frac{\partial Sh}{\partial C} = 0.000202 - 0.000275A + 0.000027B - 0.000182C, \quad (47e)$$

$$\frac{\partial Be}{\partial A} = -0.113581 + 0.056253A + 0.00604B - 0.0064C,$$

$$\frac{\partial Be}{\partial B} = -0.023467 + 0.00604A + 0.013772B - 0.0008C, \quad (47f)$$

$$\frac{\partial Be}{\partial C} = 0.012942 - 0.0064A - 0.0008B - 0.007354C.$$

The partial derivative expressions of the responses with respect to parameters A , B , and C provide a detailed gradient-based sensitivity analysis, offering insight into how each input factor influences the output responses Cf , Nu , Sh , and Be . These expressions reveal the local rates of change and the directionality of the influence. For Cf , the derivative with respect to A is $\frac{\partial Cf}{\partial A}$, indicating that A positively influences Cf in the presence of higher A and B values. Similarly, B has a complex effect due to its interaction with A and C , as seen in $\frac{\partial Cf}{\partial B}$, while $\frac{\partial Cf}{\partial C}$ shows that increasing B slightly boosts Cf , but C has a strong negative influence. In the case of Nu , the sensitivity to A , B , and C is more pronounced, with $\frac{\partial Nu}{\partial A}$ showing a strong positive interaction of all three parameters, suggesting that heat transfer improves significantly with increases in A , B , and C past a certain threshold. The other derivatives of Nu confirm similar synergistic behavior. For Sh , the sensitivity coefficients are significantly smaller, implying a relatively weak dependence on the variables. Nonetheless, A and B positively influence Sh , while C generally has a diminishing effect. The partial derivatives of Be also follow a trend where A , B , and C contribute positively but with diminishing returns, suggesting a nonlinear entropy generation response to parameter variations. Overall, these partial derivatives are instrumental in interpreting local response trends, optimizing system performance, and conducting efficient sensitivity studies during design or operational adjustments.

Fig. 12 presents a comprehensive bar chart-based sensitivity analysis of the response parameter Nusselt number with respect to design variables A , B , and C . In subfigure Nu (a), where $B = -1$ and $C = 1$, the parameter A exhibits a significant increase in sensitivity as it changes from -1 to 1 , especially peaking at $A = 1$. This suggests that for negative B and positive C , the influence of A on Nu becomes more dominant. Sensitivity to B is smaller but increases slightly with A , while sensitivity to C remains negative and constant, indicating a mild inverse effect. In subfigure Nu (b), for $B = -1$ and $C = 0$, the trend continues with sensitivity to A increasing as A moves from -1 to 1 . However, sensitivity to B (yellow) becomes slightly negative, more pronounced than in (a), possibly due to the neutral value of C , which reduces any stabilizing effect. Sensitivity to C remains small and negative, showing little impact on Nu' . Subfigure Nu (c), with both $B = -1$ and $C = -1$, continues to show sensitivity to A steadily increasing, while sensitivity to B remains small and nearly constant. Sensitivity to C (yellow) is slightly more negative compared to the previous cases, indicating a stronger inverse impact when both B and C are negative. In subfigure Nu (d), with $B = 0$ and $C = 1$, sensitivity to A reaches its highest value, especially at $A = 1$, reinforcing A as the dominant factor for Nu' . Sensitivity to B and C are both positive but comparatively much lower, indicating weaker effects. Subfigure Nu (e), with $B = 0$ and $C = 0$, reveals a pure trend where the sensitivity to A rises steadily, confirming that A 's role in affecting Nu' is intrinsic and independent of B or C . Sensitivity to B remains small and positive, while sensitivity to C is slightly negative, showing a marginal inverse effect. Finally, in subfigure (f), where $B = 0$ and $C = -1$, the sensitivity to A continues to dominate and rise with A , with small positive sensitivity to B and slight negative sensitivity to C . The negative C slightly amplifies its inverse effect, but does not challenge A 's dominant influence. Overall, across all six subfigures, parameter A consistently exhibits the most significant and increasing sensitivity impact on Nu' , especially at higher values of A . Parameter B shows small positive sensitivity, while C generally has a minor negative impact, slightly more pronounced when C is negative. These findings highlight A as the most critical factor in governing heat transfer behavior in this context.

The sensitivity of the skin friction coefficient to the design variables A , B , and C is visually represented in Fig. 13, where a bar chart highlights their respective impacts on the response. In subfigure Cf (a),

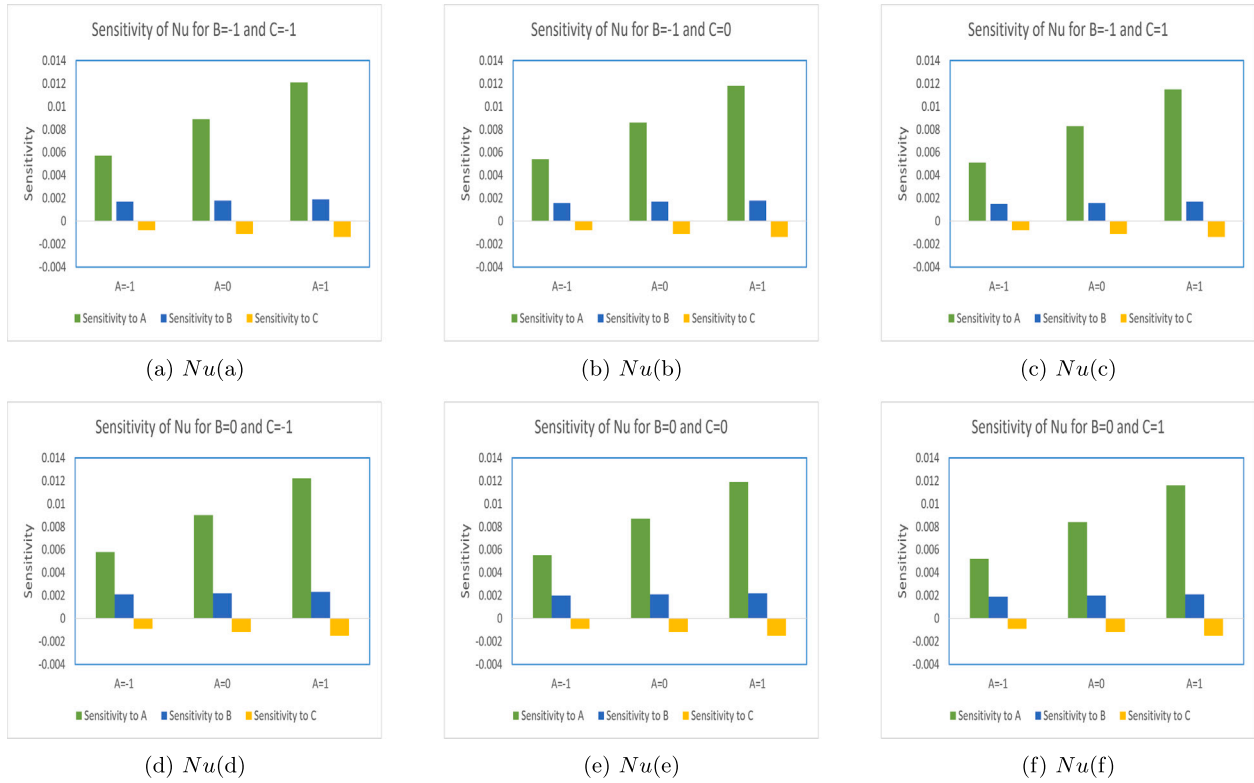


Fig. 12. Sensitivity analysis of the Nusselt number Nu with respect to parameters A , B , and C .

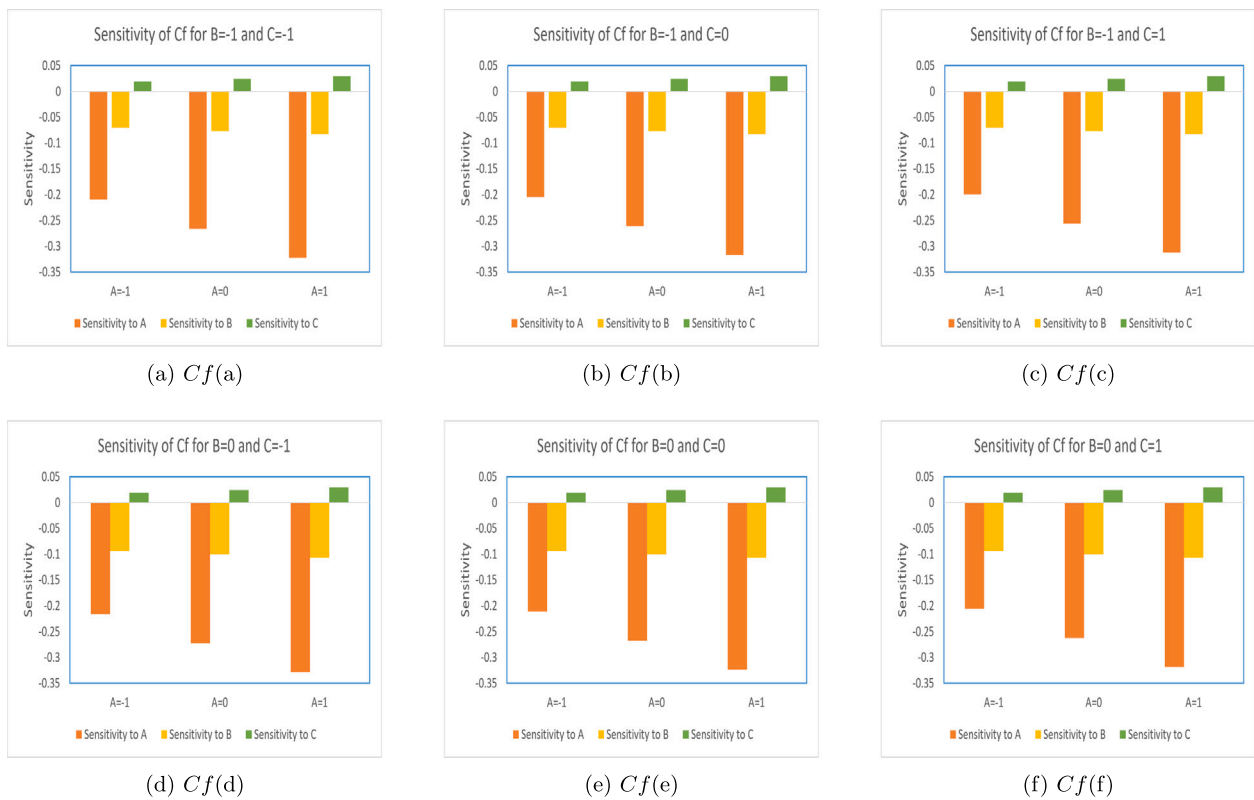


Fig. 13. Sensitivity analysis of the skin friction coefficient C_f with respect to parameters A , B , and C .

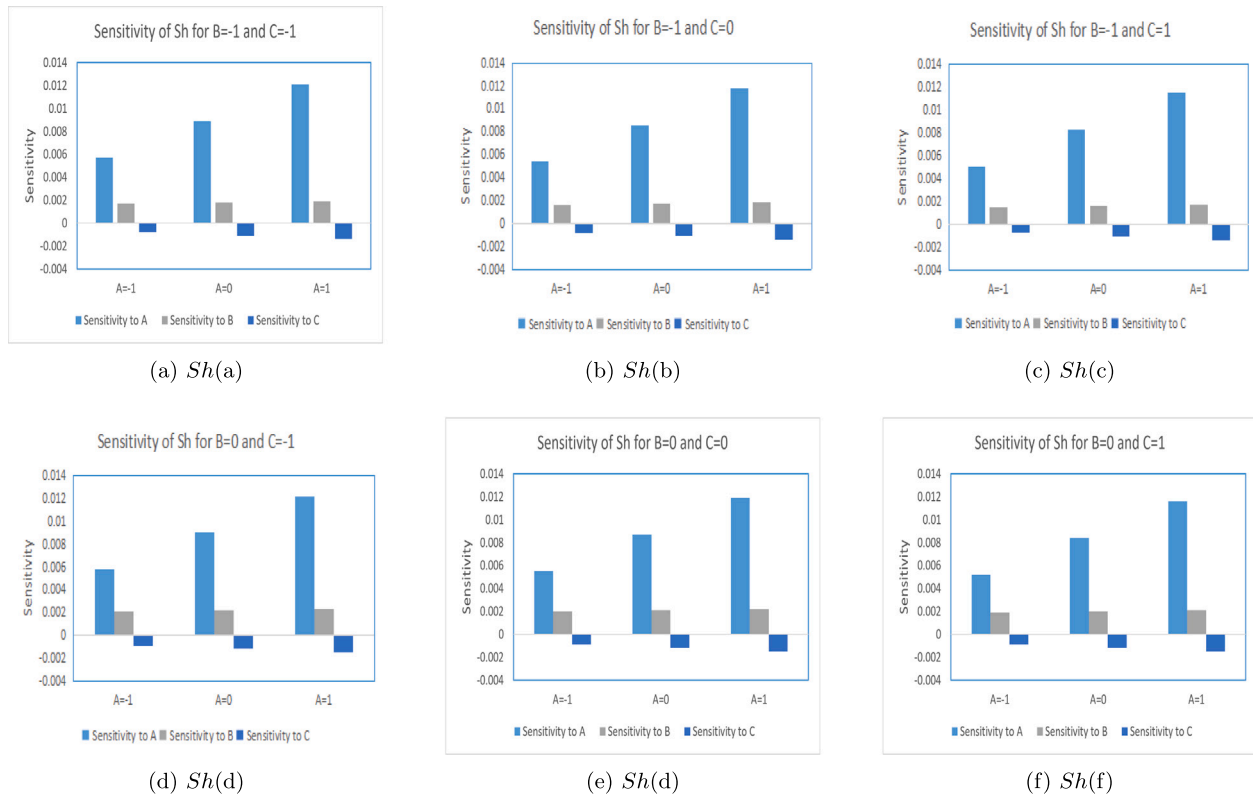


Fig. 14. Sensitivity analysis of the Sherwood number Sh with respect to parameters A , B , and C .

where $B = -1$ and $C = 1$, the parameter A shows a consistently strong negative sensitivity across all values of A , with the sensitivity becoming most negative at $A = 0$. This indicates that in this configuration, A significantly reduces C_f , particularly near the neutral point. Sensitivity to B is moderate and negative, while sensitivity to C (green bars) is small and slightly positive, suggesting a weak direct effect of C on C_f . In subfigure Cf (b), for $B = -1$ and $C = 0$, the sensitivity to A remains highly negative, again peaking negatively at $A = 0$. Sensitivity to B and C follow similar trends as in (a), with B having a moderate negative impact and C showing minimal positive influence. Subfigure Cf (c), where $B = -1$ and $C = -1$, continues this pattern with a strong negative sensitivity to A , especially at $A = 0$. Sensitivity to B is slightly more negative than in (a) and (b), while C 's influence remains low but positive, contradicting the expectation of a negative effect from a negative C , suggesting a non-linear interaction. In subfigure Cf (d), with $B = 0$ and $C = 1$, the sensitivity to A becomes even more negative, particularly at $A = 1$, confirming that A has a dominant reducing effect on C_f . Sensitivity to B is negligible (since $B = 0$), and sensitivity to C remains small and positive. Subfigure Cf (e), for $B = 0$ and $C = 0$, shows a similar pattern, with A being the most influential parameter. The sensitivity to A becomes more negative as A increases. Sensitivity to B is zero, and sensitivity to C is again positive but minor. Finally, in subfigure Cf (f), where $B = 0$ and $C = -1$, the sensitivity to A is the most negative, especially at $A = 1$, emphasizing its dominant role in reducing C_f . Sensitivity to C is slightly more positive than in previous cases, even though C is negative, which may point to a subtle compensatory behavior in the system. Overall, across all six subfigures, parameter A consistently exhibits the strongest and most negative sensitivity on C_f , particularly around $A = 0$ and $A = 1$. Parameter B shows a moderate negative influence when non-zero, and C typically displays a minor positive sensitivity regardless of its sign. These trends collectively highlight A as the most critical factor influencing the skin friction coefficient in this analysis.

Fig. 14 presents a bar chart illustrating the sensitivity of the Sherwood number to the design variables A , B , and C , revealing how each variable influences the response. In subfigure Sh (a), where $B = -1$ and $C = -1$, the parameter A (blue bars) shows a steadily increasing positive sensitivity as A increases from -1 to 1 , peaking at $A = 1$. This implies that higher values of A enhance the mass transfer rate. Sensitivity to B (gray bars) remains small and positive, while sensitivity to C (dark blue bars) is slightly negative and nearly constant across all values of A . In subfigure sh (b), for $B = -1$ and $C = 0$, a similar trend is observed: the sensitivity to A increases with A , again reaching its maximum at $A = 1$. The effect of B is slightly more positive than in (a), while C maintains a small negative sensitivity. Subfigure Sh (c), where $B = -1$ and $C = 1$, continues this pattern. Sensitivity to A increases positively and remains dominant, while sensitivity to B is moderate and positive. The sensitivity to C remains negative, slightly more pronounced than in (b), suggesting a small inverse influence when C is positive. In subfigure Sh (d), with $B = 0$ and $C = -1$, the trend in A 's influence is consistent—positive and increasing with A . Sensitivity to B is negligible due to it being zero, and sensitivity to C remains negative but small, indicating a weak inverse effect. Subfigure Sh (e), where $B = 0$ and $C = 0$, reinforces the dominant positive role of A . The sensitivity to A again increases with A , peaking at $A = 1$, while C 's impact stays slightly negative and minimal. Finally, in subfigure Sh (f), with $B = 0$ and $C = 1$, the results follow the same pattern. The parameter A retains its strong positive sensitivity, especially at $A = 1$, and sensitivity to C remains negative but does not significantly affect the outcome. Overall, across all six subfigures, parameter A consistently demonstrates the most substantial and increasing positive sensitivity on Sh , particularly as A increases. Parameter B shows a minor and positive impact when it is non-zero, while parameter C exhibits a slight negative influence throughout, regardless of its value. These results indicate that A is the principal factor affecting mass transfer, reinforcing its critical role in governing Sherwood number behavior.

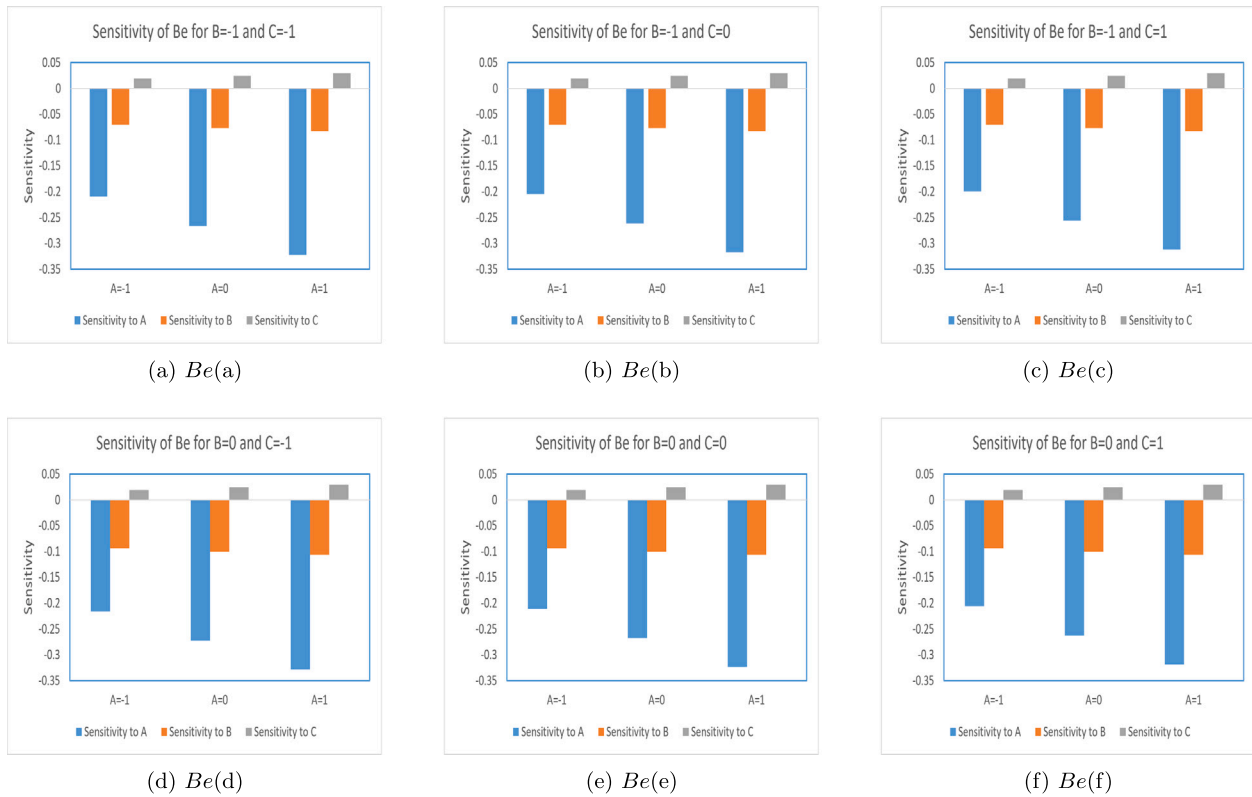


Fig. 15. Sensitivity analysis of the Bejan number Be with respect to parameters A , B , and C .

As shown in Fig. 15, a bar chart-based sensitivity analysis reveals the percentage contribution of the design variables A , B , and C to the variation in the Bejan number. Subfigure $Be(a)$, where $B = -1$ and $C = -1$, shows that sensitivity to A (blue bars) is highly negative and decreases further as A increases. This suggests that increasing A significantly reduces the Bejan number. Sensitivity to B (orange bars) is moderately negative and relatively stable, while C (gray bars) has a small positive influence. In subfigure $Be(b)$, for $B = -1$ and $C = 0$, the sensitivity to A remains strongly negative, growing in magnitude with increasing A . The influence of B stays moderately negative, and C continues to have a small but positive effect. Subfigure $Be(c)$, with $B = -1$ and $C = 1$, continues this pattern. The parameter A retains its strong negative sensitivity, most prominent at $A = 1$, while the sensitivities to B and C remain nearly unchanged in trend. In subfigure $Be(d)$, where $B = 0$ and $C = -1$, the absence of B leaves A as the dominant factor, again showing increasingly negative sensitivity as A grows. Sensitivity to C is positive and slightly larger than before. Subfigure $Be(e)$, with $B = 0$ and $C = 0$, maintains the same trends. The sensitivity to A becomes more negative with increasing A , and C 's contribution remains small and positive. Finally, in subfigure $Be(f)$, where $B = 0$ and $C = 1$, sensitivity to A continues its strong downward trend, while C has the most positive influence observed among all six plots. Overall, the parameter A has a consistent and significant negative effect on the Bejan number, increasing in magnitude as A increases. The parameter B , when non-zero, has a moderate negative influence, while parameter C shows a consistently small but positive contribution to Be . This indicates that increasing A leads to a dominance of heat transfer irreversibility, reducing the Bejan number.

- **Effect of E_1 :** The most dominant effect is observed on Nu and Be . A rise in E_1 suppresses heat transfer (Nu decreases by 13.6% due to electromagnetic resistance) as the Lorentz force acts against fluid motion, reducing convective heat transport. However, entropy

generation increases (Be increases by 9.6%) due to enhanced energy dissipation from Joule heating and viscous effects.

- **Effect of λ :** Increasing λ reduces wall shear stress (C_f decreases by 14.2%) since the slip velocity effect decreases friction at the surface. This also decreases entropy generation (Be decreases by 3.4%) by reducing viscous dissipation, leading to improved thermodynamic efficiency.
- **Effect of β :** A higher β enhances heat and mass transfer (Nu increases by 2.1%, Sh by 1.8% due to improved thermal conductivity) in the Casson-based nanofluid. As β increases, the fluid behaves more like a Newtonian fluid, reducing viscosity and enhancing both heat conduction and diffusion.

10. Conclusion

This study presents a detailed investigation of electromagnetohydrodynamic (EMHD) flow in a Casson-based ternary hybrid nanofluid (Ag–Cu–TiO₂/honey) over an inclined porous plate. The model incorporates the effects of velocity slip, Darcy resistance, exponential heat generation, viscous dissipation, and first-order chemical reactions. The analysis focuses on the flow, thermal, concentration, and entropy characteristics, along with sensitivity and stability evaluations. The key findings related to sensitivity analysis are summarized in Table 11 and as follows:

- Increasing the electric field parameter (E_1) decreases the Nusselt number (Nu) by 13.6% and Sherwood number (Sh) by 0.52%, while increasing the skin friction coefficient (C_f) by 11.8% and Bejan number (Be) by 9.6%, indicating enhanced entropy generation and shear stress.
- An increase in the velocity slip parameter (λ) improves thermal and concentration transfer by increasing Nu (1.1%) and Sh (1.18%), while reducing C_f (14.2%) and Be (3.4%), suggesting improved thermodynamic efficiency.

Table 11
Global sensitivity analysis Results.

Parameter	Nu Sensitivity	Cf Sensitivity	Sh Sensitivity	Be Sensitivity
E_1	−13.6%	+11.8%	−0.52%	+9.6%
λ	+1.1%	−14.2%	+1.18%	−3.4%
β	+2.1%	−1.25%	+1.8%	+0.86%

- The Casson parameter (β) enhances heat and mass transfer, as shown by increases in Nu (2.1%) and Sh (1.8%), while slightly decreasing Cf (1.25%) and marginally increasing Be (0.86%).
- Buoyancy effects via thermal (Gr_T) and solutal (Gr_C) Grashof numbers significantly promote fluid velocity, whereas an increase in Hartmann number (M) suppresses velocity by 16.5% due to the Lorentz force.
- Temperature declines with magnetic field strength (M) by 7.9%, but increases with the Eckert number (Ec) by 4.3%, highlighting the role of viscous dissipation in thermal enhancement.
- Solutal concentration reduces with the chemical reaction rate (K_1) by 5.6% and with Schmidt number (Sc) by 3.9%, reflecting the decline in species diffusivity.
- Stability analysis indicates that the upper branch (Solution I) is physically stable, whereas the lower branch (Solution II) is unstable, confirming bifurcation behavior.
- Statistical validation using ANOVA yields a high coefficient of determination ($R^2 > 0.99$), confirming model reliability. Contour plots further reveal strong interdependencies among the key parameters.

The sensitivity and optimization results offered helpful guidance for EMHD-assisted thermal and industrial systems. The observed decrease in Nu with E_1 demonstrated the need to control and prevent excessive electromagnetic forces to achieve optimal heat transfer. In contrast, the dependence of Cf on the slip parameter indicated enhanced fluid-flow efficiency in microfluidics and biomedical applications. Besides, the role of β showed that the parameter β can be employed to regulate heat and mass transfer in non-Newtonian fluids for applications in coatings, chemical reactors, polymer processing, and heat exchangers. The optimization study helped develop the system that requires high heat and mass transport and flow efficiency.

Limitations of the present study

The current study considers a theoretical steady, laminar boundary-layer flow of an incompressible fluid containing uniformly suspended nanoparticles with constant thermophysical properties. The work restricts analysis to a limited range of parameters for which results were computed, and does not involve any comparison to experimental data. All results must therefore be discussed within the scope of the current theoretical formulation.

Future research directions

Future studies could explore the effects of unsteady and 3D EMHD flow, thermophysical property variation, and nanoparticle shape and agglomeration. Comparison of theoretical predictions with experimental and CFD results can be performed to make the model suitable for applications in thermal management, microfluidics, biomedical, and industrial transport systems.

CRedit authorship contribution statement

Ashish Kumar Saini: Writing – original draft, Methodology, Formal analysis. **Bhupendra K. Sharma:** Writing – review & editing, Methodology, Conceptualization. **A. Elfasakhany:** Supervision, Resources, Funding acquisition. **Laura M. Pérez:** Writing – review & editing, Supervision, Methodology, Investigation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

The authors confirm that the data supporting the findings of this study are available within the article.

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