



Enhanced thermal transport and entropy reduction in bio-convective tri-hybrid MHD nanofluids through porous media under exponential heating

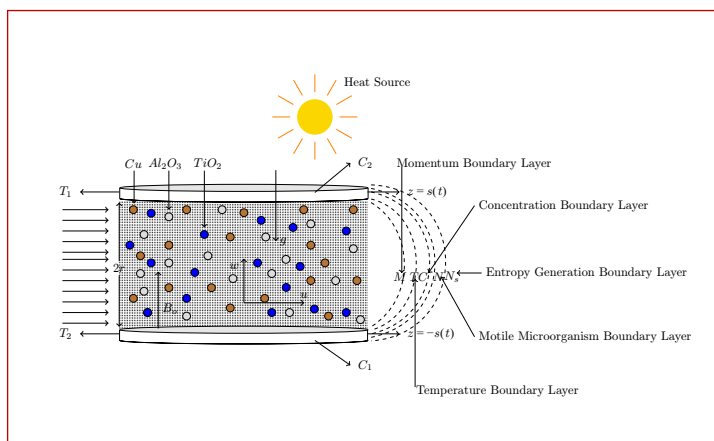
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Abstract

The current investigation explores the thermal and mass transport characteristics of a tri-hybrid nanofluid formulated by suspending copper(Cu), alumina(Al_2O_3), and titanium oxide(TiO_2) nanoparticles in water(H_2O), considering the influence of motile microorganisms. The physical model incorporates the effects of porous medium, viscous dissipation, thermophoresis, magnetic field, and an exponential heat source. The Governing partial differential equations are converted into a set of nonlinear ordinary differential equations through suitable similarity transformations and are numerically solved using MATLAB's bvp4c. This method ensures accurate resolution of boundary layer phenomena associated with the flow and thermal fields. Important engineering metrics such as the Nusselt number, skin friction coefficient, Sherwood number, and motile density number are computed numerically. The graphical results reveal that magnetic field strength and porous medium resistance significantly alter the velocity field, while Thermophoresis parameter and heat source intensity strongly influence the temperature profile. The concentration dynamics are dominantly affected by chemical reaction rate and activation energy, whereas the motile microorganism distribution is governed by bio-convective parameters. In addition, entropy generation is optimized to minimize thermodynamic irreversibilities and enhance the overall system performance. The findings demonstrate the potential applicability of the proposed model in bio-convective systems, thermal energy storage, and advanced thermal management and cooling technologies.

Graphical abstract



Results:-

- Heat generation(Q) → increase temperature
- Activation energy(E_a) → reduces concentration
- Nanoparticles volume friction → Enhances heat transfer
- Magnetic number (M) → Decreases velocity

Keywords Thermal and mass transport · Motile microorganisms · Activation energy · Entropy generation · Tri-hybrid nanofluid

Extended author information available on the last page of the article

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Introduction

Entropy generation is a thermodynamic measure of irreversibility caused by processes such as friction, heat transfer, fluid mixing, and chemical reactions, leading to energy loss and reduced system efficiency. Physically, it represents the degradation of useful energy into unavailable energy. Entropy generation analysis is widely used in heat exchangers, cooling systems, energy conversion devices, porous medium transport, and thermal optimization to improve system performance and minimize energy losses. Farooq et al. [1] used the Tiwari–Das two-phase model to investigate the boundary layer behavior of a hybrid nanofluid over a non-uniform radially diverging porous medium with impact of injection as well as suction by including the viscous dissipation through an energy dissipation term in the energy equation. This approach enabled an in-depth perspective on thermodynamic interactions within the flow system. Miles et al. [2] studied the entropy generation, heat transmission, and laminar convection flow of nanofluids inside a cylindrical shell embedded with porous material. In the current investigation, Shah et al. [3] investigated about the Prandtl–Eyring model across a heated, deformation surface within Darcy–Forchheimer media nanofluid flow which is oil-based fluid. Gandhi et al. [4] explored impact of entropy generation and geometric shape on flow of blood through an multi-branched artery with time-dependent characteristics, highlighting the complex interaction between flow dynamics and vessel geometry. In a related study, Venkateswarlu et al. [5] examined the electromagnetic tri-hybrid nanofluid over a convective extension sheet, factoring the entropy, viscosity and influence of Darcy–Forchheimer media, contributing to an in-depth perspective on energy dissipation in such systems. Entropy generation in second-grade MHD nanofluids has been optimized by incorporating slip and density effects. At the same time, mixed convection studies have addressed steady and oscillatory regimes under reduced gravity, activation energy and radiation, Mirza et al. [6].

A nanofluid is a suspension of nanoparticles within a base fluid (H_2O , $C_2H_6O_2$). Hybrid nanofluids go further by incorporating two or more types of nanoparticles within the carrier liquid, creating a synergistic effect that improves performance. The unique properties of Hybrid nanofluid make it valuable in electronics cooling, energy systems, and advanced heat exchange processes. In recent work, Arif [7] analyzed thermal effects for viscoelastic flow under thermal radiation of TRIHNF over an spinning plate. Meanwhile, Shamshuddin et al. [8] examined circulation characteristics of tri-hybrid nanofluid with polymer as the base fluid across revolving disk. Kumar et al. [9] analyzed the flow of Jeffery fluid subjected to term of

origin over a non-uniform vertically stretching plate, offering valuable perception of the complex behavior for such fluids in diverse flow configurations. In their recent work, Gandhi et al. [10] studied the contribution of EMHD to nanofluids flow in a vertically bifurcated artery, shedding light on how electromagnetic forces affect the behavior of fluids in biological systems as well as examined by Sharma et al [11]. Prasad et al. [12] explored how pollutant concentration, nanoparticle size, and inter facial layer affect nanofluid flow in convergent/divergent channels using numerical analysis. Okasha et al. [13] analyzed thermally radiative NaAlg-based ternary hybrid nanofluid flow with Christov–Cattaneo heat flux and Stefan blowing effects semi-analytically. Cao et al. [14] examined $CO_2 - CH_4$ displacement using in situ CT imaging in carbonate rocks. Mass transfer is governed by a diffusion dominated mixing front.

Viscosity is a fluid's resistance to flow due to internal friction between layers, and it significantly influences heat transfer and flow behavior. Viscous dissipation converts fluid kinetic energy into thermal energy, increasing fluid temperature and reducing efficiency. A conducting fluid generate heat due to electrical resistance, known as Joule heating. Controlling viscosity variation and Joule heating is essential for improving thermal performance, energy efficiency, and sustainable heat transfer systems. Alanazi et al. [15] have examined heat-related attributes and viscous joule on three-dimensional nanofluid flow in a hybrid state through an annular porous plate. Henda et al. [16] also examined impact of both thermophoresis and Brownian motion in behavior of Brogiorno nanofluid. Their discoveries help to understand thermal behavior and transport phenomena in nanofluids, which are crucial in improving heat transfer capabilities. Sharma et al. [17] studied the simultaneous influence of uneven heat sink or source and Joule heating over fluctuating MHD flow across a vertically expanding plate. In this research, Gandhi et al. [18] have evaluated impact of multi component nanoparticles in 2D blood circulation passes across a permeable cylindrical artery, analyzing with respect to Joule heating, external magnetic field, and viscous energy loss, along with the presence of a bell-shaped constriction. Darcian MHD flows have been analyzed under multiple slip and Joule heating effects in inclined, radiative geometries under micro gravity by Arni et al. [19].

A highly permeable material is characterized by an extensive network of interconnected pores, resulting in high surface area and permeability. These properties enhance fluid flow and thermal and material transport, making such material essential in applications like filtration, catalysis, and energy storage, where efficient transport and reaction processes are critical. In the present research, Ali et al. [20] explored the consequences of variations in energy

generation, nanoparticle size, mass flux on fluid dynamics, heat source, and the Darcy–Forchheimer cylindrical model, with particular focus on effects of radiation. Algehyne et al. [21] examined about the Jeffrey fluid model and investigated the impact of magnetohydrodynamic forces and thermal energy emission on three-dimensional Jeffrey fluid circulation across a permeable, deforming surface. Kumar et al. [22] examined the interplay of radiation, Joule heating, viscosity-induced heat generation, and Analysis of Darcy–Forchheimer interactions in non-Newtonian blood flow via an artery. Sharma et al. [23] had explored the thermal movement in a solar collector induced by a parabolic rotating tube, with a focus onto the flow behavior of the HNF throughout a Riga surface within porous domain governed by Darcy–Forchheimer law. Govind [24] examined the movement of mass and the flow of entropy in a chemically reactive cylindrical flow system, incorporating thermal radiative hybrid nanofluid, porous medium applied perpendicular to the flow. Kumar et al. [25] studied analyzes the interplay that these factors impose on flow characteristics and thermal characteristics. The thermal behavior of a two-phase model describing flow of dusty hybrid nanofluid through an elongating cylinder, incorporating Darcy–Forchheimer effects, is analyzed by Ali et al. [26]. Sun et al. [27] examined a porous electrowetting ionic liquid electrospray by voltage–current test over a long operating time.

Chemical reactions transform reactants into products through molecular rearrangement accompanied by energy exchange. In fluid and heat transfer systems, chemical reactions significantly influence temperature, concentration, and mass transport behavior. Activation energy represents the minimum energy required for reaction initiation and controls reaction rates, entropy generation, and thermal transport. These effects are widely applied in chemical reactors, biomedical processes, catalytic systems, energy conversion devices, and thermal engineering applications. Kumar et al. [28] analyzed characteristic for Arrhenius activation enthalpy as well as thermal transport over the flow behavior of tangential rheologically complex fluid on a moving extended surface. Azam et al. [29] explored impact of activation enthalpy in bioconvection transfer with gyrotactic microorganisms of chemically active nanofluid interacting with a porous cylinder. Sharma et al. [30] explored role of Arrhenius activation energy over the MHD nanofluid flow via a deforming sheet, considering effects for an exponential heat source. In another study, Sharma et al. [31] analyzed mixed impact of microorganisms along with Arrhenius activation energy onto stationary flow past an angled extending sheet with Brownian motion and a porous medium. Additionally, Karthik et al. [32] scrutinized mass transfer and thermal behavior involved into exothermic/endothermic chemical reactions, focusing upon nanofluid flow over cylindrical plate into a porous media. Abbas [33] assessed

impacts of activation energy and an inclined magnetic dipole over flow of bio-Marangoni nanofluid through cylindrical channel. Lin et al. [34] studied the two-phase cooling devices utilized with deep microgrooves to shield airflow and movement of laser-ablated product

Thermal conductivity quantifies a material's heat conduction proficiency. It represents to the rate at which heat transfers throughout a unit area of a material due to the temperature gradient. Superior thermal conductivity materials, like metals that conduct heat, are used in heat exchangers and electronics cooling. Low thermal conductivity converts energy in buildings like insulators that slow heat transfer. Accurate modeling and experimental thermal conductivity measurements are essential for optimizing nanofluids for specific applications. Sharma et al. [35] examined effect of heat conduction and temperature-dependent viscosity upon the electrically conducting MHD flow of blood over an elongating sheet, considering other variables like thermal radiation and chemical reactions. Choudhary [36] analyzed the effects of thermal radiation, resistive heating over stable 2D, axisymmetric flow of HNF on a radial stretched-out surface. Vijay [37] investigated the movement of MHD hybrid nanofluid above a spinning disk, considering Dufour phenomena, Soret. Kumawat et al. [38] analyzed the behavior of blood flow in a restricted artery under impact of a magnetic environment and heat origin. Gandhi et al. [39] explored the thermal radiation, Ohmic heating and heat source impact upon the EMHD mixed-mode convective flow in nanofluids via a vertical surface. Ramasekhar et al. [40] investigated thermal transfer in bioconvected Oldroyd-B nanofluid within variable Arrhenius activation energy and thermal conductivity over a nailed boundary surface. Khedher et al. [41] examined the effects of the Cattaneo–Christov heat flux and thermal radiation on Sutterby hybrid nanofluid flow using artificial neural networks. Wang et al. [42] investigated a latent heat coupled wellbore heat transfer model using PCMs as source term.

Mass and heat transfer are essential transport phenomena governing energy and substance movement in engineering and physical systems. Convection and buoyancy forces significantly influence fluid motion, temperature distribution, and heat transfer efficiency, while exponential heat sources intensify thermal energy generation exponentially with time. Physically, they help optimize thermal performance, improve energy utilization, and enhance the efficiency and stability of industrial and energy systems. Kumawat et al. [43] investigated into the thermal transport and fluid dynamics concerning a biphasic blood circulation model situated within a curved tube exhibiting time-dependent stenosis. Khan et al. [44] analyzed heat transfer characteristics within a model of organism-driven convection for Agrawal-type axisymmetric flow of aqueous $Cu - TiO_2$ hybrid nanofluid traversing a permeable rotating disk, starting from a situation of zero mass

flux. Li et al. [45] examined the melting thermal transmission within bioconvection nanofluid circulation over a nonlinear surface, considering the influence of microorganisms into their analysis. Ahmad et al. [46] analyzed thermally radiative effects Sutterby fluid models along a stretching surface with heat and mass transfer to non-Fourier heat conduction effects. Kumar et al. [47] numerically investigated radiative thermal transmission in flow behavior on a deformable rotating cone, and Chandan et al. [48] analyzed temperature distribution, velocity fields, and the influence of radiation fin. Aoudia et al. [49] studied the impacts of Hall current and gyrotactic microorganisms on a CNTs- TiO_2 /water tri-hybrid nanofluid for biosensor and micro mixer applications.

Bioconvection is a fluid dynamics-phenomenon driven by the convective transport induced by motile microorganisms, which swim upward in response to light or oxygen gradients. This accumulation near the fluid's surface creates unstable density stratification, triggering convective currents that significantly enhance mixing and mass transport. In systems where natural buoyancy interacts with external forces, as in mixed convection, these dynamics are essential for governing heat and fluid flow. These microbes, which orient themselves against gravity and fluid torque, have profound practical implications in biotechnology and environmental science. For instance, in industrial algal photo bioreactors, gyrotactic bioconvection creates natural plumes that ensure effective circulation for biofuel production, maximizing light harvesting without mechanical stirring. Additionally, understanding these patterns is critical for modeling marine phytoplankton patchiness, which informs our knowledge of oceanic carbon sequestration and the global carbon cycle. Rehman et al. [50] analyzed coupled mass and thermal transmission via Soret and Dufour effects over a multi-physics model of MHD Carreau fluid. Kumar et al. [47] studied and concluded that nanoparticle aggregation eliminates the implications of Brownian-induced particle diffusion in all flow regimes. Waseem et al. [51] investigated viscoelastic non-Newtonian fluid flow with thermal radiation, motile microorganisms, and chemical reaction across an exponentially stretching sheet. Jawad et al. [52] explored the dynamics of tangent hyperbolic nanofluid flow subjected to chemical reactions, bioconvection due to motile microorganisms on a stretchable surface. Farkhad et al. [53] investigated the bioconvection flow of a water-based tri-hybrid nanofluid with gyrotactic microorganisms, numerically accounting for Soret and Dufour transport effects.

When an electrically conductive fluid interacts with a magnetic field and due to this interaction nanofluid and its motion influences, known as MHD effect. This study incorporates the MHD effect into the momentum equation revealing that the magnetic field strength significantly influences fluid velocity. This interaction is used in the energy systems and thermal management. Makker et al. [54] evaluated the 3D modeling

of MHD bio-convective fluid influenced by thermal source past an exponentially expanding sheet. Mishra et al. [55] focused on entropy generation analysis in synthetic cilia with magnetized nanofluid flow past a inclined channel. Qureshi et al. [56] conducted a study about entropy production via magnetized hybrid nanofluid between a radiative coaxial porous disks. Ali et al. [57] examined the mass and thermal transport within nanofluid flow among two orthogonally aligned coaxial permeable disks. Raza et al. [58] explored impact of ternary hybrid nanofluid flow over mass and heat transmission within 2D rotating porous channels. Afzal et al. [59] examined entropy production alongside mass and heat transmission within hybrid nanofluid flow among two porous disks. Additionally, Raza et al. [60] delved into the dynamics of TRIHNF with gyrotactic microorganisms, while Raza et al. [61] explored entropy generation and heat/mass transmission in magnetized TRIHNF flow over porous disks influenced by motile microorganisms. Majeed et al. [62] studied contributing to advancements in biomedical and industrial fluid systems of thermal radiation impacts on MHD flow across nonlinear surface. Kumar et al. [63] explored the active and passive control mechanisms for enhanced flow regulation while Kumar et al. [64] studied the operational matrices to assess the thermal behavior of convex spine fins using the Vieta–Lucas method.

Existing studies on nanofluid transport mainly focus on conventional or binary hybrid nanofluids under limited physical conditions. Very few investigations have examined entropy generation in bio-convective ternary hybrid nanofluid flow between rotating porous disks, incorporating Darcy–Forchheimer porous medium effects, activation energy, an exponential heat source, Joule heating, viscous dissipation, Brownian motion, thermophoresis, and gyrotactic microorganisms simultaneously. Therefore, a comprehensive thermodynamic analysis of these coupled transport mechanisms remains lacking in the literature. To address these research gaps, the present study is the first to investigate entropy-optimized bio-convective ($Cu + Al_2O_3 + TiO_2 + H_2O$) tri-hybrid nanofluid flow between rotating porous disks by simultaneously incorporating magnetohydrodynamic effects, Darcy–Forchheimer porous medium resistance, activation energy, exponential heat source, Joule heating, viscous dissipation, Brownian motion, and motile gyrotactic microorganisms. The governing equations are transformed via similarity variables and numerically resolved with MATLAB's bvp4c solver. The model captures detailed interactions between velocity, concentration, temperature, microorganism distribution, and entropy production. The expected outcomes include improved insight into thermodynamic irreversibility and enhanced predictions of engineering quantities such as Nusselt number, motile density, skin friction, and Sherwood number. These results provide essential insights for enhancing bio-convective thermal transmission in

energy storage technologies, advanced cooling systems, and biomedical devices.

Mathematical formulation

Considering a 2-D, time varying, chemical transformation, viscous dissipation, incompressible, and joule-heating effects, energy and substance transfer of motile gyrotactic micro-organism with nano-layered entropy formation, activation energy, rotating tri-hybrid MHD nanofluid ($Cu + Al_2O_3 + TiO_2/H_2O$) between two orthogonally permeable disks and employ water, in thermal balance with the nanoparticles, as the base fluid. Applying a external magnetic field and assume a low Reynolds number allows the induced magnetic field to be disregarded. By applying an external magnetic force applied along the Z-axis across porous disks of diameter $2r$ is moving up or down uniformly at a time-dependent rate $s'(t)$. The C_1 , T_1 and N_1 denote the temperature, concentration, and microorganism at lower surface and C_2 , T_2 and N_2 denote the temperature, concentration and microorganism at upper surface. Disks are in highly porous medium and nonlinear heat source is considered with $q_o < 0$ heat sink and $q_o > 0$ heat source. u, v, w being the velocity in r, θ, z direction in cylindrical coordinate system based on above mentioned suppositions (Fig. 1) and system of Model equations [55–62]

Continuity equation:

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

Three-dimensional momentum equation:

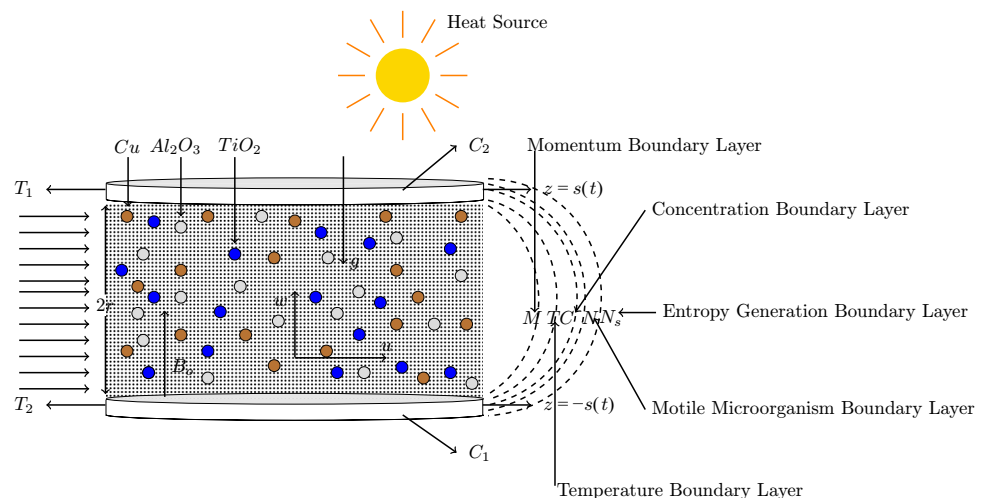
$$\begin{aligned} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} - \frac{v^2}{r} \\ = -\frac{1}{\rho_{\text{trihnf}}} \frac{\partial p}{\partial r} + \nu_{\text{trihnf}} \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2} \right) \\ - \frac{\sigma_e B_o^2}{\rho_{\text{trihnf}}} u + g\beta_T(T - T_2) + g\beta_C(C - C_2) \\ + g\beta_N(N - N_2) + \frac{\nu_{\text{trihnf}}}{k^*} u - \frac{F_o}{\sqrt{k^*}} u^2 \end{aligned} \quad (2)$$

$$\begin{aligned} \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial r} + w \frac{\partial v}{\partial z} + \frac{vu}{r} \\ = \nu_{\text{trihnf}} \left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} - \frac{v}{r^2} + \frac{\partial^2 v}{\partial z^2} \right) \\ - \frac{\sigma_e B_o^2}{\rho_{\text{trihnf}}} v \end{aligned} \quad (3)$$

$$\begin{aligned} \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} \\ = -\frac{1}{\rho_{\text{trihnf}}} \frac{\partial p}{\partial z} + \nu_{\text{trihnf}} \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2} \right) \\ - \frac{\sigma_e B_o^2}{\rho_{\text{trihnf}}} w \end{aligned} \quad (4)$$

Temperature equation:

Fig. 1 A 2D representation of Model



$$\begin{aligned}
& \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} \\
&= \alpha_{\text{trihnf}} \frac{\partial^2 T}{\partial z^2} + \frac{\mu_{\text{trihnf}}}{(\rho C_p)_{\text{trihnf}}} \left[\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right] \\
&+ \frac{\sigma_e B_o^2}{(\rho C_p)_{\text{trihnf}}} (u^2 + v^2) + \frac{Q_o(T - T_2)}{(\rho C_p)_{\text{trihnf}}} e^{-m_1 \frac{z}{s}} \\
&+ \frac{(\rho C_p)_s}{(\rho C_p)_{\text{trihnf}}} \left[(D_b)_{\text{trihnf}} \left(\frac{\partial T}{\partial z} \frac{\partial C}{\partial z} \right) \right. \\
&\left. + \frac{(D_T)_{\text{trihnf}}}{T_2} \left(\frac{\partial T}{\partial z} \right)^2 \right]
\end{aligned} \quad (5)$$

Concentration equation:

$$\begin{aligned}
& \frac{\partial C}{\partial t} + u \frac{\partial C}{\partial r} + w \frac{\partial C}{\partial z} \\
&= (D_b)_{\text{trihnf}} \frac{\partial^2 C}{\partial z^2} - K_o(C - C_2) \left(\frac{T}{T_2} \right)^m e^{\left(\frac{-E_a}{k_b T} \right)} \\
&+ \frac{(D_T)_{\text{trihnf}}}{T_2} \left(\frac{\partial^2 T}{\partial z^2} \right)
\end{aligned} \quad (6)$$

Motile-microorganism equation:

$$\begin{aligned}
& \frac{\partial N}{\partial t} + u \frac{\partial N}{\partial r} + w \frac{\partial N}{\partial z} + \frac{bW_c}{C_1 - C_2} \left(N \frac{\partial C}{\partial z} \right) \\
&= (D_n)_{\text{trihnf}} \frac{\partial^2 N}{\partial z^2}
\end{aligned} \quad (7)$$

The novelty of the present mathematical model lies in the simultaneous incorporation of several advanced physical mechanisms into the tri-hybrid nanofluid system. The term

$\frac{\sigma_e B_o^2}{(\rho C_p)_{\text{trihnf}}} (u^2 + v^2)$ represents Joule heating effects due to the applied magnetic field, while the viscous dissipation contribution is modeled through $\frac{\mu_{\text{trihnf}}}{(\rho C_p)_{\text{trihnf}}} \left[\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right]$ in the energy equation. The exponential heat source term $\frac{Q_o(T - T_2)}{(\rho C_p)_{\text{trihnf}}} e^{-m_1 \frac{z}{s}}$ introduces non-uniform thermal generation effects. Furthermore, the concentration equation includes the Arrhenius activation energy and chemical reaction term $K_o(C - C_2) \left(\frac{T}{T_2} \right)^m e^{\left(\frac{-E_a}{k_b T} \right)}$ which characterizes temperature-dependent reaction kinetics. The Darcy–Forchheimer porous medium resistance effects are incorporated through $\frac{\nu_{\text{trihnf}}}{k^*} u - \frac{F_o}{\sqrt{k^*}} u^2$. In addition, Brownian motion and thermophoresis effects are modeled in the energy and concentration equations, while microorganism transport is represented through the bioconvection term involving motile gyrotactic microorganisms. The simultaneous inclusion of these coupled effects in a ternary hybrid nanofluid flow between rotating porous disks constitutes the major novelty of the present study (Fig. 2).

where $\nu = \frac{\mu_{\text{trihnf}}}{\rho_{\text{trihnf}}}$ and $(\rho C_p)_{\text{trihnf}}$ are indicates the kinematic viscosity and specific heat capacity of TRIHNF which values are showed in table 1 and table 2 [60]:

In this research, Copper (Cu) nanoparticles are widely used in advanced thermal management systems (like cooling divece, solar collectors etc.) due to their exceptionally high thermal conductivity and excellent heat transfer characteristics. Additionally, Cu nanoparticles exhibit antimicrobial properties, making them useful in biomedical and water purification applications. Alumina (Al_2O_3) nanoparticles

Fig. 2 Schematic representation of the nanolayer effect around a nanoparticle in a base fluid

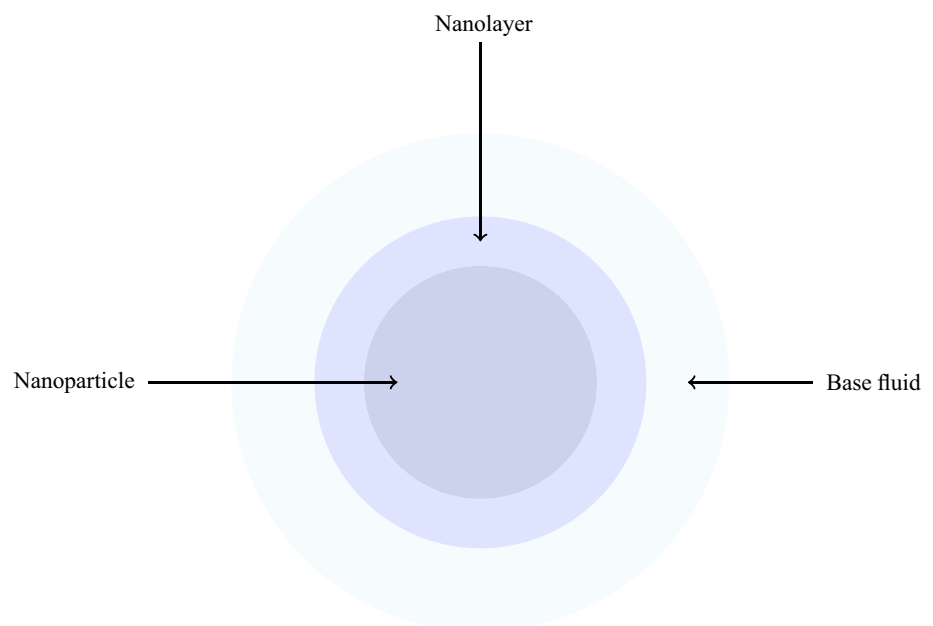


Table 1 Properties

Properties	Base fluid (H ₂ O)	Cu(ϕ_1)	Al ₂ O ₃ (ϕ_2)	TiO ₂ (ϕ_3)
C _p	4179	385	765	686.2
ρ	997.1	8933	3970	4250
κ	0.613	401	40	8.9538

possess high thermal stability, corrosion resistance, mechanical strength, and chemical inertness. These properties make them highly suitable for industrial cooling systems, nuclear reactors, and aerospace thermal protection systems. Furthermore, alumina nanoparticles improve the stability and durability of nanofluids under high-temperature operating conditions. Titanium oxide (TiO₂) nanoparticles are well known for their photocatalytic activity, chemical stability, non-toxicity, and optical properties. They are commonly utilized in solar energy systems, wastewater treatment, biomedical coatings, and photovoltaic devices. In heat transfer applications, TiO₂ nanoparticles used to increase thermal conductivity and thermal energy transport.

The combined suspension of Cu, Al₂O₃, and TiO₂ nanoparticles in the base fluid forms a tri-hybrid nanofluid with superior thermophysical characteristics compared to conventional nanofluids and hybrid nanofluids. The synergistic interaction among these nanoparticles significantly enhances thermal conductivity, heat transfer efficiency, thermal stability, and energy transport performance, making the present ternary hybrid nanofluid model highly suitable for advanced engineering and biomedical applications. Table 1 shows the thermophysical values of different NPs (Cu, Al₂O₃, TiO₂) and base fluid (H₂O) and Table 2 represents the thermophysical properties of TRIHNF. In the table,

$\phi_1, \phi_2, \phi_3, \rho_{s1}, \rho_{s2}, \rho_{s3}$, and ρ_f represent the volume fractions of the Cu, Al₂O₃, and TiO₂ nanoparticles, as well as the densities of the solid nanoparticles and the base fluid. $(dp)_1, (dp)_2$, and $(dp)_3$ denote the diameters of the different types of nanoparticles, while $(\rho C_p)_{s1}, (\rho C_p)_{s2}$, and $(\rho C_p)_{s3}$ represent the specific heat capacities of the Cu, Al₂O₃, and TiO₂ nanoparticles, as well as the liquid base fluid. $k_{s1}, k_{s2}, k_{s3}, k_{hnf}, k_{nf}$, and k_{bf} denote the thermal conductivities of the three distinct types of nanoparticles (Cu, Al₂O₃, TiO₂), solid hybrid nanofluids (HNFs), nanofluids (NFs), and the base fluid and K_{nl} represent the nanolayer thermal conductivity. In Table 2 $\lambda_1 = 1 + \frac{h}{2r}, \lambda_2 = 1 + \frac{h}{r}$, h shows the nanolayer thickness and the nanoparticle radius is denoted by r , the thermal conductivity of the nanolayer fluid, hybrid nanolayer fluid, and tri-hybrid nanolayer fluid is represented by k_{nlndf}, k_{hnlndf} , and $k_{trihnlndf}$, respectively.

The tri-hybrid nanolayer thermal diffusivity is expressed as $\alpha_{trihnf} = \frac{k_{trihnf}}{(\rho C_p)_{trihnf}}$. The imposed boundary conditions describe the transport behavior of the tri-hybrid nanofluid between two rotating porous disks located at $z = -s(t)$ and $z = s(t)$. At the lower disk, the radial velocity satisfies the no-slip condition, while the tangential and axial velocities represent disk rotation and fluid suction/injection through the porous surface. The temperature, concentration, and microorganism density are maintained at constant values (T_1), (C_1), and (N_1), respectively. Similar velocity, thermal, and concentration conditions are applied at the upper porous disk with prescribed values (T_2), (C_2), and (N_2). The flow conditions for the initial lower and upper boundaries are as follows:

Table 2 Properties of NF, HNF, and TRIHNFs

Properties	(NF,HNF,TRIHNFs)
Density(ρ)	$\rho_{trihnf} = \phi_1 \rho_{s1} + \phi_2 \rho_{s2} + \phi_3 \rho_{s3} - (1 - \phi_1 - \phi_2 - \phi_3) \rho_f$
Viscosity(μ)	$\mu_{trihnf} = \mu_f [1 + 0.1008 ((\phi_1)^{0.69574} (dp_1)^{0.44708} + (\phi_2)^{0.69574} (dp_2)^{0.44708} + (\phi_3)^{0.69574} (dp_3)^{0.44708})]$
Heat Capacity (ρC_p)	$(\rho C_p)_{trihnf} = \phi_1 (\rho C_p)_{s1} + \phi_2 (\rho C_p)_{s2} + \phi_3 (\rho C_p)_{s3} - (1 - \phi_1 - \phi_2 - \phi_3) (\rho C_p)_f$
Thermal Conductivity (k)	$\frac{k_{trihnf}}{k_{nf}} = \frac{k_{s3} + (n_3 - 1)k_{hnf} - (n_3 - 1)\phi_3(k_{hnf} - k_{s3})}{k_{s3} + (n_3 - 1)k_{hnf} + \phi_3(k_{hnf} - k_{s3})}$ $\frac{k_{hnf}}{k_{nf}} = \frac{k_{s2} + (n_2 - 1)k_{nf} - (n_2 - 1)\phi_2(k_{nf} - k_{s2})}{k_{s2} + (n_2 - 1)k_{nf} + \phi_2(k_{nf} - k_{s2})}$ $\frac{k_{nf}}{k_f} = \frac{k_{s1} + (n_1 - 1)k_f - (n_1 - 1)\phi_1(k_f - k_{s1})}{k_{s1} + (n_1 - 1)k_f + \phi_1(k_f - k_{s1})}$
Nanolayer Thermal Conductivity	$\frac{k_{trihnlndf}}{k_{nlndf}} = \frac{(k_{s3} - k_{nlr})\phi_3 k_{nlr}(\lambda_2^2 - \lambda_1^2 + 1) + (k_{nlr} + k_{s3})\lambda_2^2(\phi_3 \lambda_1^2(k_{nlr} - k_{nlndf}) + k_{nlndf})}{\lambda_2^2(k_{s3} + k_{nlr}) - (k_{s3} - k_{nlr})\phi_3(\lambda_2^2 + \lambda_1^2 - 1)}$ $\frac{k_{hnlndf}}{k_{nlndf}} = \frac{(k_{s2} - k_{nlr})\phi_2 k_{nlr}(\lambda_2^2 - \lambda_1^2 + 1) + (k_{nlr} + k_{s2})\lambda_2^2(\phi_2 \lambda_1^2(k_{nlr} - k_{nlndf}) + k_{nlndf})}{\lambda_2^2(k_{s2} + k_{nlr}) - (k_{s2} - k_{nlr})\phi_2(\lambda_2^2 + \lambda_1^2 - 1)}$ $\frac{k_{nlndf}}{k_f} = \frac{(k_{s1} - k_{nlr})\phi_1 k_{nlr}(\lambda_2^2 - \lambda_1^2 + 1) + (k_{nlr} + k_{s1})\lambda_2^2(\phi_1 \lambda_1^2(k_{nlr} - k_f) + k_f)}{\lambda_2^2(k_{s1} + k_{nlr}) - (k_{s1} - k_{nlr})\phi_1(\lambda_2^2 + \lambda_1^2 - 1)}$

$$\begin{cases} z = -s(t), u = 0, v = -\frac{rA_1s'(t)}{2s}, \\ w = -A_1s'(t), T = T_1, C = C_1, N = N_1, \\ z = s(t), u = 0, v = \frac{rA_1s'(t)}{2s}, \\ w = A_1s'(t), T = T_2, C = C_2, N = N_2 \end{cases} \quad (8)$$

Here, $s'(t)$ represents the derivative concerning time t . Following are the similarity transformations which are employed to simplify the governing equations into a solvable (dimensionless) form while preserving the essential physics [61]:

$$\begin{cases} \eta = \frac{z}{s}, u = -\frac{rv_f}{s^2}F_\eta(\eta, t), v = \frac{rv_f}{s^2}G(\eta, t), \\ w = \frac{2v_f}{s}F(\eta, t), \\ \theta(\eta, t) = \frac{T-T_2}{T_1-T_2}, \chi(\eta, t) = \frac{C-C_2}{C_1-C_2}, \\ \psi(\eta, t) = \frac{N-N_2}{N_1-N_2}, \xi = \frac{r}{R} \end{cases} \quad (9)$$

Applying these similarity transformation to Eqs. (2-8), they are simplified into the following form:

$$\begin{aligned} & \frac{v_{\text{trihnf}}}{v_f}F_{\eta\eta\eta\eta} + \alpha(3F_{\eta\eta} + \eta F_{\eta\eta\eta}) - \frac{s^2}{v_f}F_{\eta\eta t} \\ & - 2FF_{\eta\eta\eta} - 2GG_{\eta} - \frac{\rho_f}{\rho_{\text{trihnf}}}MF_{\eta\eta} \\ & - G_T\theta_{\eta} - G_C\chi_{\eta} - G_N\psi_{\eta} - \frac{v_{\text{trihnf}}}{v_f Da}F_{\eta\eta} \\ & + 2Fr\frac{\rho_f}{\rho_{\text{trihnf}}}F_{\eta}F_{\eta\eta} = 0 \end{aligned} \quad (10)$$

$$\begin{aligned} & \frac{v_{\text{trihnf}}}{v_f}G_{\eta\eta} + \alpha(2G + \eta G_{\eta}) \\ & - \frac{s^2}{v_f}G_t + 2GF_{\eta} - 2FG_{\eta} - \frac{\rho_f}{\rho_{\text{trihnf}}}MG_{\eta} = 0 \end{aligned} \quad (11)$$

$$\begin{aligned} & \theta_{\eta\eta} - \frac{v_f}{\alpha_{\text{trihnf}}}(2F - \eta\alpha)\theta_{\eta} - \frac{s^2}{\alpha_{\text{trihnf}}}\theta_t \\ & + \left[\frac{\mu_{\text{trihnf}}}{\mu_f} \left((F_{\eta\eta})^2 + (G_{\eta})^2 \right) \right. \\ & \left. + M \left((F_{\eta})^2 + G^2 \right) \right] \frac{k_f}{k_{\text{trihnf}}}EcPr + \frac{k_f}{k_{\text{trihnf}}}PrQ\theta e^{-m_1\eta} \\ & + \frac{k_f}{k_{\text{trihnf}}}Pr[N_b\theta_{\eta}\chi_{\eta} + N_t(\theta_{\eta})^2] = 0 \end{aligned} \quad (12)$$

$$\begin{aligned} & ;\chi_{\eta\eta} - \frac{v_f}{(D_b)_{\text{trihnf}}}(2F - \eta\alpha)\chi_{\eta} - \frac{s^2}{(D_b)_{\text{trihnf}}}\chi_t \\ & + \frac{N_t}{N_b}\theta_{\eta\eta} - \frac{v_f}{(D_b)_{\text{trihnf}}}K_{cr}\chi \left(1 + \frac{\theta}{\omega} \right)^m e^{\frac{-\omega E}{\theta + \omega}} = 0 \end{aligned} \quad (13)$$

$$\begin{aligned} & \psi_{\eta\eta} - \frac{v_f}{(D_n)_{\text{trihnf}}}(2F - \eta\alpha)\psi_{\eta} - \frac{s^2}{(D_n)_{\text{trihnf}}}\psi_t \\ & - \frac{bW_c}{(D_n)_{\text{trihnf}}}[\chi_{\eta}\psi_{\eta} + (\psi + N^*)\chi_{\eta\eta}] = 0 \end{aligned} \quad (14)$$

$$\begin{cases} \eta = -1, F = -Re, F_{\eta} = 0, G = -Re, \\ \theta = 1, \chi = 1, \psi = 1, \\ \eta = 1, F = Re, F_{\eta} = 0, G = Re, \\ \theta = 0, \chi = 0, \psi = 0 \end{cases} \quad (15)$$

After the similarity transformation, the non dimensional boundary conditions characterize the symmetric rotational flow, heat transfer, nanoparticle concentration, and micro-organism distribution between the two disks. Table 3 shows the dimensionless parameters [54, 55, 61, 62]. The Reynolds number (R) represents the ratio of inertial forces to viscous forces and characterizes the flow strength within the rotating porous disks. The magnetic parameter (M) quantifies the influence of the externally applied magnetic field on the electrically conducting tri-hybrid nanofluid. Larger values of M enhance the Lorentz force, which effect fluid motion and velocity distribution. The Darcy number (Da) characterizes the permeability of permeable medium, where higher values indicate lower resistance to fluid flow. The Forchheimer parameter (Fr) represents inertial drag effects in the porous structure and becomes significant at higher flow velocities. The thermal Grashof number (G_T), solutal Grashof number (G_C), and microorganism Grashof number (G_N) represent the buoyancy effects caused by temperature, concentration, and

Table 3 Dimensionless parameter

$\alpha = \frac{ss'(t)}{v_f}$	$Re = \frac{(A_1ss'(t))}{2v_f}$	$M = \frac{\sigma_e(B_0)^2s^2}{\mu_f}$
$G_T = \frac{g\beta_T(T_1-T_2)}{U^2}$	$G_C = \frac{g\beta_C(C_1-C_2)}{U^2}$	$G_N = \frac{g\beta_N(N_1-N_2)}{U^2}$
$Da = \frac{\kappa^*}{s^2}$	$Fr = \frac{F_0\xi R}{\kappa^*}$	$Ec = \frac{(T_1-T_2)(C_p)_f}{(T_1-T_2)(C_p)_f}$
$N_t = \frac{D_T(T_1-T_2)\zeta}{v_f}$	$N_b = \frac{D_b(C_1-C_2)\zeta}{v_f}$	$N^* = \frac{N_2}{N_1-N_2}$
$Pr = \frac{(\mu C_p)_f}{k_f}$	$Pe = \frac{bW_c}{(D_n)_{\text{trihnf}}}$	$K_{cr} = K_0 \frac{s^2}{(D_b)_{\text{trihnf}}}$
$Q = \frac{Q_0s^2}{(\mu C_p)_f}$	$E = \frac{E_b}{\kappa_b T_2}$	$\omega = \frac{T_2}{T_1-T_2}$
$Sn = \frac{v_f}{(D_n)_{\text{trihnf}}}$	$Sc = \frac{v_f}{(D_b)_{\text{trihnf}}}$	
$B_1 = \frac{(D_b)_{\text{trihnf}}(C_1-C_2)^2T_2^2}{k_f(\Delta T)^2C_2}$	$B_2 = \frac{(D_n)_{\text{trihnf}}(N_1-N_2)^2T_2^2}{k_f(\Delta T)^2N_2}$	$B_3 = \frac{(D_b)_{\text{trihnf}}(C_1-C_2)T_2}{k_f(\Delta T)}$
$B_4 = \frac{(D_n)_{\text{trihnf}}(N_1-N_2)T_2}{k_f(\Delta T)}$	$L_1 = \frac{v_{\text{trihnf}}}{v_f}$	$L_2 = \frac{\rho_f}{\rho_{\text{trihnf}}}$
$L_3 = \frac{(\rho C_p)_{\text{trihnf}}}{(\rho C_p)_f}$	$L_4 = \frac{\mu_{\text{trihnf}}}{\mu_f}$	$L_5 = \frac{k_f}{k_{\text{trihnf}}}$

microorganism density differences, respectively. The Prandtl number (Pr) denotes the ratio of momentum diffusivity to thermal diffusivity and determines the relative thickness of momentum and thermal boundary layers. The Eckert number (Ec) calculates the conversion of kinetic energy into internal energy due to viscous dissipation effects. The heat source parameter (Q) characterizes internal thermal energy generation or absorption within the system. The Brownian motion parameter (Nb) describes nanoparticle random motion caused by molecular collisions, whereas the thermophoresis parameter (Nt) represents nanoparticle migration induced by temperature gradients. The Schmidt number (Sc) expresses the ratio of momentum diffusivity to mass diffusivity and governs concentration boundary layer behavior. The activation energy parameter (E) characterizes sensitivity in chemical reaction rates to temperature variations according to Arrhenius theory. The chemical reaction parameter (K_{cr}) controls the rate of species consumption or generation during the reaction process. The Peclet number (Pe) represents the ratio of microorganism advection transport to microorganism diffusion transport. The bioconvection Lewis number (Sn) characterizes the diffusivity of motile microorganisms. The parameter (N^*) denotes the microorganism concentration difference parameter associated with gyrotactic microorganism distribution.

The parameters L_1, L_2, L_3, L_4 , and L_5 represent the ratios of thermophysical properties between the tri-hybrid nanofluid and the base fluid, accounting for variations in viscosity, density, heat capacity, and thermal conductivity due to nanoparticle suspension. The diffusivity of motile microorganisms and different nanoparticles is denoted by B_1, B_2, B_3 , and B_4 . The Peclet number, denoted as $Pe = PrRe$, represents the heat transfer. Consequently, we set $F = fRe$ and $G = gRe$, as done in the work of Majdalani et al. [65], where α is shown to be a constant, $f = f(\eta)$, $g = g(\eta)$, $\theta = \theta(\eta)$, $\chi = \chi(\eta)$, $\psi = \psi(\eta)$, that results in $\theta_t = 0, g_{\eta t} = 0, f_{\eta\eta t} = 0, \chi_t = 0, \psi_t = 0$ and also using the thermophysical properties of TRIHNF so that we obtain the following equations.

$$L_1 f'''' + \alpha(3f'' + \eta f''') - 2Reff''' - 2Regg' - L_2 Mf'' - [G_T \theta' + G_C \chi' + G_N \psi'] \frac{1}{Re} + \frac{L_1}{Da} f'' + 2FrReL_2 f' f'' = 0 \quad (16)$$

$$L_1 g'' + \alpha(2g + \eta g') + 2Regf' - 2Refg' - L_2 Mg = 0 \quad (17)$$

$$\theta'' - L_3 L_5 Pr(2Ref - \eta \alpha) \theta' + [L_4 ((f'')^2 + (g')^2) + M((f')^2 + g^2)] L_5 Re^2 Ec Pr + Pr L_5 Q \theta e^{-m_1 \eta} + Pr L_5 [N_b \theta' \chi' + N_t (\theta')^2] = 0 \quad (18)$$

$$\chi'' - Sc(2Ref - \eta \alpha) \chi' + \frac{N_t}{N_b} \theta'' - K_{cr} Sc \chi \left(1 + \frac{\theta}{\omega}\right)^m e^{\frac{-E\omega}{\theta + \omega}} = 0 \quad (19)$$

$$\psi'' - Sn(2Ref - \eta \alpha) \psi' - Pe[\chi' \psi' + (\psi + N^*) \chi''] = 0 \quad (20)$$

$$\begin{cases} \eta = -1, & f = -1, & f' = 0, & g = -1, \\ \theta = 1, & \chi = 1, & \psi = 1, \\ \eta = 1, & f = 1, & f' = 0, & g = 1, \\ \theta = 0, & \chi = 0, & \psi = 0. \end{cases} \quad (21)$$

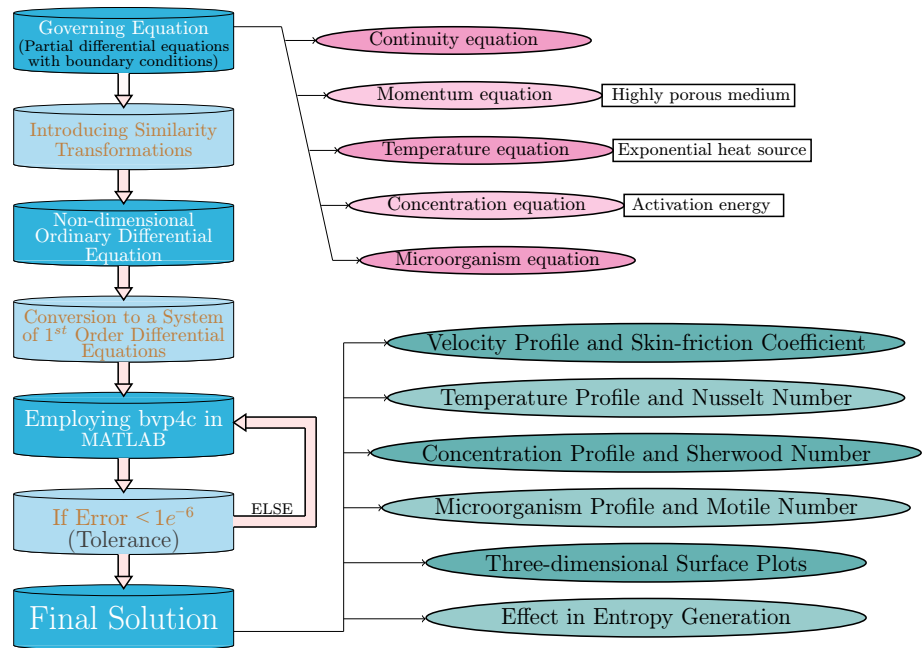
Numerical solution

The Bvp4c numerical solver that is implemented in MATLAB is specifically designed to solve boundary value problems (BVPs) for ordinary differential equations (ODEs). It approximates the solution by a shooting method where the problem is discretized into a set of algebraic equations. The equations are solved in an iterative manner so that both the differential equations and the boundary conditions are satisfied. Bvp4c is useful for solving problems where the solution is smooth and boundary conditions are specified at more than one point. It supports differential problems with two-point and multi-point boundary constraints. The function can handle singularities and offers flexibility in defining and adjusting the mesh points for better accuracy. So we convert 4th and 2nd-order derivatives into a system of equations with 1st-order derivatives. This has been done for solving the problem by using the MATLAB solver bvp4c. This transformation allows the numerical values of different parameters to be calculated in a tabular form and in graphs. The flowchart is shown in Fig. 3, describing the structure of the paper.

$$\begin{cases} f = y(1), f' = y'(1) = y(2), f'' = y'(2) = y(3), \\ f''' = y'(3) = y(4), g = y(5), g' = y'(5) = y(6), \\ \theta = y(7), \theta' = y'(7) = y(8), \\ \chi = y(9), \chi' = y'(9) = y(10), \\ \psi = y(11), \psi' = y'(11) = y(12) \end{cases} \quad (22)$$

Now equations are changed into 1st order as:

Fig. 3 Flowchart



$$\begin{aligned}
 f'''' &= y'(4) = \frac{1}{L_1} [-\alpha(3y(3) + \eta y(4)) + 2Rey(1)y(4) \\
 &+ 2Rey(5)y(6) + L_2My(3)] + \\
 &[G_Ty(8) + G_Cy(10) + G_Ny(12)] \frac{1}{ReL_1} \\
 &- \frac{1}{Da}y(3) - \frac{2FrRe}{L_1}L_2y(2)y(3)
 \end{aligned} \quad (23)$$

$$\begin{aligned}
 g'' &= y'(6) \\
 &= \frac{-\alpha(2y(5) + \eta y(6)) - 2Rey(5)y(2) + 2Rey(1)y(6) + L_2My(5)}{L_1}
 \end{aligned} \quad (24)$$

$$\begin{aligned}
 \theta' &= y'(8) = L_3L_5Pr(2Rey(1) - \eta\alpha)y(8) \\
 &- [L_4((y(3))^2 + (y(6))^2) \\
 &+ M((y(2))^2 + y(5)^2)]L_5Re^2EcPr \\
 &- PrL_5Qy(7)e^{-m_1\eta} + PrL_5[N_by(8)y(10) + N_t(y(8))^2]
 \end{aligned} \quad (25)$$

$$\begin{aligned}
 \chi'' &= y'(10) = Sc(2Rey(1) - \eta\alpha)y(10) - \frac{N_t}{N_b}y'(8) \\
 &+ K_{cr}Scy(9)\left(1 + \frac{y(7)}{\omega}\right)^m e^{\frac{-E_0}{y(7)+\omega}}
 \end{aligned} \quad (26)$$

$$\begin{aligned}
 \psi' &= y'(12) = Sn(2Rey(1) - \eta\alpha)y(12) \\
 &+ Pe[y(10)y(12) + (y(11) + N^*)y'(10)]
 \end{aligned} \quad (27)$$

Boundary conditions for MATLAB are,

$$\begin{cases} \eta = -1 = l, \quad \eta = 1 = r \\ y_l(1) + 1; y_l(2); y_l(5) + 1; y_l(7) - 1; \\ y_l(9) - 1; y_l(11) - 1; \\ y_r(1) - 1; y_r(2); y_r(5) - 1; y_r(7); \\ y_r(9); y_r(11); \end{cases} \quad (28)$$

Physical parameter

Considering some physical parameter such as skin friction, Nusselt number, Sherwood number, and Motile microorganism which are approximated for each permeable layer.

Skin friction number

The flow between the upper and lower porous disks is described in the radial and tangential directions, as represented by

$$\xi_{zf} = \mu_{trihnf} \left(\frac{\partial u}{\partial z} \right) = \mu_{trihnf} \frac{rv_f}{s^3} F_{\eta\eta} \quad (29)$$

$$\xi_{z\theta} = \mu_{trihnf} \left(\frac{\partial v}{\partial z} \right) = \mu_{trihnf} \frac{rv_f}{s^3} G_{\eta\eta} \quad (30)$$

The total shear stress denotes ξ_w ,

$$\xi_w = \sqrt{\xi_{zr}^2 + \xi_{z0}^2} \quad (31)$$

Skin friction number for lower porous disk is as

$$C_{f(\eta=-1)} = \frac{\xi_w|_{\eta=-1}}{\rho_f(s'A_1)^2} \quad (32)$$

$$C_{f(\eta=-1)} = \frac{L_1}{Re_r} \sqrt{(f'(-1))^2 + (g'(-1))^2} \quad (33)$$

Skin friction number for upper disk is as

$$C_{f(\eta=1)} = \frac{\xi_w|_{\eta=1}}{\rho_f(s'A_1)^2} \quad (34)$$

$$C_{f(\eta=1)} = \frac{L_1}{Re_r} \sqrt{(f'(1))^2 + (g'(1))^2} \quad (35)$$

The local Reynolds number is represented by $Re_r = 4 \frac{s}{r} Re$.

Nusselt number

Heat flux due to flow of fluid heat transfer is as

$$\begin{aligned} e_z|_{\eta=-1} &= -k_{\text{trihnf}} \left(\frac{\partial T}{\partial z} \right) |_{\eta=-1} \\ &= \frac{(T_1 - T_2)}{s} k_{\text{trihnf}} \theta'(-1) \end{aligned} \quad (36)$$

$$\begin{aligned} e_z|_{\eta=1} &= -k_{\text{trihnf}} \left(\frac{\partial T}{\partial z} \right) |_{\eta=1} \\ &= \frac{(T_1 - T_2)}{s} k_{\text{trihnf}} \theta'(1) \end{aligned} \quad (37)$$

The Nusselt number Nu_z for nanolayer thermal conductivity and tri-hybrid nanofluid thermal conductivity of lower porous disk is as

$$Nu_{z(\eta=-1)} = \frac{se_z}{k_f(T_1 - T_2)} |_{\eta=-1} = -\frac{k_{\text{trihnf}}}{k_f} \theta'(-1) \quad (38)$$

The Nusselt number Nu_z for nanolayer thermal conductivity and tri-hybrid nanofluid thermal conductivity of upper porous disk is as

$$Nu_{z(\eta=1)} = \frac{se_z}{k_f(T_1 - T_2)} |_{\eta=1} = -\frac{k_{\text{trihnf}}}{k_f} \theta'(1) \quad (39)$$

Sherwood number

The Nusselt number for nanolayer thermal conductivity and tri-hybrid nanofluid thermal conductivity which is denoted by $(Sh|_{\eta=-1})$ and $(Sh|_{\eta=1})$ for lower and upper porous disk.

$$Sh|_{\eta=-1} = \frac{kq_z}{(D_b)_{\text{trihnf}}(C_1 - C_2)} |_{\eta=-1} = -\chi'(-1), \quad (40)$$

$$Sh|_{\eta=1} = \frac{kq_z}{(D_b)_{\text{trihnf}}(C_1 - C_2)} |_{\eta=1} = -\chi'(1), \quad (41)$$

where

$$\begin{aligned} q_z|_{\eta=-1} &= -(D_b)_{\text{trihnf}} \left(\frac{\partial C}{\partial z} \right) |_{\eta=-1} \\ &= -(D_b)_{\text{trihnf}} \frac{(C_1 - C_2)}{k} \chi'(-1), \end{aligned} \quad (42)$$

$$\begin{aligned} q_z|_{\eta=1} &= -(D_b)_{\text{trihnf}} \left(\frac{\partial C}{\partial z} \right) |_{\eta=1} \\ &= -(D_b)_{\text{trihnf}} \frac{(C_1 - C_2)}{k} \chi'(1), \end{aligned} \quad (43)$$

Motile density number

Motile density number is represented by $Nn_{\eta=-1}$ and $Nn_{\eta=1}$ for lower and upper disks and it is expressed as

$$Nn|_{\eta=-1} = \frac{kq_n}{(D_n)_{\text{trihnf}}(N_1 - N_2)} |_{\eta=-1} = -\psi'(-1), \quad (44)$$

$$Nn|_{\eta=1} = \frac{kq_n}{(D_n)_{\text{trihnf}}(N_1 - N_2)} |_{\eta=1} = -\psi'(1), \quad (45)$$

where

$$\begin{aligned} q_n|_{\eta=-1} &= -(D_n)_{\text{trihnf}} \left(\frac{\partial N}{\partial z} \right) |_{\eta=-1} \\ &= -(D_n)_{\text{trihnf}} \frac{(N_1 - N_2)}{k} \psi'(-1), \end{aligned} \quad (46)$$

$$\begin{aligned} q_n|_{\eta=1} &= -(D_n)_{\text{trihnf}} \left(\frac{\partial N}{\partial z} \right) |_{\eta=1} \\ &= -(D_n)_{\text{trihnf}} \frac{(N_1 - N_2)}{k} \psi'(1), \end{aligned} \quad (47)$$

Entropy generation

The primary factors contributing to entropy generation in the medium are mass and heat transmission, chemical reactions, Joule heating, fluid friction, and diffusion. These processes irreversibly degrade the system's energy, leading to increased entropy. According to the second law of thermodynamics, the entropy generation is given by the following expression [61]:

$$\begin{aligned}
s'''_{\text{Gen}} = & \frac{K_{\text{trihnf}}}{T_2^2} \left(\frac{\partial T}{\partial z} \right)^2 + \frac{\mu_{\text{trihnf}}}{T_2} \left[\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right] \\
& + \frac{\sigma_e B_o^2}{T_2} (u^2 + v^2) + \frac{(D_b)_{\text{trihnf}}}{C_2} \left(\frac{\partial C}{\partial z} \right)^2 \\
& + \frac{(D_n)_{\text{trihnf}}}{N_2} \left(\frac{\partial N}{\partial z} \right)^2 \\
& + \frac{(D_b)_{\text{trihnf}}}{T_2} \left(\frac{\partial C}{\partial z} \frac{\partial T}{\partial z} \right) + \frac{(D_b)_{\text{trihnf}}}{T_2} \left(\frac{\partial N}{\partial z} \frac{\partial T}{\partial z} \right)
\end{aligned} \quad (48)$$

Since initially entropy generation is $(s'''_{\text{Gen}})_0 = \frac{k_f(\Delta T)^2}{s^2 T_2^2}$. Now using the similarity transformation to get dimensionless expression for entropy generation where $N_s = \frac{(s'''_{\text{Gen}})}{(s'''_{\text{Gen}})_0}$ then equation of total entropy is written as:

$$\begin{aligned}
N_s = & \frac{1}{L_5} (\theta')^2 + \text{Re}^2 \text{EcPr}\omega L_4 [(f'')^2 + (g')^2] \\
& + \text{Re}^2 \text{MEcPr}\omega [(f')^2 + (g')^2] + B_1 (\chi')^2 \\
& + B_2 (\psi')^2 + B_3 (\chi' \theta') + B_4 (\psi' \theta')
\end{aligned} \quad (49)$$

for numerical solution;

$$\begin{aligned}
N_s = & \frac{1}{L_5} (y(8))^2 + \text{Re}^2 \text{EcPr}\omega L_4 [(y(3))^2 + (y(6))^2] \\
& + \text{Re}^2 \text{MEcPr}\omega [(y(2))^2 + (y(5))^2] + B_1 (y(8))^2 \\
& + B_2 (y(12))^2 + B_3 y(10)y(8) + B_4 (y(12)y(8))
\end{aligned} \quad (50)$$

Results and discussion

The current investigation proposes a detailed numerical analysis of a ternary hybrid nanofluid flow system ($\text{Cu} + \text{Al}_2\text{O}_3 + \text{TiO}_2/\text{H}_2\text{O}$) between two permeable rotating disks, incorporating the complex interplay of motile microorganisms, exponential heat source, activation energy, magnetic field, and porous medium effects. By modeling energy, momentum, and mass transfer under bio-convective and magnetohydrodynamic (MHD) influences, the research aims to optimize entropy generation and evaluate thermal-hydraulic performance in a multi-physics environment relevant to biomedical and energy systems. For precise outcomes and robustness of the proposed model, the governing partial differential equations were converted into a system of nonlinear ordinary differential equations through similarity transformations and subsequently solved using numerically by MATLAB's high-precision `bvp4c` solver. A wide range of relevant physical parameters, such as Reynolds number (Re), viscous dissipation (Ec), Darcy number (Da), magnetic parameter (M), Brownian motion (Nb), thermophoresis (Nt), and activation energy (Ea), were varied systematically as listed in Table 4. The accuracy of the numerical method was validated through the maximum residual errors of the governing equations, which were found to be on the order of 10^{-7} , as presented in Table 5. To further validate the accuracy and reliability of the present numerical model, a comparative analysis was performed by evaluating the computed values of the Nusselt number, Sherwood number, and Skin Friction coefficient against the benchmark results reported by Raza et al. [61]. The results, presented in Table 6, exhibit excellent agreement across a range of Reynolds numbers, with negligible deviations observed between the present work and the existing literature. This close correspondence

Table 4 Range of the values of flow parameters

Parameter	Range	Default	References	Parameter	Range	Default	References
Re	[0-8]	0.5	[55]	α	[-5,5]	1	[61]
M	[0-10]	2	[55]	G_T	[0-10]	0.5	[55]
G_C	[0-10]	0.5	[55]	G_N	[0-10]	0.5	[55]
Da	[0-1]	0.02	[55]	Fr	[0-20]	0.5	[55]
Ec	[0-8]	0.5	[55]	Pr	[0-20]	0.5	[55]
Pe	[0-8]	1	[61]	Kcr	[0-15]	0.5	[55]
Nb	[0-5]	0.5	[55]	Nt	[0-5]	0.5	[55]
Sc	[0-10]	0.5	[55]	Sn	[0-10]	0.5	[55]
N^*	[0-8]	0.5	[61]	Q	[-10,10]	0.5	[55]
B1	[0-10]	0.5	[61]	B2	[0-10]	0.5	[61]
ϕ_1	[0-1]	0.02	[61]	ϕ_2	[0-1]	0.02	[61]
ϕ_3	[0-1]	0.02	[61]	dp_1	[0-1]	0.5	[61]
dp_2	[0-1]	0.5	[61]	dp_3	[0-1]	0.5	[61]
Ea	[-10,10]	0.5	[55]	ω	[0-1]	1	[55]

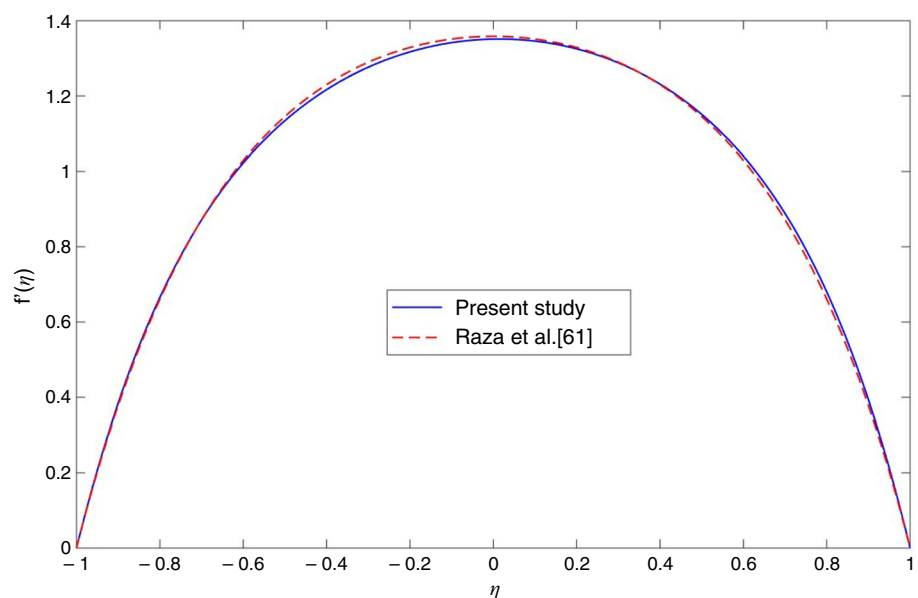
Table 5 Comparison of present results with Raza et al. [61] for validation (selected parameters: $M = 1$, $Ec = 0.2$, $Pr = 6.2$)

Variable	Raza et al. [61]	Present study	Error/ %
$f''(0)$ (velocity gradient)	1.234	1.219	1.22
$-\theta'(0)$ (Nusselt number)	0.842	0.853	1.31
$-\phi'(0)$ (Sherwood number)	0.416	0.423	1.68

confirms the adopted numerical approach's consistency, robustness, and precision and highlights its suitability for analyzing complex hybrid nanofluid flows in porous media environments with motile microorganisms and heat generation effects. Figures 4 and 5 also provide comparative graphical validation of radial velocity and concentration profiles with parameter Re and K_{cr} , respectively. These validations confirm the consistency and reliability of the numerical model, enabling a meaningful exploration of the thermo-physical behavior and entropy characteristics across varying flow and thermal conditions.

Table 6 Comparison of Nusselt number, Sherwood number, and Skin Friction number with Raza et al. [61]

Re	Nu (Present)	Nu [61]	Sh (Present)	Sh [61]	C_f (Present)	C_f [61]
-5	7.6400	7.6399	-0.3959	-0.3958	3.6506	3.6504
-3	3.9059	3.9058	-0.3659	-0.3659	2.9415	2.9414
-1	2.0736	2.0736	-0.3377	-0.3377	2.3740	2.3740
0	1.5339	1.5337	-0.3247	-0.3245	2.1379	2.1377
1	1.1462	1.1462	-0.3115	-0.3115	1.9293	1.9292
3	0.6609	0.6609	-0.2873	-0.2873	1.5844	1.5844
5	0.3978	0.3978	-0.2647	-0.2646	1.3172	1.3172

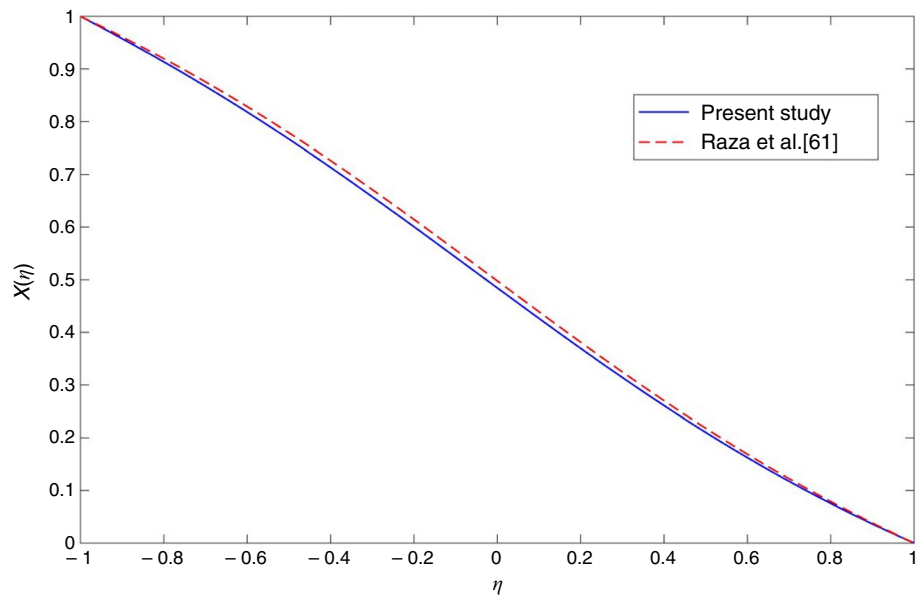
Fig. 4 Comparative analysis of Radial velocity

In this section, we assess the combined influence of various factors, including activation energy, Darcy–Forchheimer drag, an exponential heat source, chemical reaction, magnetic field interactions, thermophoresis, viscous dissipation, Brownian motion, Joule heating, and the associated heat and mass transfer processes. This analysis provides a comprehensive understanding of how each parameter interacts within the system. Graphs representing different flow parameters, including velocity, concentration, and the dynamics of motile microorganisms are presented for further analysis.

Velocity profile

Figures 6 and 7 show how the expansion ratio parameter affects the radial and tangential velocities. On increasing the expansion ratio parameter, the radial velocity will increase and decrease after its peak point, and the tangential velocity will increase. This occurs because an increase in the conduction area allows the fluid to spread over a wider region, promoting its expansion. This extended area is the region for changing fluid velocity. It is a result of momentum transfer. Figures 8 and 9 depict the impact caused by Reynolds value

Fig. 5 Comparative analysis of Concentration



on the velocity. As the Re enhances, velocity also increases. The radial velocity is found to be lowest at both ends of the porous disk and highest at center. This distribution is primarily influenced by the fluid's interaction with the surface geometry. When the Reynolds number increases, the radial velocity grows at central region of the wall. Tangential velocity starts low at the initial point of the porous disk and progressively increases as it moves further along. Figure 10 exemplifies the impact of the magnetic field parameter (M) on the radial velocity, revealing a significant alteration as the parameter fluctuates. The magnetic field manifests the ramifications of magnetic forces on the dynamics of the fluid. The selected range elucidates the intensity of the magnetic field. The fluid exhibits increased velocity in proximity to the disk surface with the escalation of the M value. Figures 11, 12, and 13 depict the influence of the thermal Grashof number, solute Grashof number, and microorganism Grashof number on the velocity profile. With increasing values of these Grashof numbers, flow at surface of the porous disk accelerates and eventually attains a higher peak velocity. This enhancement in fluid motion arises from the increased buoyancy forces generated by thermal, concentration, and microorganism density differences. Physically, stronger buoyancy effects intensify the convective transport mechanism, enabling the fluid to overcome viscous resistance more effectively and thereby augmenting the overall flow strength within the boundary layer. Figures 14 and 15 demonstrate the relationship between Darcy number and Forchheimer number at velocity profile. The fluid flow is faster on the disk's surface than on the wall's middle. Velocity has a peak velocity in the middle. An expansion of the value of Darcy number results in a corresponding rise in

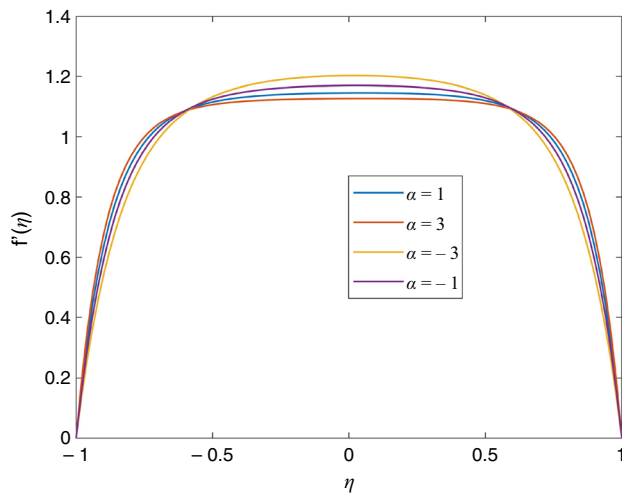
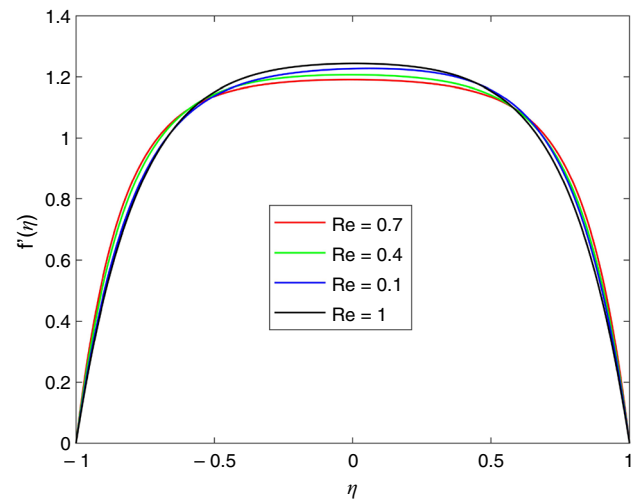
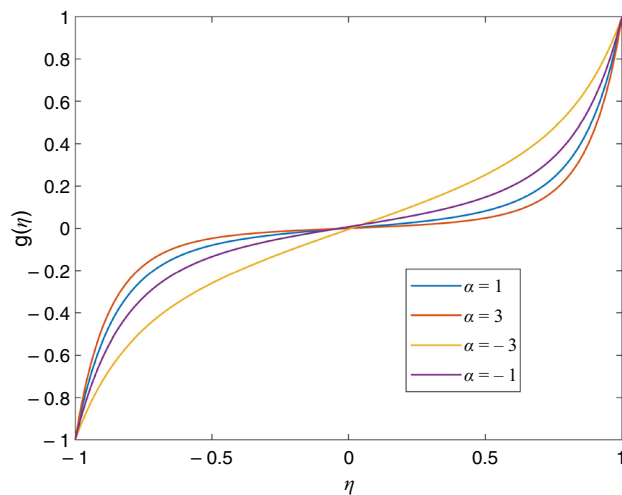
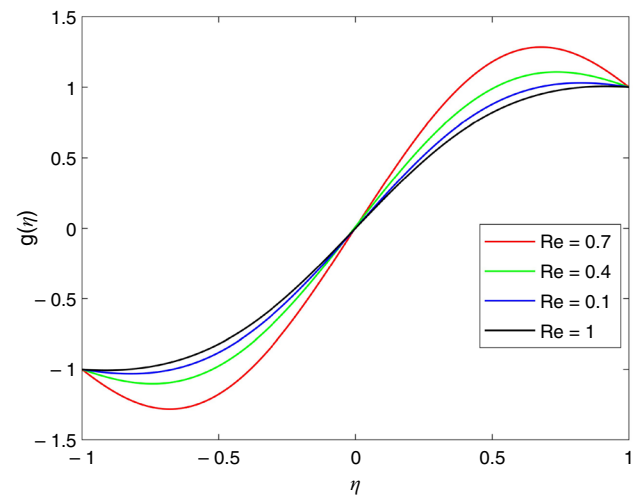
peak velocity, while an escalation in the Forchheimer number produces a diminishment in the peak velocity.

Temperature profile

Figure 16 suspects the α parameter on the temperature distribution within the fluid. The fluctuation in the intersecting area of fluid flow is governed by the α parameter, with an increased value of α reduces the thermal surface layer on both disks, leading to enhanced heat transfer efficiency. Thermal boundary layer refers region adjacent to the wall where heat is exchanged among surface and fluid. A thinner thermal boundary layer is good and has less resistance for a Greater efficient heat transfer. Subsequently, the temperature gradient becomes more uniform. Figure 17 clarifies the impact of Re on thermal distribution. An increase in the Re causes the thermal boundary layer to thicken. This augmentation in boundary layer thickness results in a more pronounced temperature gradient throughout the flow, thereby enhancing thermal efficiency and facilitating improved overall heat exchange. Figure 18 is find the difference of magnetic field parameter on the temperature distribution. When the M parameter intensifies, the thermal boundary layer extends and becomes more pronounced, initiating an enhancement in heat exchange among the fluid and the disks. Additionally, this effect is crucial in optimizing the heat transfer rate, particularly in systems where magnetic fields are actively applied. Figure 19 emphasizes the Eckert number's influence on the temperature gradient. An improvement in the Eckert number suggests a more robust heat transfer within the fluid and the surface. Figure 20 shows how temperature $\theta(\eta)$ changes with varying nanolayer thickness h . As h increases from 0.1 to 0.4, the

Table 7 Parametric study of entropy generation contributions for varying Ec and M

Ec	M	$N_{s,\text{total}}$	Joule/ %	Viscous/ %	Thermal/ %	Solutal/ %	Micro./ %
0.1	1	1.24	22.6	18.3	38.4	12.1	8.6
0.2	1	1.59	21.1	26.2	34.7	10.4	7.6
0.2	3	2.21	35.4	25.5	27.3	7.8	4.0
0.3	3	2.89	34.2	32.7	23.6	6.3	3.2

**Fig. 6** Effect of the non-dimensional expansion ratio parameter on radial velocity distribution**Fig. 8** Effect of Reynolds number on radial velocity**Fig. 7** Influence of the dimensionless expansion ratio on the tangential velocity profile**Fig. 9** Reynolds number impact in Tangential velocity

peak temperature gradually decreases. This trend suggests that thinner nanolayers promote better thermal conductivity by enhancing the interaction effects between nanoparticles and the carrier fluid. Consequently, smaller nanolayer thicknesses result in boosted heat transfer capabilities in the ternary hybrid nanofluid. Figure 21 presents the temperature

variation $\theta(\eta)$ for different nanoparticle radii r . As the radius increases from 0.1 to 0.4, the peak temperature value noticeably. This behavior suggests that larger nanoparticles improve heat retention by offering a greater surface area for thermal interaction. Consequently, increasing the particle size enhances the overall thermal conductivity of the TRIHNF. Figure 22 illustrates role of Peclet number in

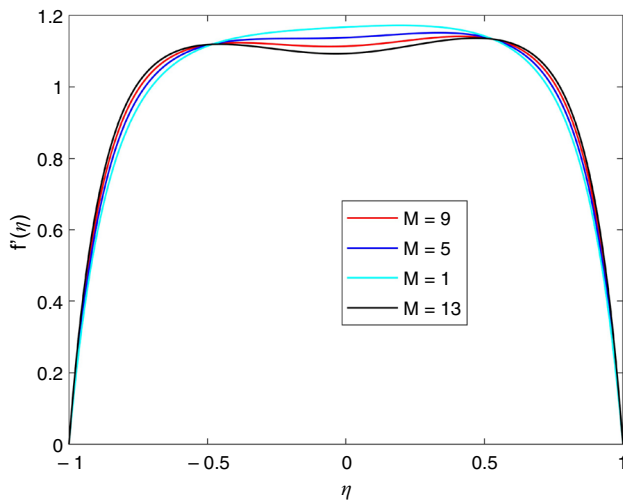


Fig. 10 Magnetic field parameter impact in Radial velocity

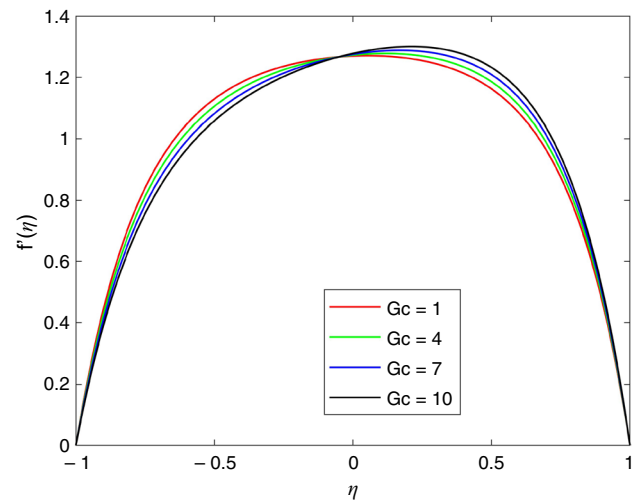


Fig. 12 Radial velocity VS Solute grashof number

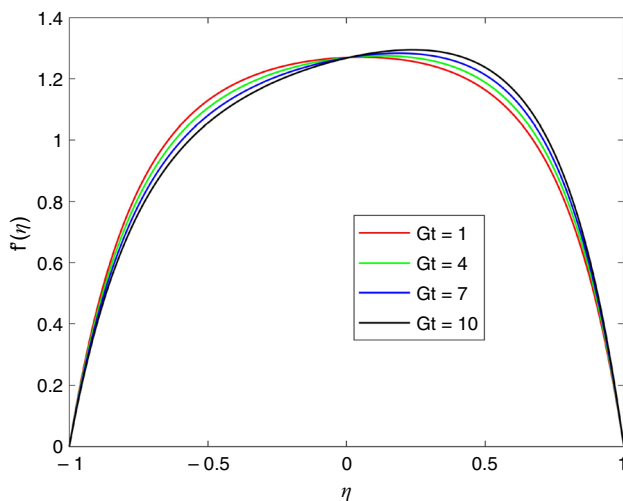


Fig. 11 Thermal Grashof number VS Radial velocity

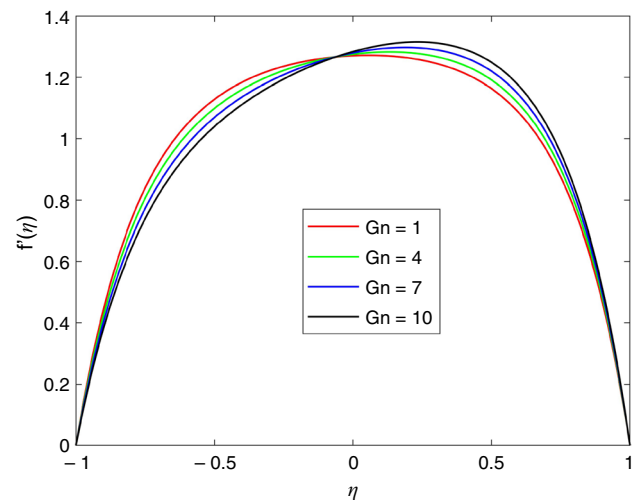


Fig. 13 Radial velocity VS Microorganism grashof number

determining temperature distribution. With the escalation of Peclet number, the heat transfer for both boundary layers increases. This leads to an enhanced thermal transfer process between the fluid and the surface. Figures 23 and 24 display the role of the Brownian motion as well as Thermophoresis indices on the temperature distribution. This leads to a more efficient thermal conduction process, thus enhancing the overall thermal efficacy of the system. Figure 25 illustrates the impact of the heat source number upon the temperature distribution. When the heat source strength enhances, the temperature distribution becomes higher. This increase occurs because a more substantial heat source triggers more heat at the surface, which consequently enhances the thermal energy transferred to the fluid. As shown in Fig. 26,

the temperature profile is influenced by the Prandtl number. When the Prandtl number increases, the temperature also elevates, indicating enhanced thermal diffusion relative to momentum diffusion, which leads to a steeper temperature gradient near the surface.

Concentration profile

Figure 27 finds the influence of the α parameter on the concentration distribution within fluid. With increasing values of α , there is a noticeable shift in concentration distribution, emphasizing the parameter's role in shaping the fluid's concentration dynamics. This movement of particles, also known as mass transport, is guided by concentration gradients within the flow. A higher expansion ratio results in

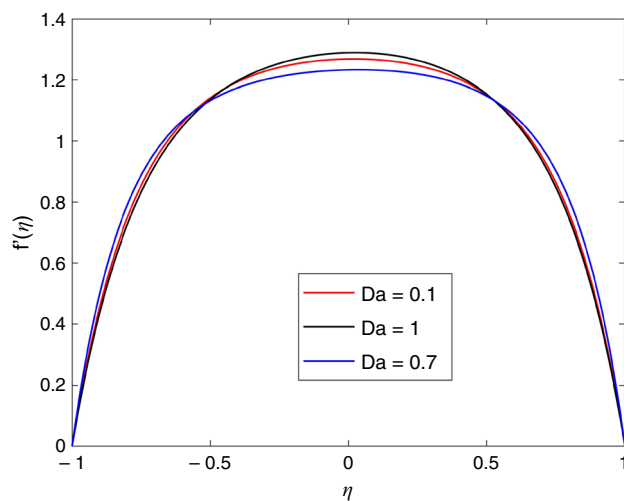


Fig. 14 Radial velocity VS Darcy number

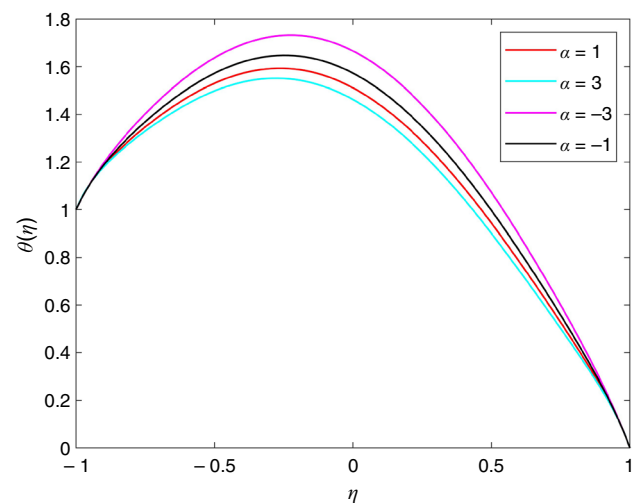


Fig. 16 Non-dimensional expansion ratio parameter impact in Temperature distribution

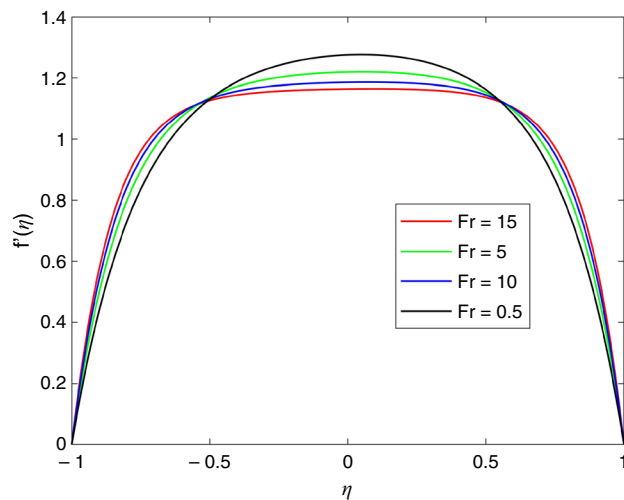


Fig. 15 Radial velocity VS Forchheimer number

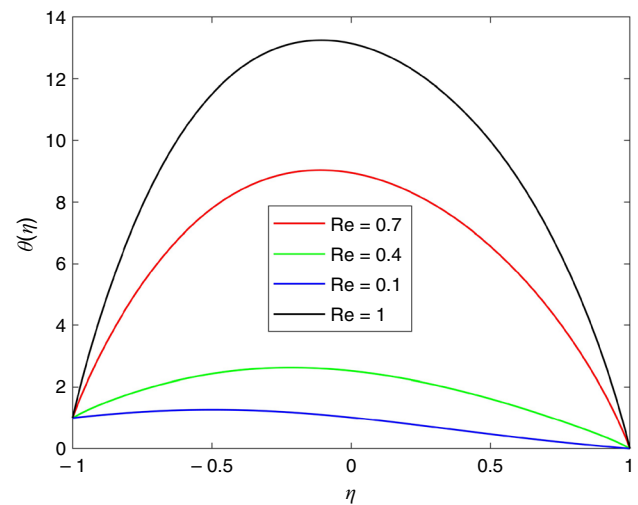


Fig. 17 Reynolds number impact in Temperature distribution

a narrower flow passage, reducing mass transfer near the upper disk. Meanwhile, the broader area in the lower disk facilitates more incredible particle migration and aggregation, enhancing material distribution within the fluid. The concentration profile's response to variations in Brownian motion number and Thermophoresis parameter is depicted in Figs. 28 and 29. As the value of the parameter N_b rises, particles concentration will decrease on upper disk and increase on the lower disk because the porous medium. As the Thermophoresis parameter increase, the mass transfer is decrease on upper disk because of thinner flow passage. Figure 30 shows activation energy on particle concentration. Activation energy provides the necessary energy threshold for chemical reactions, leading to higher concentration levels within the fluid. It acts as a potential barrier separating

different energy states. Indicating improved mass transport and reaction dynamics. Physically, larger activation energy values suppress the consumption rate of reactant species, allowing a greater amount of the species to remain within the boundary layer. Consequently, the concentration boundary layer thickens, resulting in enhanced species accumulation and diffusion near the porous disks. Figure 31 demonstrates the influence of the chemical reaction parameter on the concentration distribution. In a physical sense, the chemical reaction parameter plays a crucial role in regulating mass transfer and particle concentration within the flow field. Consequently, higher values of the chemical reaction parameter reduce species accumulation near the disk surfaces, thereby weakening concentration gradients

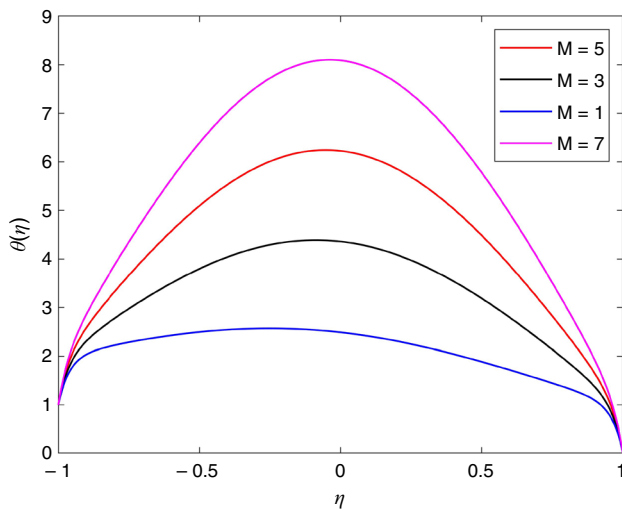


Fig. 18 Magnetic field parameter impact in temperature profile

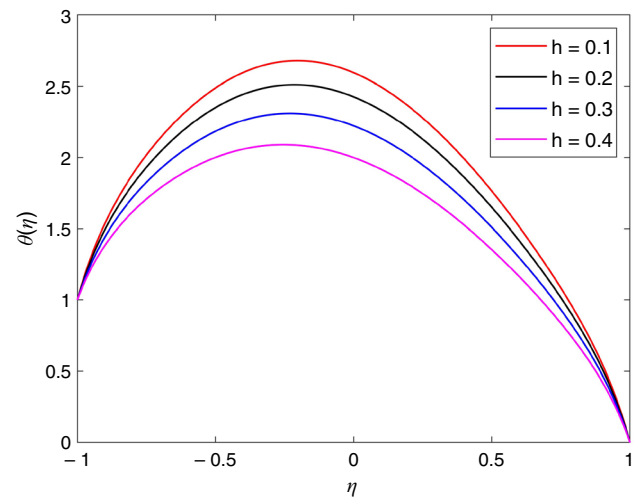


Fig. 20 Nanolayer effect in Temperature distribution

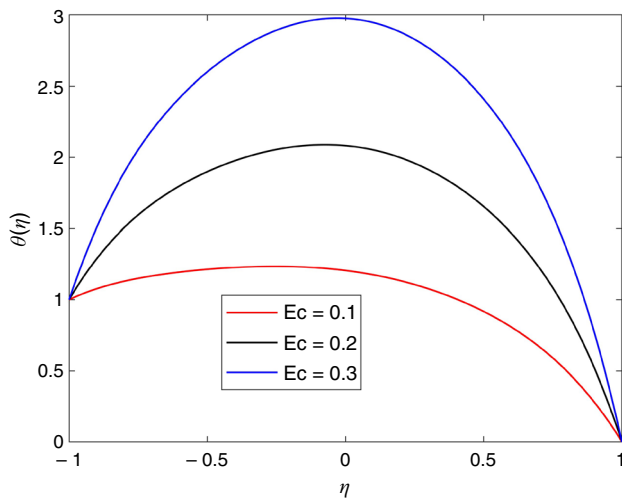


Fig. 19 Eckert number effect on Temperature

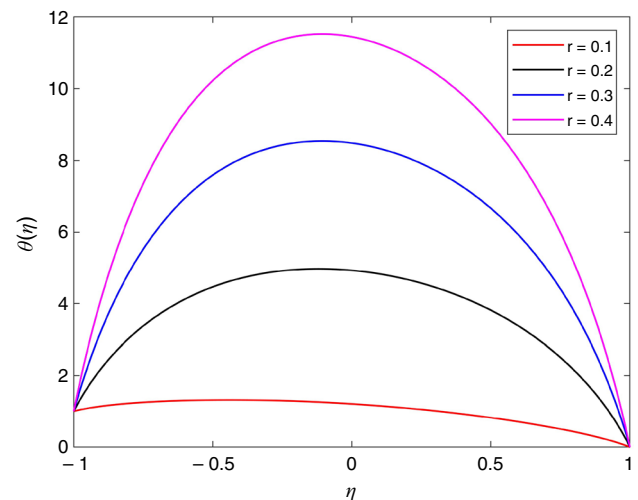


Fig. 21 Nanolayer effect impact on Temperature distribution

and limiting diffusive transport. Figure 32 demonstrates the impact of the Schmidt number over the concentration profile. The Schmidt number quantifies the ratio of momentum diffusion to mass diffusion. As the Schmidt number escalates, the mass transfer within the concentration boundary layer on both porous disks declines due to the predominance of momentum diffusion over mass diffusion. Consequently, this engenders a reduction in the concentration gradient along with a decrease in the concentration profile.

Motile microorganism

The influence of the expansion ratio parameter on the profile of motile microorganisms is illustrated in Fig. 33. An upward trend in the expansion ratio brings about a drop of the momentum layer thickness, leading to uniform momentum distribution. Specifically, an enhancement in the expansion ratio parameter gives rise to a decrease in the momentum layer thickness, which in turn facilitates a more uniform momentum distribution throughout the flow. Consequently, this phenomenon fosters a greater homogeneity in distribution of motile microorganisms throughout the fluid medium. The enhancement of the expansion ratio parameter results in a diminished momentum layer

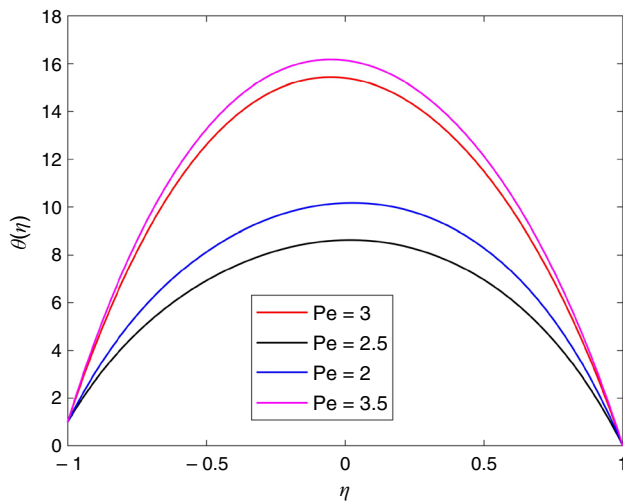


Fig. 22 Peclet number impact in Temperature distribution

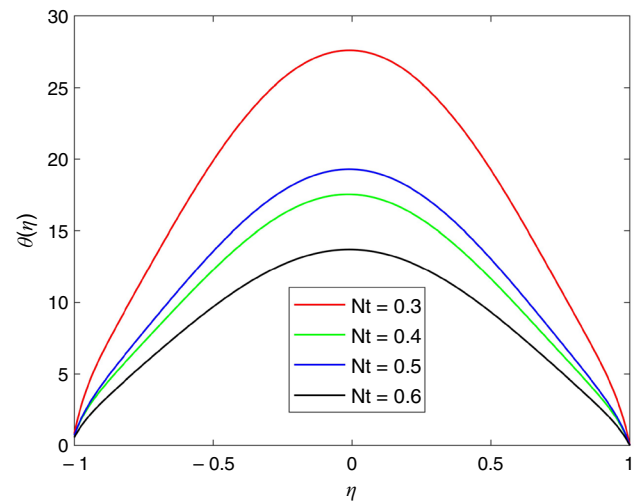


Fig. 24 Thermophoresis number effect on Temperature profile

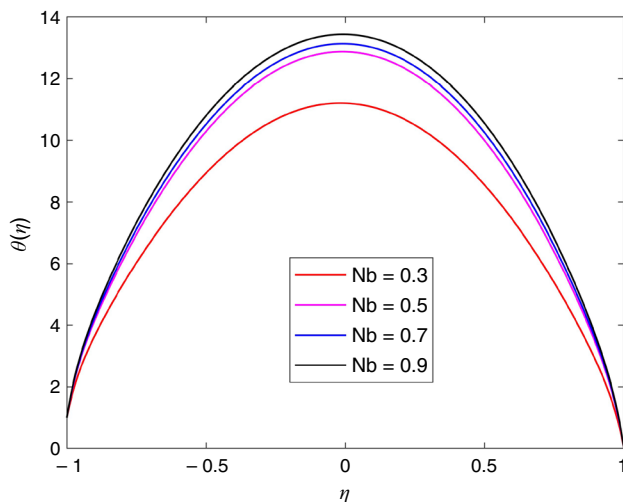


Fig. 23 Brownian motion number effect on Temperature profile

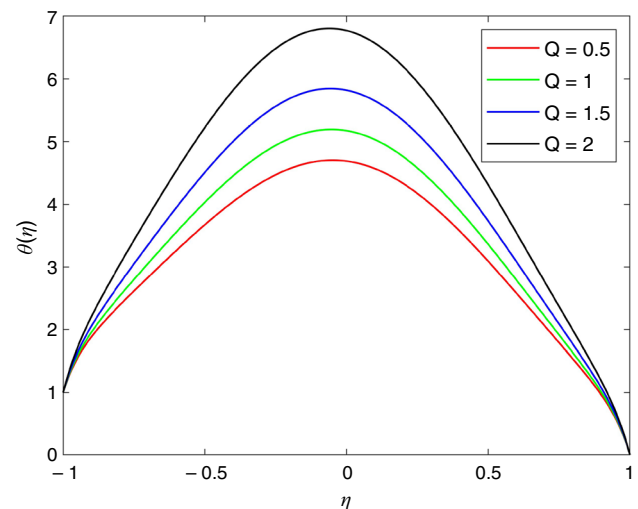


Fig. 25 Heat source number impact in Temperature profile

thickness, thereby facilitating a more uniform distribution of momentum throughout the flow. Consequently, this results in a more consistent dispersion of motile microorganisms within the fluid environment. Figure 34 demonstrates the effect of Re on the distribution of motile microorganisms. As the Reynolds number increases, the momentum boundary layer becomes thicker, enhancing the transport of motile particles within the fluid medium. The increase in the Reynolds number also enlarges the thermal, momentum, and motile microorganism boundary layers, thereby promoting faster fluid motion and more efficient particle mixing. Physically, Re allowing microorganisms to be convected more effectively by the fluid and resulting in a more uniform spatial distribution near the disks. Figure 37 presents the response of the microorganism

concentration profile to variations in the Schmidt number. Higher Schmidt numbers indicate lower mass diffusivity, which restricts the dispersion of motile microorganisms within the flow field. This behavior reduces the thickness of the microorganism boundary layer adjacent to both disks. Figure 35 presents the Peclet number's effect on motile microorganisms' behavior. As the Peclet number increases, the boundary layer becomes thicker. The augmented depth of the boundary interface leads to dominance over the viscous forces, influencing the overall flow characteristics. The impact of the density ratio parameter on the motile microorganism profile is shown in Fig. 36. A higher motile number density ratio leads to an increased thickness of the boundary layer surrounding motile microorganisms. It can impact the transport and diffusion of

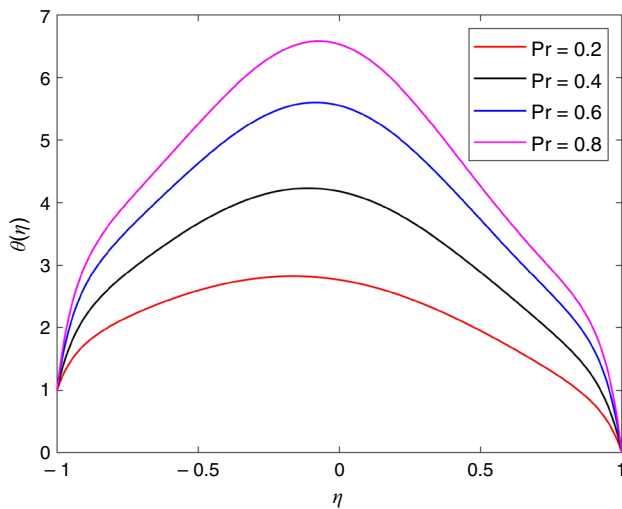


Fig. 26 Prandtl number effect on Temperature profile

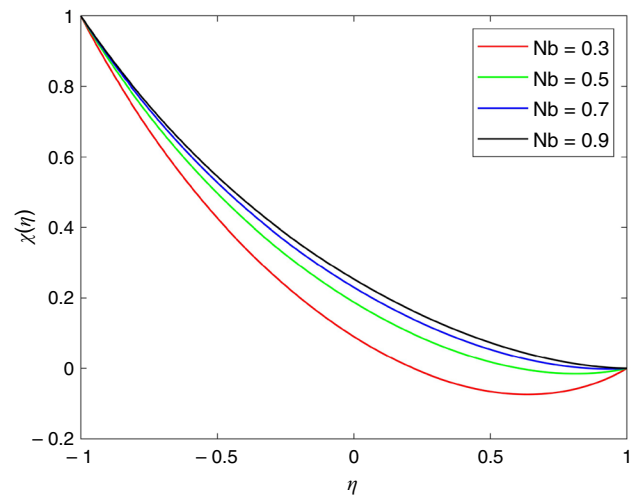


Fig. 28 Brownian motion number effect on Concentration

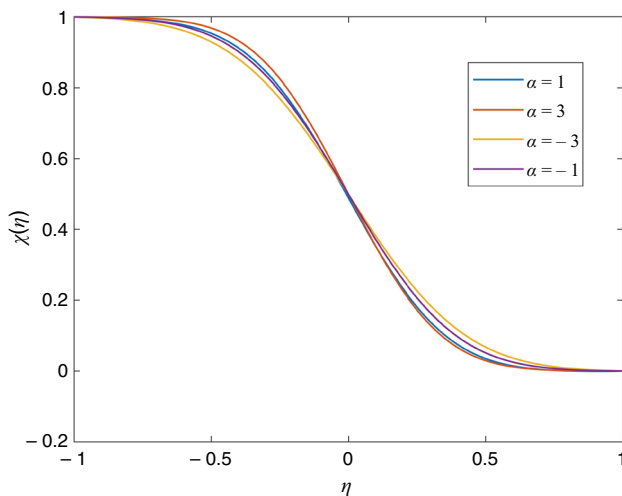


Fig. 27 Non-dimensional expansion ratio parameter impact in Concentration profile

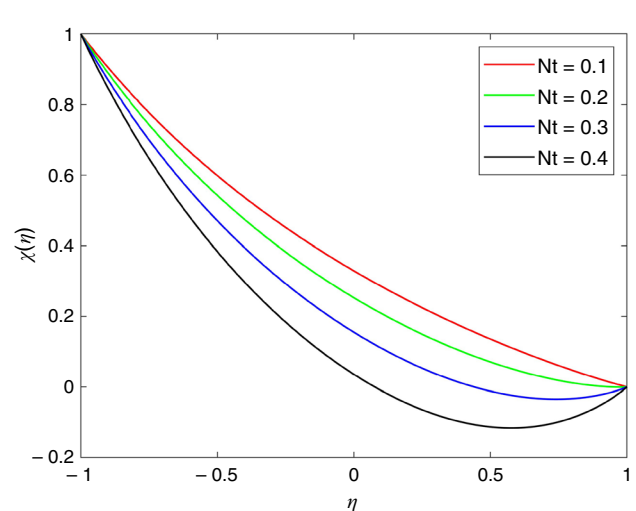


Fig. 29 Thermophoresis number effect on Concentration profile

surrounding substances. Figure 37 presents the response of the microorganism concentration profile to variations in the Schmidt number. Higher Schmidt numbers indicate lower mass diffusivity, which restricts the dispersion of motile microorganisms within the flow field. This reduces the thickness of the microorganism boundary layer adjacent to both disks.

Entropy generation

Figure 38 displays the interaction between the Eckert number and entropy generation. As Eckert's number enhanced, more heat transferred from the entropy boundary layer. This region has higher kinetic energy of the

fluid flow than enthalpy. Figure 39 shows how the Peclet number influences entropy generation. As the Peclet number grows, the upper boundary layer thickness of entropy generation reduces, but the lower boundary layer thickness enhances, with fluid motion becoming more significant than viscous forces. This leads to a more efficient thermal and mass transmission process, as the dominance of fluid motion enhances overall system performance. The impact of different diffusivity parameters on entropy generation, as shown in Figs. 40 and 41, reflects how variations in mass and heat diffusion rates influence the irreversibility of the fluid flow. Higher diffusivity leads to more uniform energy and particle distribution, reducing entropy generation, while lower diffusivity increases

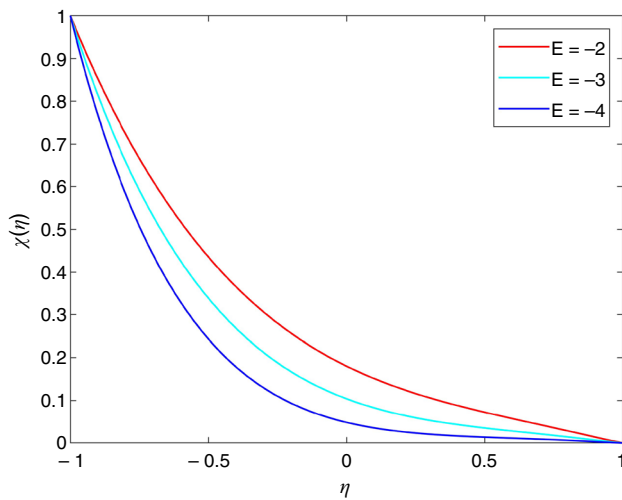


Fig. 30 Activation energy number VS Concentration

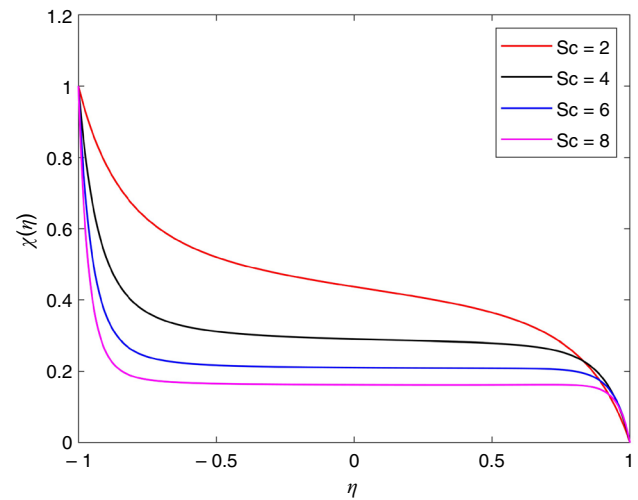


Fig. 32 Schmidt number effect on Concentration profile

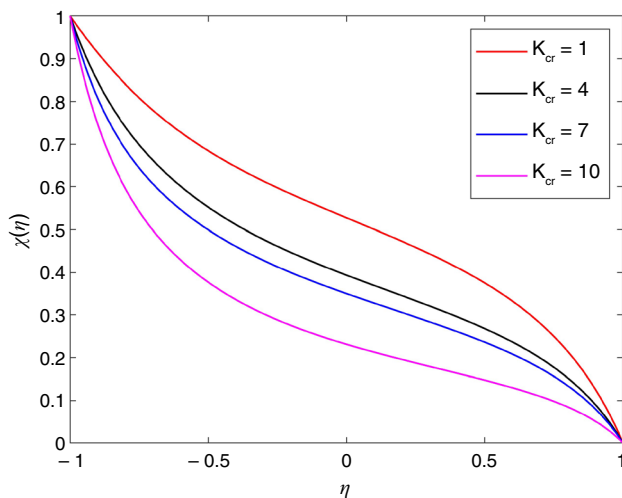


Fig. 31 Chemical reaction parameter impact in Concentration

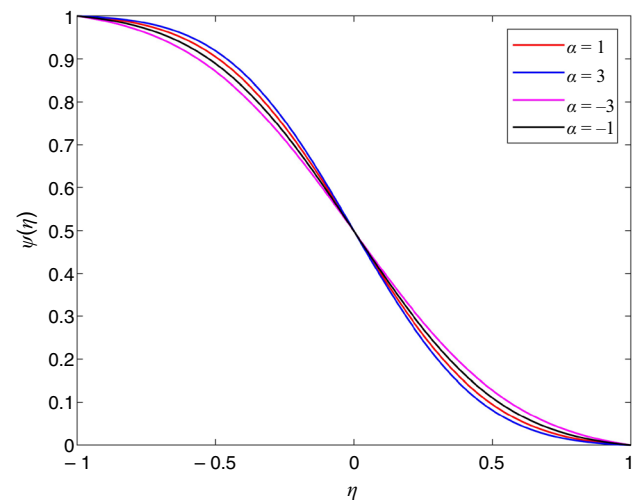


Fig. 33 Non-dimensional expansion ratio parameter impact in Micro-organisms distribution

energy dissipation and enhances entropy production. When diffusivity parameters enhance, the entropy generation levels increase. To better understand the role of various irreversibility mechanisms, we quantitatively assessed entropy generation arising from Joule heating, viscous dissipation, thermal conduction, mass diffusion, and microbial motion. Table 7 highlights how different mechanisms contribute to total entropy generation (N_s) under varying Eckert numbers (Ec) and magnetic parameters (M). As Ec increases, the contribution from viscous dissipation significantly rises, indicating that higher fluid friction enhances irreversibility. Conversely, thermal

conduction becomes less dominant at higher Ec , showing a shift toward kinetic-energy-driven entropy generation. Joule heating plays a more substantial role when M increases, as electromagnetic effects intensify entropy production. Additionally, contributions from solutal diffusion and microbial motion decrease with higher Ec and M , implying that molecular and biological effects become secondary under strong thermal and magnetic influences. These findings from Table 7 suggest that optimizing Ec and M is crucial for managing entropy sources and improving thermal system performance. Additionally, thermal conduction emerges as the primary source of entropy generation in regimes characterized by low Peclet and Eckert numbers, signifying that heat transfer is

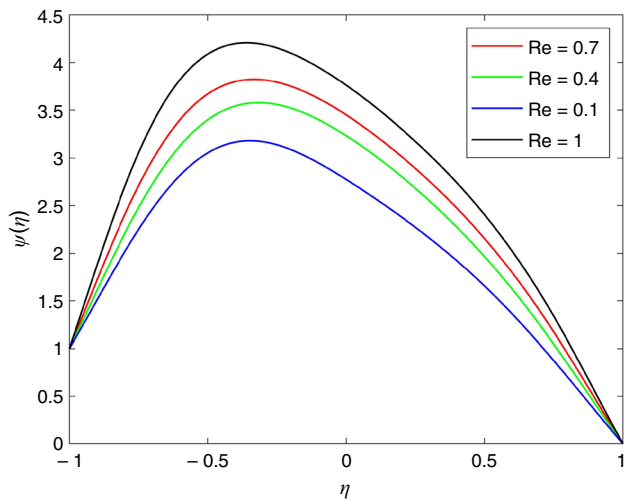


Fig. 34 Reynolds number impact in Motile microorganism distribution

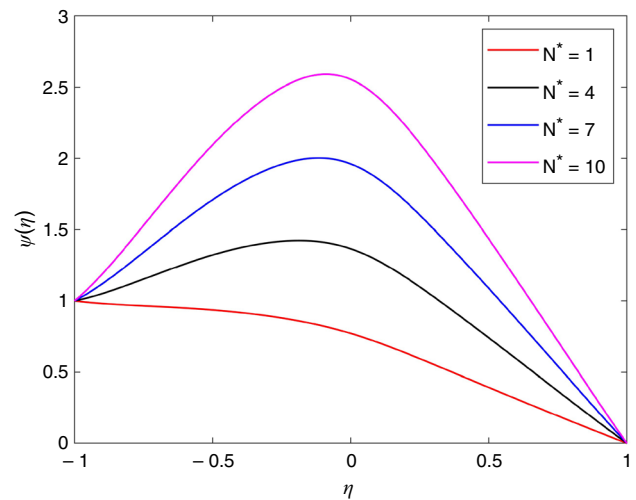


Fig. 36 Density ratio of motile effect on microorganism profile

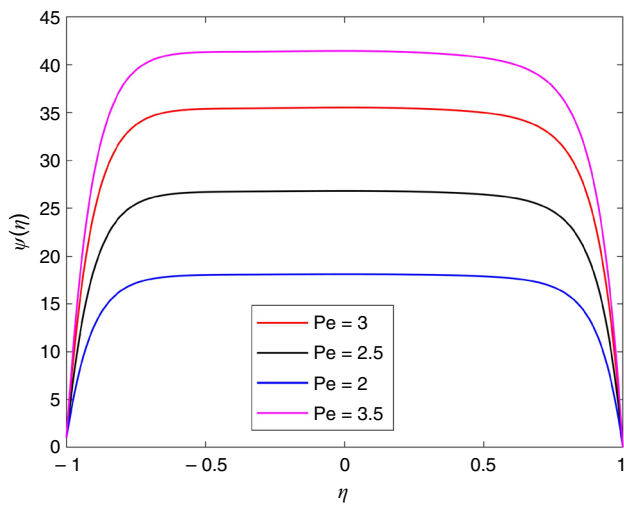


Fig. 35 Peclet number impact in Microorganism profile

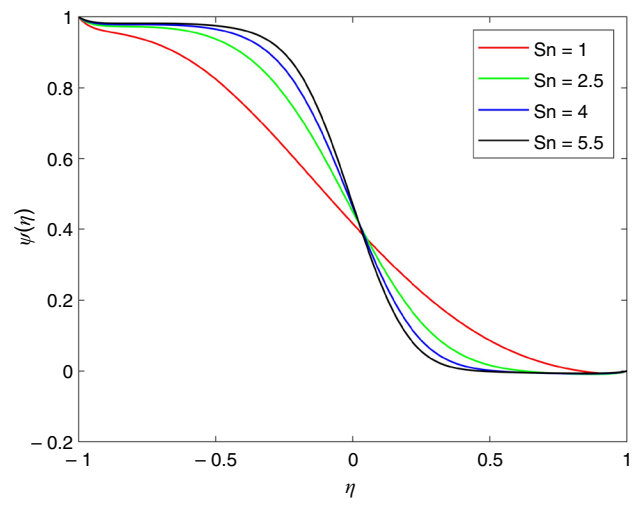


Fig. 37 Schmidt number of motile transfer impact in microorganism profile

predominantly conduction-driven under such conditions. As the activity of motile microorganisms increases, especially at higher motile density parameters, their influence on entropy generation becomes more significant due to

intensified bio-convective mixing. This reveals a trade-off between enhanced mixing efficiency and increased thermodynamic irreversibility, highlighting the importance of optimizing parameters in bio-thermal fluid systems.

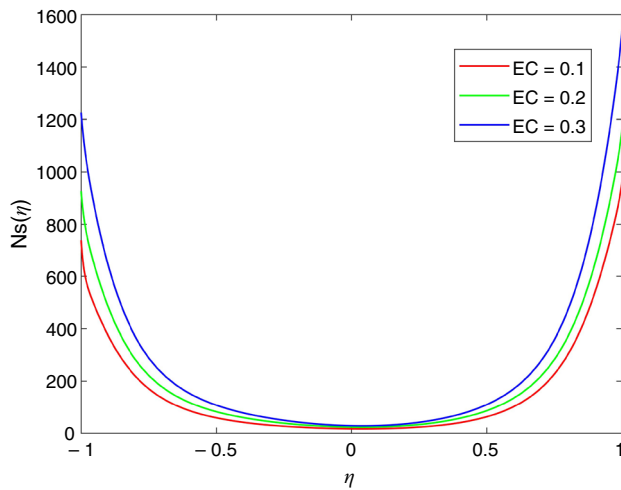


Fig. 38 Eckert number effect on Entropy generation

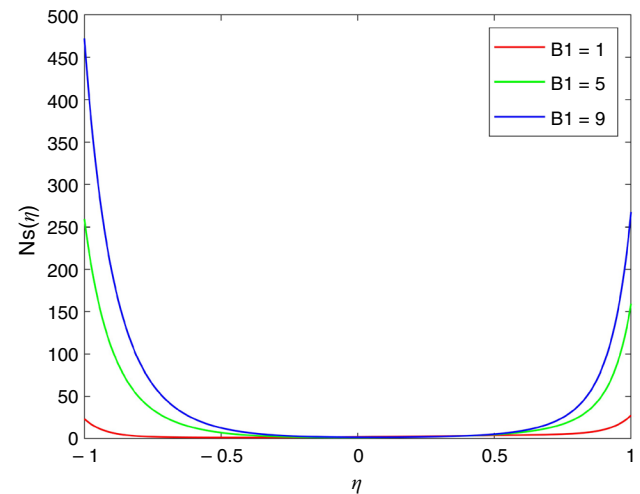


Fig. 40 Diffusivity parameter impact in entropy generation

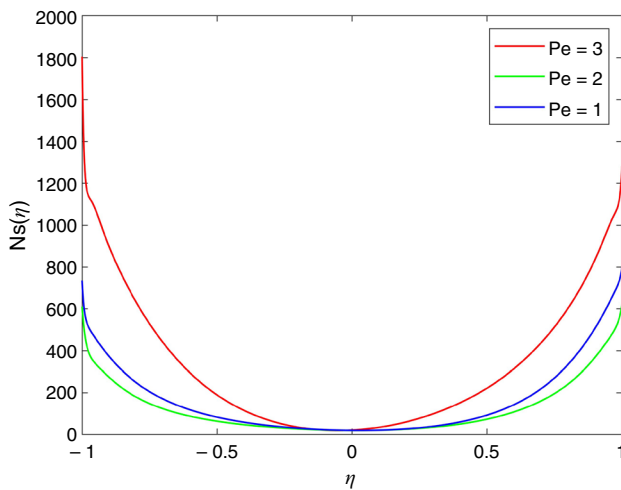


Fig. 39 Peclet number impact in Entropy generation profile

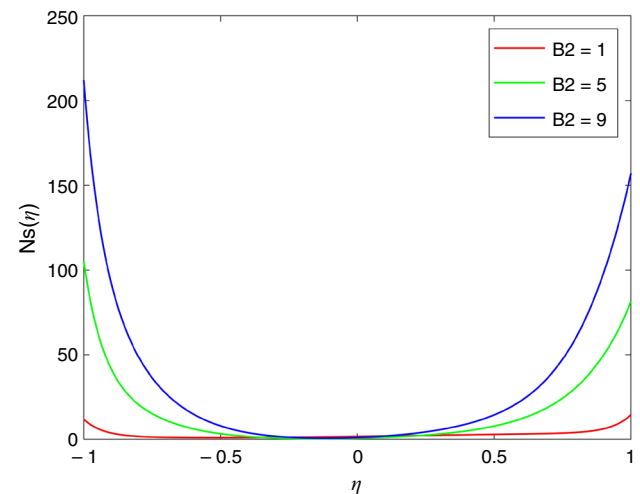


Fig. 41 Diffusivity parameter impact in entropy generation

Surface plot

This section highlights the significance of crucial engineering quantities, such as motile number, Nusselt number, Sherwood number, and skin friction, in heat transfer and fluid dynamics. It examines their relationships and how various physical parameters influence them. Figures 42 and 43 are located to determine the effect of the Solute graph of number Gc and Magnetic field parameter M against Skin friction C_f . Figures 44 and 45 illustrate the impact of the Ec and Pr on the Nusselt number (Nu). The Nu number is crucial in determining heat transfer, as it quantifies the rate of convective heat transfer relative to conduction. Higher values of Nu signify more

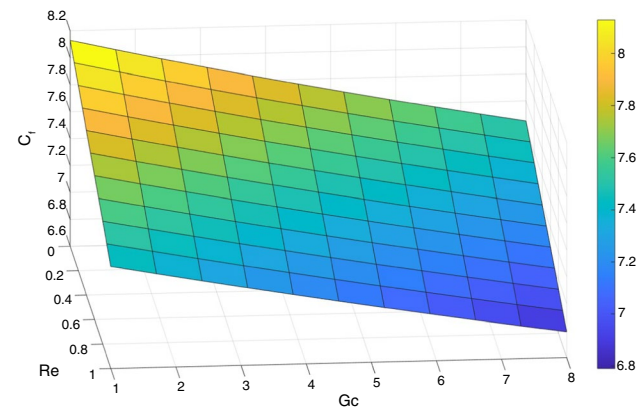


Fig. 42 Velocity surface plot for solutal Grashof number

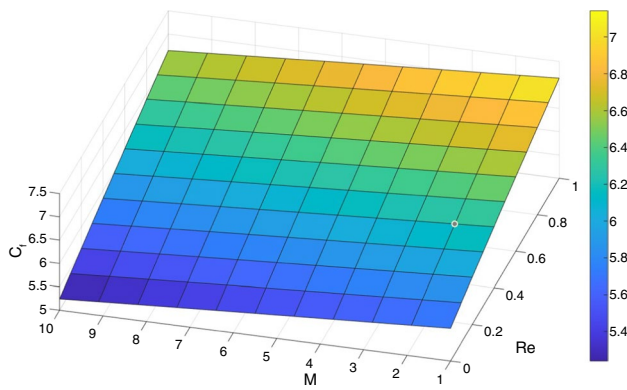


Fig. 43 Velocity surface plot for Magnetic number

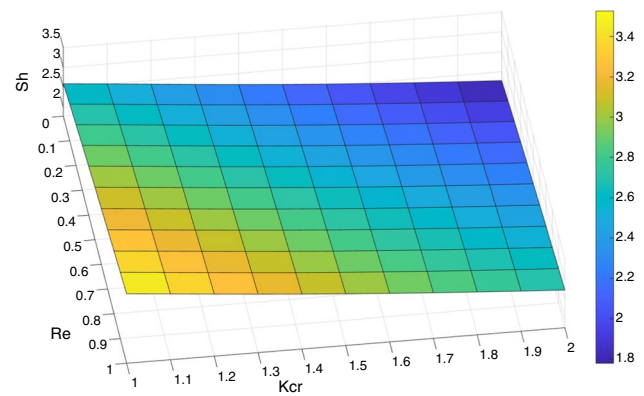


Fig. 46 Concentration surface plot for chemical reaction parameter

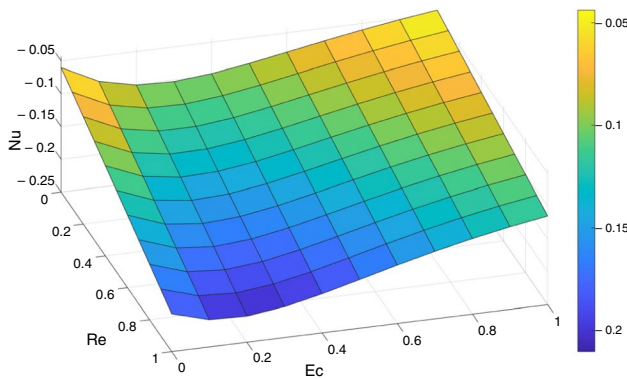


Fig. 44 Temperature surface plot for Eckert number

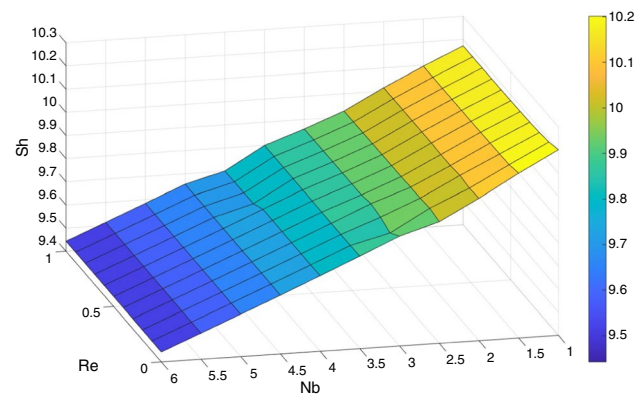


Fig. 47 Concentration surface plot for Brownian motion parameter

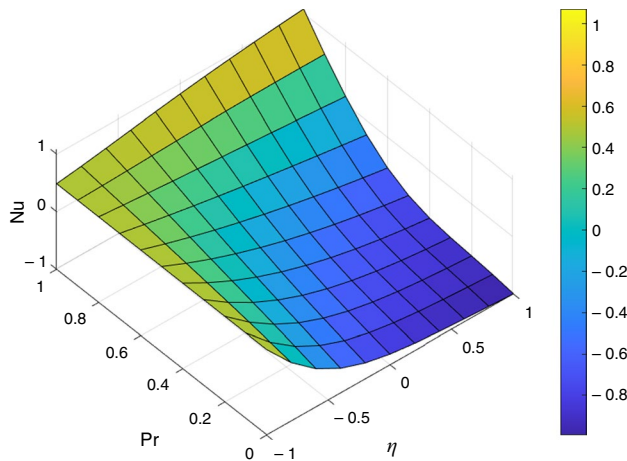


Fig. 45 Temperature surface plot for Prandtl number

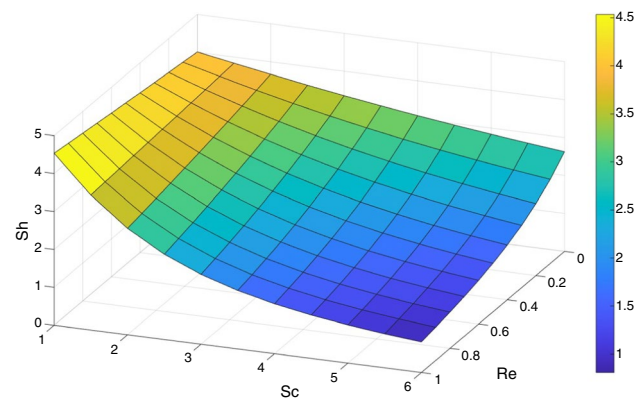


Fig. 48 Concentration surface plot for Schmidt number of mass transfer

efficient heat transfer. Figures 46–48 show the influence of the chemical reaction parameter (K_{cr}), Brownian motion parameter (N_b), and Schmidt number (Sc) on the Sherwood number (Sh), which characterizes the rate of mass transfer at the surface. The Sherwood number

increases as mass transfer is intensified, particularly in the presence of stronger Brownian motion or more pronounced chemical reactions. Physically, a larger Sherwood number signifies steeper concentration gradients

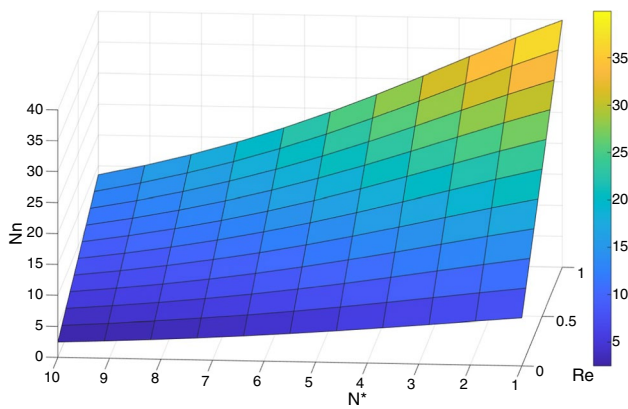


Fig. 49 Microorganism surface plot for density ratio of motile

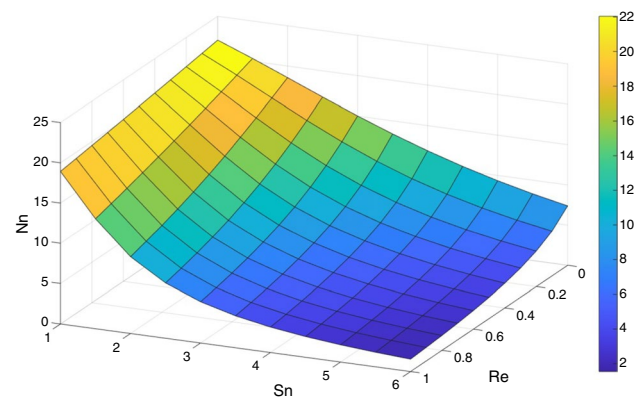


Fig. 51 Microorganism surface plot for Schmidt number of motile transfer

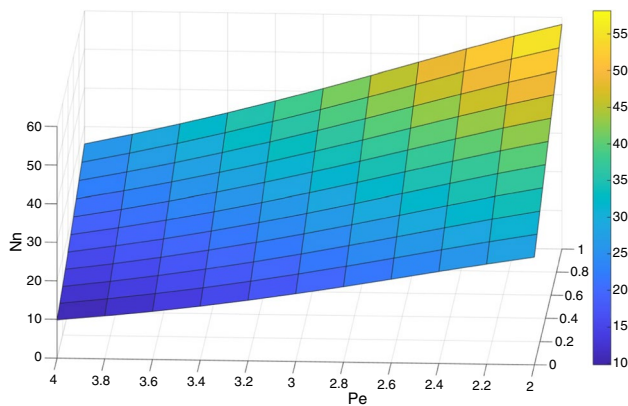


Fig. 50 Microorganism surface plot for Peclet number

near the disk surface, indicating more efficient species transport from the boundary layer into the surrounding fluid. Finally, Figs. 49–51 reveal how the motile number (N), Pe , and Schmidt motile transfer number affect the motile number (Nn), which governs the movement and

dispersion of motile microorganisms in the fluid. These results provide a comprehensive view of how various fluid dynamic factors influence thermal and mass transport processes. These findings offer a thorough understanding of how different fluid dynamic factors impact thermal and mass transport processes, providing essential insights for optimizing engineering systems. Recognizing these interrelationships can accelerate the creation of highly efficient thermal and fluid management strategies in real-world applications. The insights gained from these parameters are crucial for enhancing the design and management of thermal-fluid systems, particularly in contexts involving bio-convective flows, nanofluids, and reactive porous materials. Furthermore, recognizing the primary parameters allows for targeted adjustments to the system, improving energy efficiency and transport effectiveness in both microscale and industrial applications.

Table 8 Nomenclature

Symbol	Description with S.I. units	Symbol	Description with S.I. units
(r, z)	Cylindrical coordinates system [M]	Ec	Eckert number
(u, v, w)	Velocity component along z axis [MT^{-1}]	Pr	Prandtl number
p	Pressure [$ML^{-1}T^{-2}$]	Q	Heat generation parameter
B_0	Uniform magnetic field [$MT^{-1}I^{-1}$]	N_t	Thermophoresis parameter
g	Gravitational acceleration [LT^{-2}]	N_b	Brownian motion parameter
T	Temperature [Θ]	K_{cr}	Chemical reaction parameter
C	Concentration [Here $C=1$]	E	Activation energy parameter
N	Motile Microorganism [L^{-3}]	Pe	Peclet number
k^*	Permeability of porous medium [L^2]	Sc	Schmidt number of mass transfer
F_0	Porous medium coefficient	Sn	Schmidt number of motile transfer
Q_0	Heat source [$ML^{-1}T^{-3}$]	N_s	Entropy generation
m_1	Exponential index	C_f	Skin friction number
D_T	Thermophoretic diffusion coefficient [L^2T^{-1}]	Nu	Nusselt number
D_b	Brownian diffusion coefficient [L^2T^{-1}]	Sh	Sherwood number
K_0	Chemical reaction constant	Nn	Motile number
m	Fitting constant		
E_a	Activation energy [ML^2T^{-2}]	Greek Symbols	
k_b	Boltzmann constant [$ML^2T^{-2}\Theta$]	ρ	Density [ML^{-3}]
W_c	Swimming velocity of cells [LT^{-1}]	η	Independent similarity variable
D_n	Diffusivity of microorganism [L^2T^{-1}]	$\theta(\eta)$	Temperature profile
C_p	Specific heat [$L^2T^{-1}\Theta$]	$\chi(\eta)$	Concentration profile
K	thermal conductivity [$MLT^{-3}\Theta$]	$\psi(\eta)$	Microorganism profile
h	Nanolayer thickness [L]	σ_e	Electrical conductivity [$M^{-1}L^{-3}T^3I^2$]
r	Radius of nanoparticle [L]	ϕ	Volume fraction
dp	Diameter of nanoparticles [L]	μ	Dynamic viscosity [$ML^{-1}T^{-1}$]
$s(t)$	Time-dependent coefficient [L]	ν	Kinematic Viscosity [L^2T^{-1}]
A_1	Dimensionless parameter	β_T	Volumetric thermal coefficient [Θ]
N^*	Ratio of motile microorganism	β_C	Volumetric concentration coefficient
F_η	Dimensionless Radial velocity	β_N	Volumetric microorganism coefficient [N^{-1}]
G_η	Dimensionless tangential velocity		
M	Magnetic number	Subscripts	
Re	Reynolds number	s	Solid
G_T	Thermal Grashof number	f	Fluid
G_C	Solute Grashof number	bf	Base fluid
G_N	Microorganism Grashof number	nf	Nanofluid
Da	Darcy number	hnf	Hybrid nanofluid
Fr	Forchheimer number	trihnf	Tri-hybrid nanofluid

Conclusion

In this paper, we have studied the THNF, which is a mixture of copper (Cu), aluminum oxide (Al_2O_3), and titanium dioxide (TiO_2) with water (H_2O) as a base fluid time-dependent, 3-D flow for the combined effect of highly porous medium, Joule heating, thermophoresis, chemical reaction, Brownian motion, magnetic field, exponential heat source/sink, activation energy, and viscous dissipation through orthogonally moving disks are examined. Velocity, temperature, concentration, motile

micro-organisms and energy generation, Nusselt number, motile number, Sherwood number, skin friction coefficient, and mass and thermal transfer rates are discussed and used the thermo-physical properties of tri-hybrid nanofluid to apply thermal and mass transfer equation and examine thermal conductivity and analyzer thermal conductivity of the model.

- A increase in the Re , Fr and Da leads to a lower fluid radial velocity. Initially, it increases, but later decreases

for improved G_T , G_C , and G_N . Radial velocity reflects contrast behavior for enhancing magnetic numbers.

- Tangential velocity shows rise in domain $[-1,0)$ and decline in domain $(0,1]$ with increase in α , whereas it shows opposite behavior for Re .
- The temperature increases for enhancing Ec , M , Q , N_b , Pe , Re and Pr , whereas decreases for rising N_t .
- The mass transfer reduces as N_t , Kcr , and Sc increase. At the same time, it becomes more pronounced with the rise in the E as well as Nb .
- Motile microorganisms increase when the density ratio N^* , Pe , Re and Sn of motile transfer are enhanced.
- The entropy improves when enhancing the Re , G_C , Pr , and Kcr , while it decreases when increasing M due to suppression of thermal gradients by the magnetic field..
- Surface plots resembling characteristics of engineering parameters such as C_f , Sh , Nu and Nn .

Model limitations and future work

Although the present model incorporates many physical effects, certain assumptions limit its general applicability. The analysis assumes a laminar, incompressible, and Newtonian base fluid, neglecting turbulence and non-Newtonian behavior that may arise in practical applications. Moreover, thermal radiation and nanoparticle aggregation effects have not been explicitly included. Future studies may extend this model to turbulent regimes, consider temperature-dependent viscosity and thermal conductivity, and employ machine learning-based surrogate models to enhance predictive accuracy and computational efficiency. Although the present model incorporates many physical effects, certain assumptions limit its general applicability. The analysis assumes a laminar, incompressible, and Newtonian base fluid, neglecting turbulence and non-Newtonian behavior that may arise in practical applications. Moreover, thermal radiation and nanoparticle aggregation effects have not been explicitly included. Future studies may extend this model to turbulent regimes, consider temperature-dependent viscosity and thermal conductivity, and employ machine learning-based surrogate models to enhance predictive accuracy and computational efficiency. Experimental validation is also recommended to confirm the numerical findings and guide practical implementation in industrial and biomedical systems. Experimental validation is also recommended to verify the numerical findings and guide practical implementation in industrial and biomedical systems. The complete nomenclature of the symbols employed in this work is shown in Table 8.

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Data Availability The authors affirm that all data supporting the conclusions of this study are comprehensively presented within the article.

Declarations

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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