

Thermal Performance Analysis of Magnetohydrodynamics with Carbon Nanotubes on a Stretching/Shrinking Porous Sheet

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Received 6 September 2025

Accepted 14 January 2026

Published 19 February 2026

Objectives: Carbon nanotubes (CNTs) can enhance heat transfer due to their superior thermal properties. Thermal performance of magnetohydrodynamics (MHD) with CNTs is increasingly important to improve energy efficiency of nanotechnology in a vast range of industrial, pharmaceutical and energy conversion. In this study, we investigate the effects of inclined MHD and slip on viscoelastic CNT flow with radiation and heat source/sink over a stretching sheet embedded in porous media. The suction/injection through the porous medium is considered. **Methods:** The governing PDEs describing the flow were converted into a system of nonlinear ordinary differential equations through the application of a similarity transformation and then solved analytically. The influences of the Hartmann number, inverse Darcy number, viscoelastic, mass suction/injection effect, radiation parameter and heat source/sink parameters on velocity and temperature profiles are graphically presented and thoroughly discussed. **Findings:** The results show that the fluid flow increases as the Hartmann number and inverse Darcy number increase, whereas reverse effects are observed in the viscoelastic parameter and solid volume fraction. At the larger inversed Darcy number and the Hartmann number, the axial velocity for single-wall carbon nanotubes (SWCNTs) becomes larger than that of multiwall carbon nanotubes (MWCNTs). Also, the temperature profile increases as the values of the

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thermal radiation, heat sources, and the Hartmann number increase. The temperature of the nanofluids (NFs) with SWCNT is larger than that with MWCNT when the thermal radiation and heat sources are the same. This shows that the SWCNT NFs can improve thermal performance better. The produced velocities of NFs with MWCNT are higher than those with SWCNT, leading to lower heat convection. *Applications:* CNTs can enhance heat transfer due to their superior thermal properties. The thermal performance of MHD with CNTs is increasingly important to improve energy efficiency of nanotechnology in a vast range of industrial, pharmaceutical and energy conversion.

Keywords: Thermal radiation; MHD; heat source/sink; nanofluids; carbon tube; porous medium.

Nomenclature

Symbol	Description	S. I. Unit
a	Constant	(-)
b	Constant	(-)
Bi	Biot number	(-)
B_0	Magnetic flux	(Tesla)
c	Constant	(-)
c_1, c_2, c_3, c_4, c_5	Dummy variables	(-)
C_P	Specific heat capacitance	(Jkg ⁻¹ K)
d	Stretching/shrinking sheet parameter	(-)
Da^{-1}	Inverse Darcy number	(-)
K	Permeability	(NA ⁻¹)
k^*	Mean absorption	(c/m)
l	Velocity slip	(m/s)
L	First order slip	(m/s)
M	Hartmann number	(-)
N_r	Radiation parameter	(-)
N_I	Heat source/sink	(Joule)
Pr	Prandtl number	(-)
q_r	Radiative heat flux	(Wm ⁻²)
$\vec{q}$	Velocity vector	(m/s)
T	Fluid temperature	(K)
T_w	Wall temperature	(K)
T_∞	Far temperature	(K)
(u, v, w)	Fluid velocity in (x, y, z) directions	(m/s)
(x, y, z)	Coordinates	(m)
V_C	Mass suction/injection parameter	(-)

Greek Symbols

$\alpha_1 \alpha_1$	Material constants	(-)
β	Viscoelastic parameter	(-)
μ_f	Dynamic viscosity of fluid	(Ns/m ²)
ρ_f	Density of the fluid	(Kg/m ³)
σ_f	Electrical conductivity of fluid	(S/m)
κ_f	Thermal conductivity of fluid	(W/mK)

ν_f	Kinematic viscosity of fluid	(m ² /s)
σ^*	Stefan–Boltzmann constant	(-)
Γ	Porosity	(-)
τ	Inclined angle	(Radian)
η	Similarity variable	(-)
ξ	Substitution variable	(-)

Abbreviations

NF	Nanofluids	(-)
CNTs	Carbon nanotubes	(-)
MHD	Magnetohydrodynamics	(-)
PDEs	Partial differential equations	(-)
ODEs	Ordinary differential equations	(-)

1. Introduction

Magnetohydrodynamic (MHD) of nanofluids (NFs) with carbon nanotubes (CNTs) on a porous stretching or shrinking sheet is of importance in a vast range of equipment and devices for the capacity of their high energy conversion and heat transfer in industrial, pharmaceutical and metallurgical processes.^{1–3} The presence of magnetic fields could produce a drag force called the Lorentz force on magnetic NFs. This drag force can regulate the fluid to move more slowly, raising its temperature as well as concentration. Thus, magnetic NFs have advantages over conventional fluids because their performance can be regulated using a magnetic field.

A number of studies have been performed to evaluate the MHD effect on fluid flow.^{4–9} For example, Reddappa *et al.*¹⁰ investigated the 2D flow of MHD Jeffery nanoparticle through a penetrable stretching surface. Munivenkatappa *et al.*¹¹ examined numerically performed MHD flow through a permeable stretching surface with convective boundaries. Gupta *et al.*¹² performed a numerical solution of magnetized GP-MoS₂/C₂H₆O₂-H₂O unsteady flow over a stretching surface. Furthermore, the thermal radiation and heat energy were elevated for the fluid flow temperature. Sulochana *et al.*¹³ evaluated the role of radiation on MHD nanoparticle flow through an elongated sheet. They obtained dual-nature solutions for both Newtonian and non-Newtonian cases. Sharma *et al.*¹⁴ studied the slip flow of MHD hybrid NF through an exponentially stretching/shrinking sheet with radiation effect. They discovered that the local Nusselt number is decreased by the accumulation of the

thermal slip effect. The radiation influence on the boundary layer flow of non-Newtonian fluids due to stretching/shrinking sheet has been portrayed by several researchers.^{15,16} Jha and Samalla¹⁷ analyzed the effects of MHD and heat source/sink on NF flow movement through a channel. Mishra and Samantara¹⁸ performed numerical analysis of the dusty fluid flow through a uniform stretching sheet in the presence of radiation and nonuniform heat source/sinks. Viscoelastic behavior and characteristics were investigated by Gizachew and Shankar.¹⁹ Das²⁰ numerically examined MHD of viscoelastic fluid flow on a circular stretching surface. Vishalakshi *et al.*²¹ analyzed MHD of non-Newtonian fluid flow through a permeable elongated/contracting sheet in the existence of thermal radiation. Recently, several researchers^{22,23} have examined various performances of viscoelastic fluid flow.

The CNTs often consist of carbon allotropes in a tube-shaped material with a cylindrical nanostructure. They have remarkable features of their superior thermal conductivities, suspensions, and are of a particular application in material sciences and nanotechnology, such as chemical manufacturing, cooling of microelectronics, optics and cancer treatment. Depending on the structure and number of graphene sheets, the CNTs are classified into two types: single-wall carbon nanotubes (SWCNTs) and multiwall carbon nanotubes (MWCNTs). SWCNTs are made of a graphene layer seamlessly rolled into a tube.²⁴ In contrast, a concentric arrangement of MWCNTs consists of multiple layers of graphene seamlessly rolled into a concentric tubular structure. Many researchers presented the advantages of the NF flow with CNTs

under various boundary layer problems.^{25–29} Yamada-Ota and Xue models have been extended to hybrid NFs on converging boundary layers or a stretching sheet, magneto-convective flow of hybrid NFs with CNT suspensions over an isothermal rotating plate, Darcy's law, peristaltic flow of CNT-based hybrid NF in the annular region of eccentric cylinders, and between two permeable disks with suspension of CNTs.^{30–34} Haq *et al.*³⁵ explored the thermophysical properties of CNT nanoparticles on a stretching surface. They showed that engine oil-based CNT has higher skin friction and heat transfer rate when compared to water and ethylene glycol-based CNT. Hayat *et al.*³⁶ reported the Darcy–Forchheimer flow of water-based SWCNT and MWCNT over a rotating disc, which was heated convectively. They found that larger values of nanoparticle volume fraction could increase the skin friction coefficients and local Nusselt number. Kandasamy *et al.*³⁷ studied the effects of MHD and thermal radiation on SWCNT nanoparticle movement through a penetrable wedge. They found that the thermal boundary layer thickness of water-based SWCNT is larger than that of copper-based NF. Shah *et al.*³⁸ illustrated the action of radiation on the MHD rotating movement of CNTs past a stretching sheet and they found that temperature enhancement is provided by thermal radiation. Mahanthesh *et al.*³⁹ compared the velocities and temperature of Marangoni convective MHD flow of SWCNT and MWCNT nanoflows on a rotating disk with solar radiation and an irregular heat source. The velocity is slightly greater in case of MWCNT-water nanoliquid than SWCNT-water nanoliquid, but the Nusselt number of SWCNT-water nanoliquid is greater than that of MWCNT-water nanoliquid. Recently, they studied the effects of cross-diffusion, heat source and magnetic field on the slip flow on a rotating deformable disk. Their numerical solutions showed that MWCNT-nanoliquid velocities are higher than SWCNT-nanoliquid. The temperature field for SWCNT-nanoliquid is higher than that of MWCNT-nanoliquid. However, there is acknowledge gap in how CNT types in MHD affect thermal and flow performance of MHD over a permeable stretching/shrinking square sheet. The effects of square stretching/shrinking porous sheet on 3D MHD nanoflow with viscoelastic CNTs are of importance in improving thermal efficiency of many applications, such as drug delivery and electronic devices.^{32,33,40–43} Although both NF with CNTs and NFs on permeable stretching sheets have

been studied, they are separated. To the best of our knowledge, non-Newtonian MHD fluid flow with CNTs on a permeable stretching/shrinking surface has been explored.

A review of the currently available literature sheds light on research gaps and inadequate research. As far as the author's knowledge, there has been no study on 3D viscoelastic CNTs flowing past a permeable stretching/shrinking sheet while considering the inclined MHD, radiation, heat source/sink, suction/injection, slip condition and mass suction/injection effects into account. To fill this gap, the novelty of the current research is to analytically examine the effect of radiation, heat source/sink, slip condition, and mass suction/injection effects on 3D inclined MHD flow of viscoelastic CNTs over a stretching/shrinking sheet. Then, with the help of similarity variables, the given PDEs are turned into ODEs. Finally, the ODEs will be solved analytically. An analytical solution has advantages over numerical solutions because the effects of physical parameters can be easily implemented. Furthermore, we plotted graphs to show the effect of pertinent parameters on the action of the variables of state, such as momentum and temperature profiles, and then we reviewed them physically.

2. Mathematical Analysis

The magnetohydrodynamics of the non-Newtonian fluid flow and heat transfer over a porous medium with CNTs is investigated on a porous stretching/shrinking sheet with a heat source/sink in Cartesian coordinates (Fig. 1). The non-Newtonian fluids with porous CNTs are on a porous stretching/shrinking sheet. A magnetic field is applied to the non-Newtonian fluids at an inclined angle, τ .

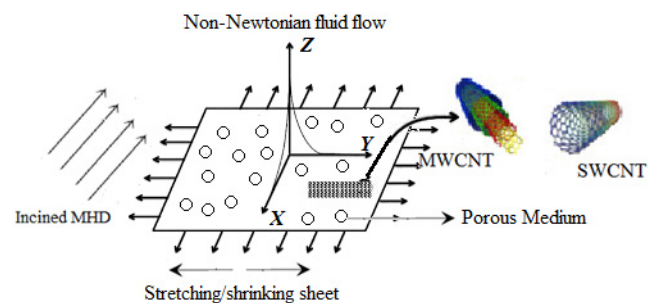


Fig. 1. The configuration of the flow field on stretching/shrinking sheet.

2.1. Mass and energy conservation

2.1.1. Mass conservation⁴⁴

$$\nabla \vec{q} = 0. \quad (1)$$

2.1.2. Momentum conservation

Equation for the momentum conservation is defined as follows⁴⁴:

$$(\vec{q}_t + (\vec{q} \cdot \nabla) \vec{q}) = \nu_{nf} \nabla^2 \vec{q} - \frac{\mu_{nf}}{K \rho_{nf}} \vec{q} - \frac{\sigma_{nf} B_0^2}{\rho_{nf}} \sin^2(\tau) \vec{q} + \nabla \delta + \gamma_0 \vec{q} \quad (2)$$

where δ is the Cauchy stress tensor. t is time, μ_{nf} , ρ_{nf} and σ_{nf} are the dynamic viscosity, density and electrical conductivity of the CNTs. For second-grade fluid, δ is defined as follows⁴⁴:

$$\delta = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 \quad (3)$$

where A_1 is the first order Rivlin–Ericksen tensor, and A_2 is the second-order Rivlin–Ericksen tensor. Moreover α_1 and α_2 are the material constants for viscoelastic fluid satisfying the following constraints:

$$\alpha_1 \geq 0, \alpha_1 + \alpha_2 = 0.$$

For the current fluid problem, the velocity vector is defined as follows⁴⁴:

$$\vec{q} = [u(x, y, z), v(x, y, z), w(x, y, z)] \quad (4)$$

2.1.3. Heat conservation

The heat equation in the presence of radiation and heat source/sink is as follows:

$$\frac{\partial T}{\partial t} + \vec{q} \cdot \nabla T = \frac{\kappa_{nf}}{(\rho C_P)_{nf}} \nabla^2 T - \frac{1}{(\rho C_P)_{nf}} \frac{\partial q_r}{\partial y} + \frac{Q_0(T - T_\infty)}{(\rho C_P)_{nf}}. \quad (5)$$

The following is a possible formulation for the boundary layer equations^{45,46} in Cartesian coordinate system:

$$\left. \begin{aligned} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} &= 0, \\ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} &= \nu_{nf} \frac{\partial^2 u}{\partial z^2} + \gamma_0 u \end{aligned} \right\} \quad (6)$$

$$\left. \begin{aligned} & - \left(\frac{\mu_{nf}}{K \rho_{nf}} + \frac{\sigma_{nf} B_0^2}{\rho_{nf}} \sin^2(\tau) \right) u \\ & + k \left\{ u \frac{\partial^2 u}{\partial x \partial z^2} + w \frac{\partial^3 u}{\partial z^3} \right. \\ & - \left(\frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial z^2} + \frac{\partial u}{\partial z} \frac{\partial^2 w}{\partial z^2} \right. \\ & \left. \left. + 2 \frac{\partial u}{\partial z} \frac{\partial^2 u}{\partial x \partial z} + 2 \frac{\partial w}{\partial z} \frac{\partial^2 u}{\partial z^2} \right) \right\} \end{aligned} \right\} \quad (7)$$

$$\left. \begin{aligned} & u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \\ & = \nu_{nf} \frac{\partial^2 v}{\partial z^2} + \gamma_0 v \\ & - \left(\frac{\mu_{nf}}{K \rho_{nf}} + \frac{\sigma_{nf} B_0^2}{\rho_{nf}} \sin^2(\tau) \right) v \\ & + k \left\{ v \frac{\partial^3 v}{\partial y \partial z^2} + w \frac{\partial^2 v}{\partial z^3} \right. \\ & - \left(\frac{\partial v}{\partial y} \frac{\partial^2 v}{\partial z^2} + \frac{\partial v}{\partial z} \frac{\partial^2 w}{\partial z^2} \right. \\ & \left. \left. + 2 \frac{\partial v}{\partial z} \frac{\partial^2 v}{\partial y \partial z} + 2 \frac{\partial w}{\partial z} \frac{\partial^2 v}{\partial z^2} \right) \right\} \end{aligned} \right\} \quad (8)$$

$$\left. \begin{aligned} u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} &= \frac{\kappa_{nf}}{(\rho C_P)_{nf}} \frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho C_P)_{nf}} \frac{\partial q_r}{\partial y} \\ & + \frac{Q_0(T - T_\infty)}{(\rho C_P)_{nf}}, \end{aligned} \right\} \quad (9)$$

where u, v and w indicate the velocity components in x, y and z directions, respectively.

ν_{nf} signifies the effective viscosity, σ_{nf} the electrical conductivity, ρ_{nf} the density, μ_{nf} the dynamic viscosity, B_0 present the uniform magnetic strength and k the viscoelastic factor,

T is the temperature, κ_{nf} is the thermal conductivity, (ρC_P) is the heat capacity, $q_r = -\frac{16\sigma^*}{3k^*} \frac{\partial T^4}{\partial y}$ indicates heat fluctuation and Q_0 depict the heat source/sink parameter.

2.1.4. Velocity and temperature boundary conditions^{26,45}

$$\left. \begin{aligned} u &= dax + l \frac{\partial u}{\partial z}, \quad v = by + l \frac{\partial v}{\partial z}, \quad w = w_0, \\ -k \frac{\partial T}{\partial z} &= h(T_w - T), \quad \text{at } z = 0 \\ u &\rightarrow 0, \quad \frac{\partial u}{\partial z} \rightarrow 0, \quad v \rightarrow 0, \\ \frac{\partial v}{\partial z} &\rightarrow 0, \quad T \rightarrow T_\infty \quad \text{as } z \rightarrow \infty \end{aligned} \right\} \quad (10)$$

2.2. Normalized mass and energy conservations

2.2.1. Dimensionless variables

The subsequent dimensionless variables are depicted to mathematical study of the problem⁴⁶:

$$\left. \begin{aligned} \eta &= \sqrt{\frac{a}{\nu}} z, u = axf_{\eta}(\eta), \quad v = ayg_{\eta}(\eta), \\ w &= -\sqrt{a\nu}[f(\eta) + g(\eta)], \quad \theta(\eta) = \frac{T - T_{\infty}}{T_w - T_{\infty}}, \end{aligned} \right\}, \quad (11)$$

where η is a similarity variable, a is a constant and $f(\eta)$ and $\theta(\eta)$ are dimensionless velocity and temperature, respectively.

2.2.2. Normalized mass and energy conservation equations

By substituting Eq. (11) in Eqs. (6)–(9), the governing PDEs are converted into the following forms^{15,16}:

$$\begin{aligned} \frac{c_1}{c_2} f_{\eta\eta\eta} + f_{\eta\eta}(f + g) - f_{\eta}^2 + \Gamma f_{\eta} \\ - \left(\frac{c_1}{c_2} Da^{-1} + \frac{c_3}{c_2} M \sin^2(\tau) \right) f_{\eta} \\ + \beta \{ f_{\eta\eta\eta\eta}(f + g) + g_{\eta\eta}(g_{\eta\eta} - f_{\eta\eta}) \\ - 2g_{\eta\eta\eta}(f_{\eta} + g_{\eta}) \} = 0, \end{aligned} \quad (12)$$

$$\begin{aligned} \frac{c_1}{c_2} g_{\eta\eta\eta} + g_{\eta\eta}(f + g) - g_{\eta}^2 + \Gamma g_{\eta} \\ - \left(\frac{c_1}{c_2} Da^{-1} + \frac{c_3}{c_2} M \sin^2(\tau) \right) g_{\eta} \\ + \beta \{ g_{\eta\eta\eta\eta}(f + g) + g_{\eta\eta}(g_{\eta\eta} - f_{\eta\eta}) \\ - 2g_{\eta\eta\eta}(f_{\eta} + g_{\eta}) \} = 0, \end{aligned} \quad (13)$$

$$(c_5 + N_R)\theta_{\eta\eta} + Pr c_4 \theta_{\eta}(f + g) + N_I Pr \theta = 0, \quad (14)$$

where $Da^{-1} = \frac{\mu_f}{\rho_f Ka}$, signifies the inverse Darcy number, $M = \frac{\sigma_f B_0^2}{\rho_f a}$ is the Hartman number, $N_I = \frac{Q_0}{a(\rho C_p)_f}$ is the heat source/sink, $N_R = \frac{16\sigma^* T_{\infty}^3}{3k^* \kappa_f}$ is the radiation parameter, $\beta = \frac{ka}{\nu}$ is the viscoelastic parameter, $\Gamma = \frac{\gamma_0}{a}$ is the porosity, $L = l\sqrt{\frac{a}{\nu}}$ denote the first order slip and $Pr = \frac{(\mu C_p)_f}{\kappa_f}$ is the Prandtl number.

2.2.3. Boundary equations

$$\left. \begin{aligned} f_{\eta}(0) &= d + Lf_{\eta\eta}(0), \quad f(0) = V_C, \\ f_{\eta}(\infty) &\rightarrow 0, \quad f_{\eta\eta}(\infty) \rightarrow 0, \\ g_{\eta}(0) &= c + Lg_{\eta\eta}(0), \quad g_{\eta}(\infty) \rightarrow 0, \quad g_{\eta\eta}(\infty) \rightarrow 0, \\ \theta_{\eta}(0) &= -Bi[1 - \theta(0)], \quad \theta(\infty) \rightarrow 0, \end{aligned} \right\}, \quad (15)$$

where $V_C = -\frac{w_0}{\sqrt{a\nu}}$ stands for the mass suction/injection, $V_C > 0$ represents the suction and $V_C < 0$ represents the injection.

2.2.4. The thermophysical properties of the CNTs

For the equivalent electrical conductivity, the Maxwell model is used. The expressions for the CNTs thermophysical properties used in the present study⁴⁷ are defined as follows:

$$\text{Density: } \frac{\rho_{nf}}{\rho_f} = 1 - \phi + \phi \left(\frac{\rho_{CNT}}{\rho_f} \right)$$

$$\text{Dynamic viscosity: } \frac{\mu_{nf}}{\mu_f} = \frac{1}{(1 - \phi)^{2.5}}$$

$$\text{Heat capacitance: } \frac{(\rho C_p)_{nf}}{(\rho C_p)_f} = 1 - \phi + \phi \left(\frac{(\rho C_p)_{CNT}}{(\rho C_p)_f} \right),$$

$$\text{Thermal conductivity:}$$

$$\frac{\kappa_{nf}}{\kappa_f} = \frac{1 - \phi + 2\phi \left(\frac{\kappa_{CNT}}{\kappa_{CNT} - \kappa_f} \right) \ln \left(\frac{\kappa_{CNT} + \kappa_f}{2\kappa_f} \right)}{1 - \phi + 2\phi \left(\frac{\kappa_f}{\kappa_{CNT} - \kappa_f} \right) \ln \left(\frac{\kappa_{CNT} + \kappa_f}{2\kappa_f} \right)}$$

$$\text{Electrical conductivity: } \frac{\sigma_{nf}}{\sigma_f} = \frac{3\phi \left(\frac{\sigma_{CNT} - 1}{\sigma_f} \right)}{\left(\frac{\sigma_{CNT}}{\sigma_f} - 2 - \left(\frac{\sigma_{CNT}}{\sigma_f} - 1 \right) \phi \right)}$$

where κ_{nf} is the electrical conductivity of the NF, and κ_{np} is the electrical conductivity of the nanoparticle. All the expressions for the nanofluidic thermophysical properties depend on particle volume fraction (ϕ).

3. Analytical Solutions

3.1. Velocity solutions

Moreover, it is interesting to note that Eq. (12) has a solution of the form

$$f(\eta) = V_C + \frac{d(1 - \exp(-\lambda\eta))}{\lambda(1 + L\lambda)} \quad (16)$$

and

$$g(\eta) = \frac{d(1 - \exp(-\lambda\eta))}{\lambda(1 + L\lambda)}. \quad (17)$$

The solutions defined in Eqs. (16) and (17) satisfy Eq. (15), then inserting these solutions into Eq. (12), we get a remarkable equation

$$\lambda = \pm \sqrt{\frac{2dc_2}{(c_1 - 2\beta dc_2)}}, \quad (18)$$

$$\begin{aligned} -2d(1 + \beta\lambda^2)c_2 + (1 + L\lambda)(\Gamma c_2 - (c_1 Da^{-1} + c_3 M \sin^2(\tau))) \\ - \lambda(V_C - c_1 \lambda + c_2 \beta V_C \lambda^2) = 0. \end{aligned} \quad (19)$$

After simplifying Eqs. (18) and (19), we get the following results:

$$\lambda = \sqrt{\frac{2c_2}{2\beta c_2 - c_1}},$$

$$V_C = \frac{-2dc_2(1 + \beta\lambda^2) - (c_1 Da^{-1} + c_3 M \sin^2(\tau) - \Gamma c_2)}{\lambda(1 + \beta\lambda^2)(1 + L\lambda)}.$$
(20)

Additionally, the local skin friction coefficient can be deliberated as

$$f_{\eta\eta}(0) = g_{\eta\eta}(0) = -\frac{\lambda d}{(1 + L\lambda)}.$$
(21)

3.2. Temperature solutions

In the presence of a temperature profile with radiation and absorption/generation, the energy equation is defined by using a new transformation

$$\xi = -\frac{Pr}{\lambda^2} \exp(-\lambda Y),$$
(22)

The substitution of Eq. (22) into Eq. (14) yields

$$(c_5 + N_R)\xi\Theta_{\xi\xi} + \left[(c_5 + N_R) - c_4 Pr \left(\frac{V_C \lambda(1 + L\lambda) + d}{\lambda^2(1 + L\lambda)} \right) + \frac{dc_4}{1 + L\lambda} \xi \right] \Theta_{\xi} + \left(\frac{N_I}{\lambda^2} \xi^{-1} \right) \Theta = 0.$$
(23)

The boundary condition reduces to

$$\frac{Pr}{\lambda} \left(\theta_{\eta} \left(\frac{Pr}{\lambda^2} \right) \right) = -Bi \left(1 - \theta \left(\frac{Pr}{\lambda^2} \right) \right), \quad \theta(0) = 0.$$
(24)

Equation (25) satisfying the boundary condition (24) is obtained in the form of Kummer's function:

$$\theta(\eta) = \frac{Bi \exp(-m\lambda\eta) F_1 \left[Pm, k + 1, \frac{Pr}{\lambda^2} \exp(-\lambda\eta) \right]}{(Bi + m\lambda) F_1 \left[Pm, k + 1, \frac{Pr}{\lambda^2} \exp(-\lambda\eta) \right]} - \left(\frac{Pm}{1+k} \right) \left(-\frac{Pr}{\lambda} \right) F_1 \left[Pm + 1, k + 2, \frac{Pr}{\lambda^2} \exp(-\lambda\eta) \right]$$
(25)

where Bi is the Biot number.

$$m = \frac{c_4 Pr A + \sqrt{(c_4 Pr A) - 4(c_5 + N_R) \frac{N_I Pr}{\lambda^2}}}{2(c_5 + N_R)},$$

$$k = \frac{\sqrt{(c_4 Pr A) - 4(c_5 + N_R) \frac{N_I Pr}{\lambda^2}}}{(c_5 + N_R)}$$

$$A = \frac{V_c \lambda(1 + L\lambda) + d}{\lambda^2(1 + L\lambda)},$$

$$P = -\frac{dc_4}{(1 + L\lambda)}.$$

4. Results and Discussion

The three-dimensional MHD flow with heat transfer through a stretching/shrinking sheet with radiation as well as heat source/sink has been presented in this work. Mathematically, we express a set of systems of PDEs (1)–(3), which are then converted into a set of nonlinear ODEs using nondimensional variables solved in the analytical method for velocity and temperature profile. Now, we present results on the stretching/shrinking sheet problem with heat transformers. Furthermore, a correlation of the transverse and axial velocity of SWCNT as well as MWCNT on the same figure will help to understand the effect of the CNTs, where solid volume fraction will remain constant, while dashed as well as solid lines represent the solution for SWCNTs and MWCNTs, respectively. The thermophysical properties of CNT are shown in Table 1, and the Prandtl number for water is kept at 6.2.

4.1. Effects of physical parameters, Da^{-1} , M and Γ on velocity profiles

Figure 2 demonstrates the effects of the inverse Darcy number, Da^{-1} , on the mass suction/injection V_c against the viscoelastic parameter β on the stretching/shrinking sheet when $M = \Gamma = 1$, $L = \varphi = 0.1$ and $\tau = 45^\circ$. The red solid lines represent MWCNT, and the black dotted lines represent SWCNT. $V_C > 0$ represents the suction, and $V_C < 0$ does the injection. It can be seen that V_c on the stretching sheet reduces as the Da^{-1} increases, keeping other parameters fixed. Particularly, V_c reduces to -19 from 10 at $\eta = 6$ when Da^{-1} increases from -10 to 19 . This can be explained by

Table 1. Physical properties of the base fluid and CNTs.⁴⁸

Physical properties	Fluid phase (water)	SWCNT	MWCNT
C_P (J/KgK)	4179	425	796
ρ (Kg/m ³)	997.1	2600	1600
κ (W/mK)	0.613	6600	3000
$\sigma(\Omega/m)^{-1}$	0.05	48 000 000	38 000 000

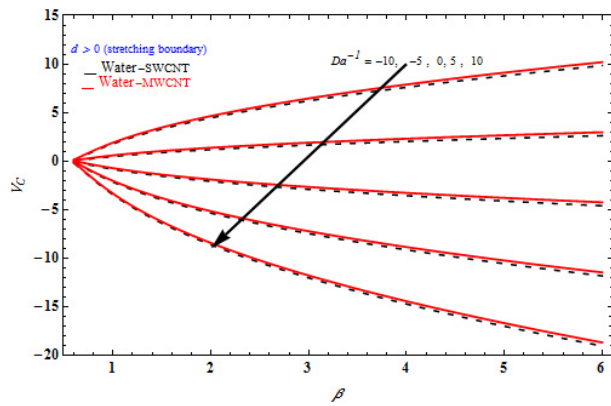


Fig. 2. Impacts of Da^{-1} on the mass transportation V_c versus β when $M = \Gamma = 1$, $L = \varphi = 0.1$ and $\tau = 45^\circ$.

the fact that the porous boundary layer becomes injected from the suction. Thus, the boundary layer thickness also decreases when the inverse Darcy number increases. However, there is no change at $\eta = 0$. A very small change is also found between SWCNT and MWCNT.

Figure 3 demonstrates the effects of the first-order slip (L) and the mass suction/injection V_c versus viscoelastic parameter β when stretching and shrinking sheet. The red solid lines indicate the flow patterns at $L = 0$, and the black dashed lines indicate the flow patterns at $L = 1$. It can be found that the inverse Darcy number (Da^{-1}) has significant effects on the mass suction/injection V_c . The mass flow can change from injection to suction when the inverse Darcy number increases to greater than 3 on stretching sheets [Fig. 3(a)]. Furthermore, it can be found that all the mass velocities shift down on the stretching sheet when L increases to 1 from 0. However, all the mass transpiration velocities shift up for the shrinking sheet when L increases to 1 from 0

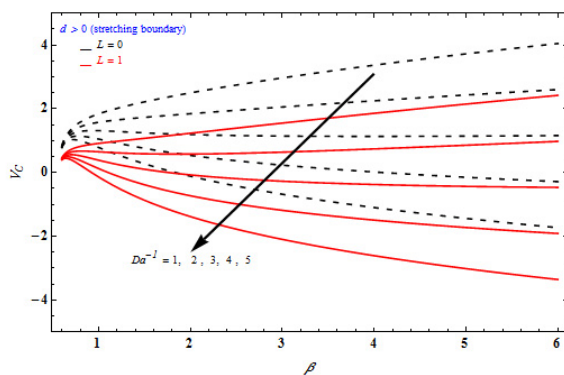
[Fig. 3(b)]. Except for a small region of η , the V_c is in the injection for most of the inverse Darcy number.

Figure 4 demonstrates the effects of the Hartmann number (M) on the mass transpiration V_c versus the viscoelastic parameter β on a stretching sheet. The mass transpiration V_c decreases as M increases. It should be noted that the decreases of V_c increases as η increases. This is because the Hartmann Number is an important criterion in magneto-hydrodynamics and expresses the ratio of the induced electrodynamic (magnetic) force to the hydrodynamic force of viscosity. When the Hartmann Number increases, the boundary layer thickness is enhanced, leading to the decreases of the velocities. Furthermore, MWCNT increases slightly the velocities compared to SWCNT.

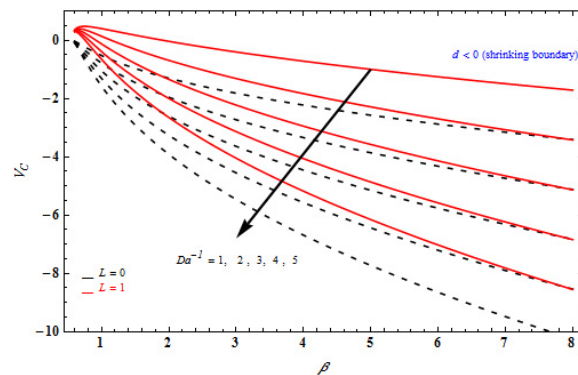
Figure 5 depicts the effects of the porosity parameter (Γ) on the mass suction/injection V_c versus viscoelastic parameter β for stretching sheet when $Da^{-1} = M = 1$, $L = \varphi = 0.1$ and $\tau = 45^\circ$. As the porosity parameter (β) increases, the mass suction/injection V_c increases. For stretching/shrinking sheet with different parameters, $Da^{-1} = M = 1$, $L = \varphi = 0.1$ and $\tau = 45^\circ$. When comparing the stretching sheet condition to the wall shrinking sheet condition, it appears that a domain exists. From Fig. 5, it can be noted that SWCNT attains a higher impact than SWCNT.

4.2. Effects of physical parameters, Da^{-1} , Γ , and M , on the normalized axial velocity $f(\eta)$ against η

Figure 6 shows the effects of the inverse Darcy number (Da^{-1}) on the normalized axial velocity $f(\eta)$ when $Da^{-1} = M = \beta = L = 1$, $\varphi = 0.1$ and



(a) stretching sheet ($d > 0$)



(b) shrinking sheet ($d < 0$)

Fig. 3. Effects of the first-order slip (L) and the inverse Darcy number (Da^{-1}) on the mass suction/injection V_c when $\Gamma = M = 1$, $L = \varphi = 0.1$ and $\tau = 45^\circ$: (a) stretching and (b) shrinking sheet.

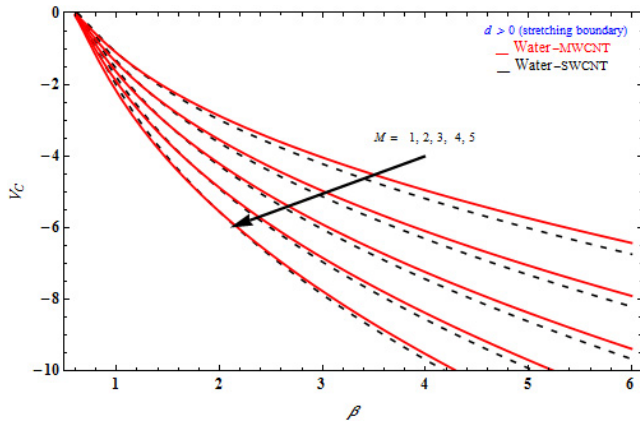


Fig. 4. Effects of the Hartmann number (M) on the mass suction/injection V_c versus β when $Da^{-1} = \Gamma = 1$, $L = \varphi = 0.1$ and $\tau = 45^\circ$.

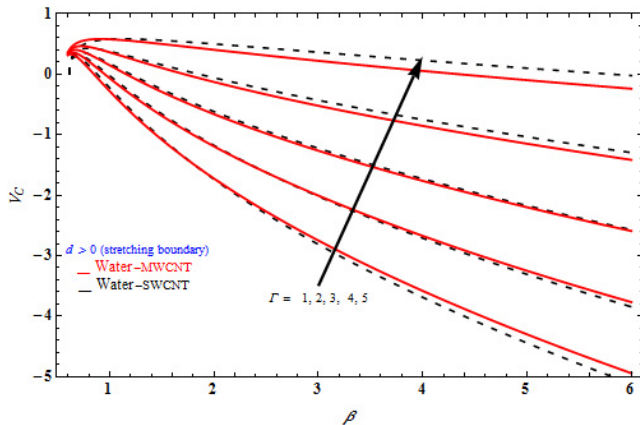


Fig. 5. Effects of the porosity parameter (Γ) on the mass suction/injection V_c versus viscoelastic parameter β for stretching sheet when $Da^{-1} = M = 1$, $L = \varphi = 0.1$ and $\tau = 45^\circ$.

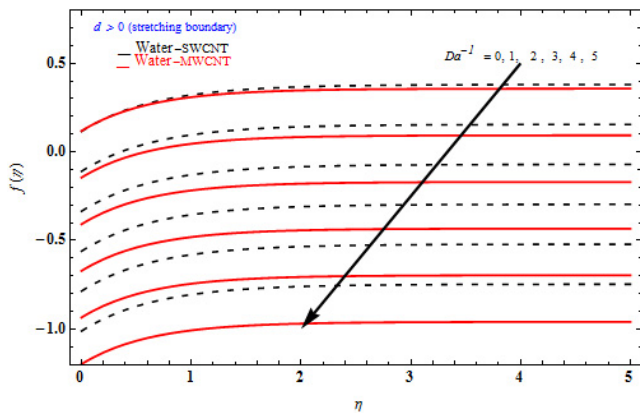


Fig. 6. Effects of the inverse Darcy number (Da^{-1}) on the normalized axial velocity $f(\eta)$ when $Da^{-1} = M = \beta = L = 1$, $\varphi = 0.1$ and $\tau = 45^\circ$.

$\tau = 45^\circ$. The normalized axial velocity decreases as the inverse Darcy number Da^{-1} increases. It can be seen that the normalized axial velocity reduces from 0.25 to -1 when Da^{-1} increases from 0 to 5. Furthermore, the normalized axial velocity is larger for SWCNT than for MWCNT. This is because fluids through higher permeability porous media diffuse faster than lower permeability, leading to a more gradual reduction of the velocity profile with distance. Fluid movement through porous materials is involved in various processes, such as the extraction in enhanced oil recovery, the water permeability of reservoir rocks and the permeability of aquifers in groundwater hydrology.

Figure 7 shows the effects of the Hartmann number M on the normalized axial velocity $f(\eta)$ versus η . The normalized axial velocity reduces as the Hartmann Number increases. This can be explained by the Hartmann number M increase the boundary layer thickness, leading to a decrease in velocity. Furthermore, a higher M parameter in magnetohydrodynamic flows enhances the Lorentz force, which inhibits fluid movement. As a result, the flow slows down more, causing a more swifter reduction of the velocity profile with distance. Figure 8 shows the effects of the porosity parameter (Γ) on the normalized axial velocity $f(\eta)$ when $Da^{-1} = M = \beta = L = 1$, $\varphi = 0.1$ and $\tau = 45^\circ$. The normalized axial velocity reduces as the porosity parameter, Γ , increases. This is because increasing the porosity parameter, Γ , increases the boundary layer thickness. Furthermore, the normalized axial velocities are larger for SWCNT than MWCNT.

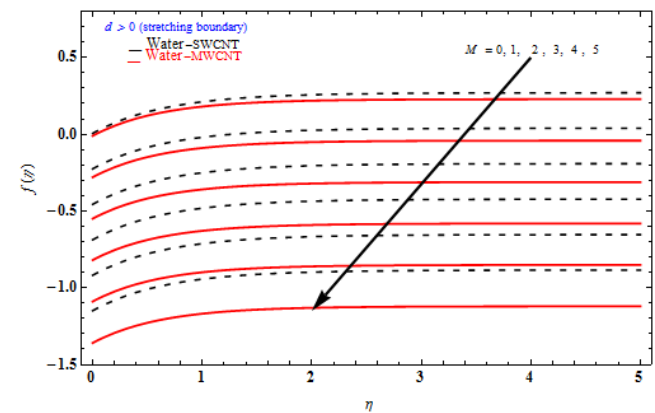


Fig. 7. Effects of the Hartmann number (M) on the normalized axial velocity $f(\eta)$ when $Da^{-1} = M = \beta = L = 1$, $\varphi = 0.1$ and $\tau = 45^\circ$.

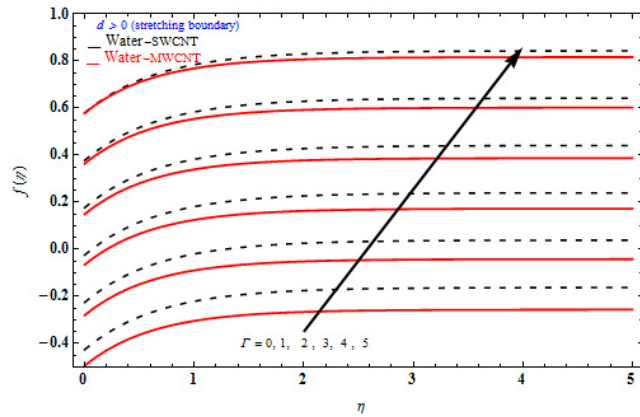


Fig. 8. Effects of the porosity parameter (Γ) on the normalized axial velocity $f(\eta)$ versus viscoelastic parameter β for stretching sheet when $Da^{-1} = M = 1$, $L = \varphi = 0.1$ and $\tau = 45^\circ$.

The difference between SWCNT and MWCNT increases as M increases and L decreases.

4.3. Effects of thermal radiation N_R , heat source N_I and M on $\theta(\eta)$ versus η

Figure 9 demonstrates the influence of thermal radiation (N_R) on the temperature, $\theta(\eta)$, versus when $\eta Da^{-1} = M = \beta = \Gamma = Bi = L = 1$, $N_I = -2$, $\varphi = 0.1$ and $\tau = 90^\circ$. This can be seen that $\theta(\eta)$ increases with raising the values of N_R . The higher values of N_R enhance the heat transfer in the fluid movement due to an enhanced thermal boundary layer. Furthermore, a higher radiation parameter could absorb more radiant energy, and more heat is transferred through radiation.

Figure 10 shows the effects of the heat source/sink (N_I) on the temperature, $\theta(\eta)$, versus when

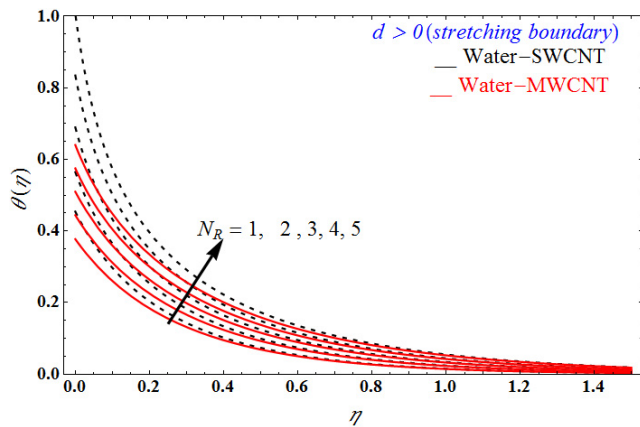


Fig. 9. Effects of thermal radiation (N_R) on the temperature, $\theta(\eta)$, versus when $\eta Da^{-1} = M = \beta = \Gamma = Bi = L = 1$, $N_I = -2$, $\varphi = 0.1$ and $\tau = 90^\circ$.

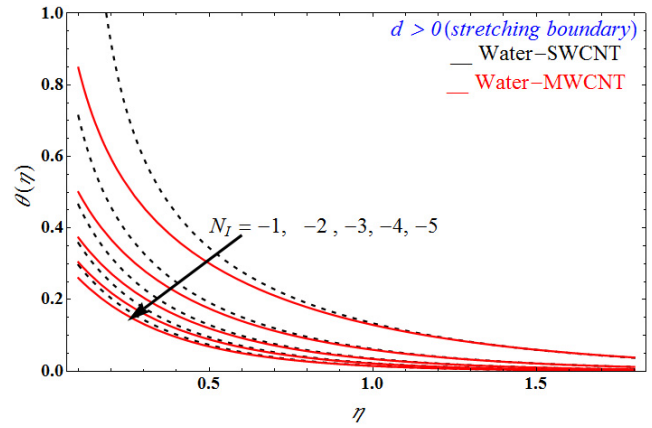


Fig. 10. Impact of N_I on $\theta(\eta)$ against η several values when $Da^{-1} = M = \beta = \Gamma = N_R = L = 1$, $Bi = 1$, $\varphi = 0.1$ and $\tau = 90^\circ$.

$\eta Da^{-1} = M = \beta = \Gamma = Bi = L = 1$, $N_I = -2$, $\varphi = 0.1$ and $\tau = 90^\circ$. The temperature decreases as the heat source/sink (N_I) decreases. This can be attributed to the energy increase from increasing heat sources. Figure 11 represents the effect of the Hartmann number, M , on $\theta(\eta)$. The temperature increases as the quantity of M increases. The higher Hartmann number means the higher Lorentz force and a kind of friction on the motion. This, in turn, generates more heat energy, which eventually enhances the temperature in the flow. Furthermore, increasing N_R , and M can increase the rate of heat transmission, which is true for both stretching and shrinking surfaces. The temperature for SWCNT is larger than for MWCNT for both thermal radiation and heat sources. The difference between SWCNT and MWCNT increases as η decreases.

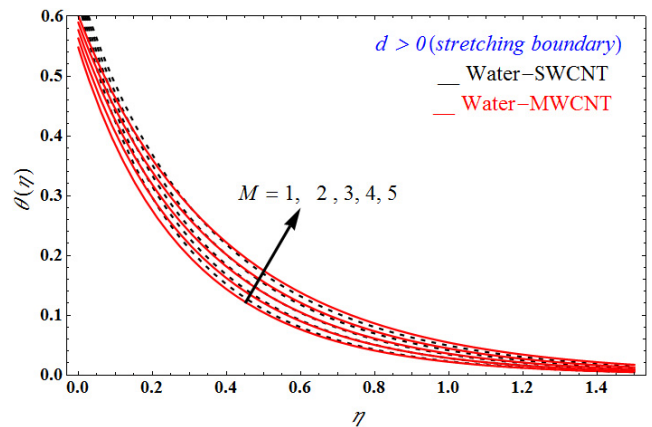


Fig. 11. Impact of M changes on $\theta(\eta)$ at $Da^{-1} = \beta = \Gamma = Bi = L = 1$, $N_I = -2$, $\varphi = 0.1$ and $\tau = 90^\circ$.

Table 2. Comparison of the current research and relevant studies by other authors.

Authors	Fluids	Value of λ
Maranna <i>et al.</i> ²⁶	Non-Newtonian	$f(n) = V_C + \frac{(1-e^{-\lambda\eta})}{\lambda(1+L\lambda)}$
Mahabaleshwar <i>et al.</i> ⁵⁶	Non-Newtonian	$f(n) = V_C + \frac{(1+d)(1-e^{-\lambda\eta})}{\lambda(1+L_1\lambda-L_2\lambda^2)}$
Present work	Non-Newtonian	$f(\eta) = V_c + \frac{d(1-\exp(-\lambda\eta))}{\lambda(1+L\lambda)}$

4.4. Discussion

Non-Newtonian fluids have difficulties in numerical modelling due to their yield stress and discontinuous features. Thermal performance of nano-non-Newtonian fluid MHD on permeable surface is increasingly important in a wide range of applications, such as nanotechnology, drug delivery and enhanced thermal management of electronic devices.¹² However, their thermal performance is uncertain due to complex interactions among NFs, porous media, flow and heat transfer. Electronics cooling ensures reliable performance in devices like smartphones and servers, porous coating enhances durability and functionality in aerospace and biomedical implants, and drug delivery systems enable precise, targeted therapies such as cancer treatment or sustained-release medications.

The components of electronic devices have integrated at higher levels. However, the high integrations can lead to poor heat dissipation, heat accumulation, and elevated temperatures, reducing the efficiency and reliability of electronic devices. Permeable thermal interface materials are used to fill the air gaps, and liquid phase change materials (PCMs) to establish an effective heat conduction channel between electronic components and reduce the heat transfer resistance, and improve the heat dissipation performance.^{41,42,49,50} Key design considerations of permeable conductors and materials are to endow flexible and stretchable conductors with permeability.⁴³ Therefore, studies of thermal performance of nano fluid MHD on permeable surface can be devoted to understanding the thermal performance of permeable conductors and materials in electronic devices. However, this study is limited to a simple permeable stretching sheet. Further research remains to be done into real phase materials.

Drug delivery brings therapeutic agents to the tissue, organ or cell of interest for release and absorption.⁵¹ The primary purposes of drug delivery systems are to improve pharmacological activity and drug absorption across a cell membrane and tissue. The absorption rate depends on the drug

particles, transport, permeability of tissue, concentration gradient, degree of protein binding and lipid solubility of the drug. Therefore, therapeutic drugs could be encapsulated to protect their activation, thereby enhancing their oral bioavailability and therapeutic index.⁵² Heat transfer affects drug delivery in several ways, including how drugs are released from carriers, how they diffuse through the body and how they are absorbed through the skin. No matter what the absorption site, the drug must penetrate the cell membrane to enter the systemic circulation.^{53–55} Fick's second law could represent the diffusion of drug agents between the capsule core and the surrounding medium through the membrane. Nanotechnology has the potential to enhance the delivery of therapeutic agents into the organs.^{40,52} Different CNTs can affect the thermophysical properties, such as thermal conductivity, diffusivity, viscosity, slip and adsorption/desorption. Therefore, studies of thermal performance of nano fluid MHD on permeable surface would be useful for understanding the drug absorption process and the affecting factors. This is essential in achieving better drug delivery, the concentration-effect relationship, establishment of dosage regimens, drug absorption and therapeutic effects.

5. Conclusions

In this study, we carry out a theoretical study of inclined MHD and radiation effects on viscoelastic CNT flow across a porous stretching/shrinking sheet. Also, the mass suction/injection parameter is considered under Navier slip conditions. The governing PDFs of non-Newtonian fluid flow and heat transfer are established over a permeable stretching/shrinking surface. Using similarity variables, the given PDEs are turned into ODEs and then solved analytically. The present nanoflow model has potential applications in healthcare materials, medical devices and biosensors due to their higher thermal efficiency, chemical compatibility with

biomolecules, i.e., proteins and DNA. The key conclusions are as follows:

- It can be found that the inverse Darcy number has significant effects on the mass suction/injection V_c , and the mass flow can change from injection to suction.
- The results show that the first-order slip (L) could significantly affect the mass V_c .
- The entire mass suction/injection velocities shift up for the shrinking sheet when L increases to 1 from 0, but shift down for the stretching sheet.
- Furthermore, the CNT type slightly affects the suction/injection velocity but affects the normalized axial velocity.
- The larger the inverse Darcy number and the Hartmann number, the larger the difference of the axial velocity between SWCNT and MWCNT becomes.
- Thermal radiation and heat sources have significant effects on the temperature profiles. The temperature increases as thermal radiation and the heat source/sink increase. The temperature of the NFs with SWCNT is larger than that with MWCNT when the thermal radiation and heat sources are the same. This shows that the SWCNT NFs can improve thermal performance better. The produced velocities of MWCNT deferred NFs are higher than those of SWCNT deferred NFs, leading to lower heat convection.

Finally, it can be found that the fluid flow in SWCNT has a higher temperature than MWCNT by using the induced slip condition. For the velocity and temperature behavior in the absence of viscoelastic fluid, porous medium and heat source/sink, the present results can be reduced to those of Maranna *et al.*²⁶ and Mahabaleshwar *et al.*⁵⁶ Table 2 shows a comparison with previous results.

Acknowledgments

This research was supported by Project FONDECYT 1240985 and Centers of Excellence with BASAL/ANID financing CIA250002. G. M. Sachin would like to thank the financial assistance received from Karnataka Science and Technology Society (KSTePS) under the program of Karnataka DST-Ph.D. fellowship for Science and Engineering: DST/KSTePS/Ph.D.Fellowship/MP-09/2024-25.

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